Sticky Leverage

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Preliminary: Comments Welcome

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Abstract

This paper considers the macroeconomic effects of allowing for nominal debt contracts in the context of a quantitative business cycle model with financial market frictions. In our setting, as in reality, corporations fund themselves by choosing the appropriate mix of nominal defaultable debt and equity securities to issue in every period. Corporate debt is priced fairly taking into account default and inflation risk, but is attractive because of the tax-deductibility of interest payments. We show that in this world unanticipated shocks to inflation will both change the real burden of corporate debt and, more significantly, also distorts corporate investment and production decisions. Unlike sticky prices and wages, the effects of having nominal leverage can be both large and persistent if the model is calibrated to match empirically observed debt levels and maturities. Finally, we show that the adoption of a standard Taylor-rule for nominal interest rates can substantively stabilize this economy, supporting perhaps its current popularity with monetary policy makers.

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1 Introduction

Since the onset of the financial crisis in 2008 monetary policy has staged the most aggressive response in at least 30 years. At the same time, financial markets now occupy a much more prominent role in macroeconomic theory. Typical models of financial frictions focus on debt and identify leverage as both a source and an important mechanism of transmission of economic fluctuations.[1] Surprisingly, the fact that debt contracts are almost all denominated in nominal terms is usually not given a lot of attention or often ignored in the literature. Yet, nominal debt creates a direct link between monetary policy and the real economy, and a potentially important source of monetary nonneutrality, even with fully flexible prices.[2]

Our goal in this paper is to develop a tractable general equilibrium model that captures the interplay between nominal debt, inflation, and real aggregates, and to examine its quantitative implications. The model embeds the dynamic investment and financing decisions of firms into a general equilibrium macroeconomic environment.

In our setting, as in reality, corporations fund themselves by choosing the appropriate mix of nominal defaultable debt and equity securities to issue in every period. Corporate debt is priced fairly by bondholders, who take into account default and inflation risk, but is attractive to issue because of the tax-deductibility of interest payments. Macroeconomic quantities are obtained by aggregating across the optimal decisions of each firm and ensuring consistency with the consumption and savings choices of a representative household/investor. As a result, our model endogenously links movements in aggregate quantities such as investment and output to changes in corporate leverage and defaults.

Because debt is written in nominal terms, unanticipated changes in inflation have real effects. In particular, lower than expected inflation increases the real value of debt, worsens

[1]Some examples include Kiyotaki and Moore (1997), Carlstrom and Fuerst (1997), Bernanke, Gertler and Gilchrist (1999), Cooley, Marimon and Quadrini (2004), Gourio (2012), Jermann and Quadrini (2012), and Gomes and Schmid (2013)

[2]Among the rare exceptions are Christiano, Motto, and Rostagno (2009), Fernandez-Vilaverde (2010), Bhamra et al (2011), and De Fiore et al. (2011).

firms' balance sheets, and makes them more likely to default. When defaults and bankruptcies have resource costs, this immediately and adversely impacts output. More importantly, even surviving firms will begin to cut future investment and production plans, as the increased (real) debt lowers the rewards to their equity owners. This *debt overhang* phenomenon has been emphasized in several empirical studies of financial crisis (e.g. Reinhart and Rogoff (2011)) and significantly propagates the initial change in inflation, accounting for most of its effects on the economy.

By adding a monetary policy rule linking short term nominal interest rates with inflation and output, this setting offers insights into the ongoing monetary stimulus around the world. We find that a standard Taylor rule parameterization implies that Central Banks produce significant inflation in response to adverse real shocks, such as declines in productivity and especially wealth.

We believe this environment with long-term nominal debt contracts offers a clearer understanding of financially driven recessions than traditional models emphasizing sticky prices and wages. While those models often imply that adverse shocks would be mitigated if prices were allowed to fall, our setting suggests the exact opposite. As Fisher (1933) suggests, deflation would only magnify the real burden of debt and further worsen economic activity. The monetary policy implications are also subtly different. In our model, Central Banks should respond to episodes of excessive leverage not just by lowering the effective real interest rate in the economy but by actually creating inflation.

While the notion that a debt deflation may have significant macroeconomic consequences goes back at least to Fisher (1933), it has not been incorporated into the modern quantitative macroeconomic literature until quite recently. Christiano, Motto, and Rostagno (2009) embed nominal short-term entrepreneurial loans into a medium-scale DSGE model with sticky prices. In contrast to them, we stress the importance of long-term debt and show how this itself gives rise to a monetary non-neutrality without relying on the stickiness of prices. Fernandez-Villaverde (2010) considers short-term nominal government within a model with

sticky prices.

De Fiore et al. (2011) also examine optimal monetary policy when firms have nominal debt with default risk. Theirs is not a quantitative analysis, and their model considers only one-period debt and has no capital choice. Kang and Pflueger (2012) study the pricing of nominal corporate debt within a model driven by productivity and inflation shocks. However, their model has constant labor and only considers two-period debt. Occhino and Pescatori (2012) consider the effects of debt overhang with one-period debt in a real model. Macro economic analyses with real long-term debt and default are Gomes and Schmid (2013), Miao and Wang (2010), and Gourio (2012).

In corporate finance, the notion of debt overhang has been extensively studied at the firm level since Myers (1977). More recently Hennessy (2004), Moyen (2005), and Chen and Manso (2010) study its implications using dynamic models of optimal firm investment. Like most corporate finance papers, their focus is on real quantities alone.

More broadly, our paper expands on the growing literature on the macroeconomic effects of financial frictions. This includes Kiyotaki and Moore (1997), Carlstrom and Fuerst (1997), Bernanke, Gertler and Gilchrist (1999), Cooley, Marimon and Quadrini (2004), and Jermann and Quadrini (2012).

The next two sections describe our model and show some of its key properties regarding the real effects of inflation. Section 4 discusses the calibration of our baseline model, and Section 5 shows our quantitative findings. The final section contains concluding remarks.

2 The Model

The novel aspects of our model center around firm investment and financing. Firms own the productive technology and the capital stock in this economy. They are operated by owners or equity holders but partially financed by defaultable debt claims. The firms' optimal choices are distorted by taxes and default costs. Households consume the firms' output and invest

any savings in the securities issued by firms. The government plays a minimal role: it collects taxes on corporate income and rebates the revenues to the households in lump-sum fashion.

2.1 Firms

We now begin by describing the behavior of firms and its investors in detail. At any point in time production and investment take place in a continuum of measure one firms, indexed by j. Some of these firms will default on their debt obligations, in which case they are restructured before resuming operations as before. Hence firms will remain on-going concerns at all times, so that their measure remains unchanged. Although this is not an essential assumption, we adopt it to preserve tractability and rely on an environment where all firms make identical choices.

2.1.1 Technology

Each firm produces according to the function:

$$y_t^j = A_t F\left(k_t^j, n_t^j\right) = A_t \left(k\right)^{\alpha} n^{1-\alpha}. \tag{1}$$

where A_t is aggregate productivity. Solving for the static labor choice to get the firms' operating profit:

$$R_t k_t^j = \max_{n_t^j} A_t F\left(k_t^j, n_t^j\right) - w_t n_t^j \tag{2}$$

where $R_t = \alpha y_t / k_t$ is the implicit equilibrium "rental rate" on capital. Given constant returns to scale, all firms chose identical ratios k^j/n^j, so R_t is identical across firms.

Firm level profits are also subject by additive idiosyncratic shocks, $z_t^j k_t^j$, so that operating profits are equal to:

$$\left(R_t - z_t^j\right) k_t^j \tag{3}$$

We assume that z_t^j is i.i.d across firms and time, has mean zero, and cumulative distribution $\Phi(z)$ over the interval $[\underline{z}, \bar{z}]$, with $\int_{\underline{z}}^{\bar{z}} \phi(z)\, dz = \int d\Phi(z)$. We prefer to think of these as direct shocks to firms operating income and not necessarily output. They summarize the overall firm specific component of business risk. Although they average to zero in the cross section, they can potentially be very large for any individual firm

Finally, firm level capital accumulation is given by the identity:

$$k_{t+1}^j = \left(1 - \delta + i_t^j\right) k_t^j = g(i_t^j) k_t^j \tag{4}$$

where i_t^j denotes the investment to capital ratio.

2.1.2 Financing

Firms fund themselves by issuing both equity and defaultable nominal bonds. Let B_t^j denote the stock of outstanding defaultable corporate nominal bonds at the beginning of period t. In what follows we work with the real value of this debt, which is denoted as b_t^j/μ_t where $\mu_t = P_t/P_{t-1}$ is the economy wide rate of inflation between period $t-1$ and t while P_t is the overall price level in period t.

To capture the fact that outstanding corporate debt is of finite maturity, we assume that in every period t a fraction λ of the principal is paid back, while the remaining $(1-\lambda)$ remains outstanding. This means that the corporate debt has an expected life of $1/\lambda$. In addition to principal amortization, the firm is also required to pay a periodic coupon c per unit of outstanding debt.

Thus, letting p_t^j denote the market price of each unit of debt in terms of consumption goods, it follows that the (real) market value of new debt issues during period t is given by:

$$p_t^j (B_{t+1}^j - (1-\lambda) B_t^j)/P_t = p_t^j (b_{t+1}^j - (1-\lambda) b_t^j/\mu_t) \tag{5}$$

We will work with the real value of these outstanding liabilities throughout the remainder of this paper.

2.1.3 Dividends and Equity Value

In the absence of new debt issues, (real) distributions to shareholders will be equal to:

$$(1-\tau)\left(R_t - z_t^j\right)k_t^j - ((1-\tau)c+\lambda)\frac{b_t^j}{\mu_t} - i_t^j k_t^j + \tau\delta k_t^j$$

The first term captures the firm's operating profits, to which we then deduct the required debt repayments and investment expenses and add the tax shields accrued through depreciation expenditures. This expression for equity distributions is consistent with the fact that only interest payments may be tax deductible.

It follows that the value of the firm to its shareholders, denoted by the function $E(\cdot)$, is the present value of these distributions plus the value of any new debt issues. It is useful to write this value function in two parts, as follows:

$$E\left(k_t^j, b_t^j, z_t^j, \mu_t\right) = \max\left[0, (1-\tau)\left(R_t - z_t^j\right)k_t^j - ((1-\tau)\,c+\lambda)\frac{b_t^j}{\mu_t} + V\left(k_t^j, b_t^j, \mu_t\right)\right] \quad (6)$$

where the continuation value $V(\cdot)$ obeys the following Bellman equation:

$$V\left(k_t^j, b_t^j, \mu_t\right) = \max_{b_{t+1}^j, k_{t+1}^j}\left\{\begin{array}{c} p_t^j\left(b_{t+1}^j - (1-\lambda)\frac{b_t^j}{\mu_t}\right) - (i_t^j - \tau\delta)k_t^j \\ +\mathrm{E}_t M_{t,t+1}\int_{\underline{z}}^{\bar{z}} E\left(k_{t+1}^j, b_{t+1}^j, z_{t+1}^j, \mu_{t+1}\right)d\Phi\left(z_{t+1}\right) \end{array}\right\} \quad (7)$$

and summarizes the effects of the decisions about future investment and financing on equity values.

Several observations about the value of equity (6) will be useful later. First, limited liability implies that equity value, $E(\cdot)$, is bounded and will never fall below zero. This

implies that equity holders will default on their credit obligations whenever their idiosyncratic profit shock z_t^j is above a cutoff level $z_t^* \leq \bar{z}$, defined by the expression:

$$(1-\tau)\left(R_t - z_t^{j*}\right)k_t^j - ((1-\tau)c+\lambda)\frac{b_t^j}{\mu_t} + V\left(k_t^j, b_t^j, \mu_t\right) = 0 \tag{8}$$

It is this value z_{t+1}^{j*} that truncates the integral in the continuation value of (7).

Second, the stochastic discount factor $M_{t,t+1}$ is exogenous to the firm and must be determined in equilibrium, in a manner consistent with the behavior of households/investors. Third, the value function is homogenous of degree one in capital k_t^j and debt b_t^j and thus so is the default cutoff z_t^{j*}.[3]

Finally, the equity value $E(\cdot)$ is decreasing in the value of outstanding real debt obligations b_t^j/μ_t. In particular:

$$\begin{aligned}\frac{\partial E(\cdot)}{\partial b_t^j} &= -((1-\tau)c+\lambda)\frac{1}{\mu_t} + \frac{\partial V(\cdot)}{\partial b_t^j} \\ &= -\left((1-\tau)c+\lambda+p_t^j(1-\lambda)\right)\frac{1}{\mu_t} \leq 0\end{aligned}$$

Intuitively, higher debt reduces both the value of the current distribution to equity holders and, if $\lambda < 1$, the continuation value, $V(\cdot)$.

2.1.4 Optimal Policies and the Debt Overhang

Given the expression for the value function of the owners of the firm we can describe the optimal decisions regarding investment and borrowing by the following first order conditions:

$$1 = \mathrm{E}_t M_{t,t+1}\frac{\partial E(k_{t+1}^j b_{t+1}^j, z_{t+1}^j, \mu_{t+1})}{\partial k_{t+1}^j} \tag{9}$$

$$\frac{\partial p_t^j}{\partial b_{t+1}^j}\left(b_{t+1}^j - (1-\lambda)\frac{b_t^j}{\mu_t}\right) + p_t^j = -\mathrm{E}_t M_{t,t+1}\frac{\partial E(k_{t+1}^j b_{t+1}^j, z_{t+1}^j, \mu_{t+1})}{\partial b_{t+1}^j} \tag{10}$$

[3] We will see below that the equilibrium prices p and M are homogenous of degree zero in these variables.

The condition for investment is fairly standard. It equates the required reduction in current equity distributions, associated with a marginal increase in next period's stock of capital, k_{t+1}, with the discounted expected future marginal increase in equity value.

The equation for optimal debt, b_{t+1}, is more novel and at the heart of our results. It compares the marginal benefit of issuing new debt, on the left hand side, with the expected reduction in future equity values on the right. Importantly, the expression for the marginal benefit in equation (10) recognizes that the debt price, p, might change when new debt is issued. This is because an increase in the amount of debt outstanding will likely increase the probability of default.

The magnitude of the effect however depends on the (real) value of currently outstanding liabilities, $(1-\lambda)b_t^j/\mu_t$, so that it is in effect relatively cheaper to issue new debt when the existing stock is already high. It is this property of the model that creates our persistence or debt overhang results below.

2.1.5 Default and Credit Risk

We now turn to the problem facing the firm's creditors. These agents buy corporate liabilities, at price p_t^j, and collect regular coupon and principal payments, $(c+\lambda)\frac{b_{t+1}^j}{\mu_{t+1}}$, until the firm defaults. In this case, shareholders walk away from the firm, and creditors take over and restructure the firm. Creditors become the sole owners and investors of the firm. Accordingly, upon default creditors collect the after tax operating income $(1-\tau)\left(R_{t+1}-z_{t+1}^j\right)k_{t+1}^j$.

After this restructuring these now owners of the firm will sell off the equity portion to new owners while continuing to hold the remaining debt. This means that in addition to the current cash flows, the creditors have a claim that equals the total enterprise, or asset, value, $V\left(k_{t+1}^j, b_{t+1}^j\right)+p_{t+1}^j\left(1-\lambda\right)b_{t+1}^j$.[4]

[4] This is only one of several equivalent ways of describing the bankruptcy procedures that yields the same payoffs for shareholders and creditors upon default. Equivalently we could assume that they sell debt and continue to run the firm as the new equity holders.

Restructuring is not entirely costless however and there is a separate loss, in the amount ξk^j_{t+1}, where $\xi \in [0,1]$.[5]

With these assumptions, the creditor's valuation of their holdings of corporate debt at the end of period t is:[6]

$$b^j_{t+1} p^j_t = \mathrm{E}_t M_{t,t+1} \left\{ \begin{array}{c} \Phi(z^{j*}_{t+1}) \left[c + \lambda + (1-\lambda) p^j_{t+1} \right] \frac{b^j_{t+1}}{\mu_{t+1}} + \int_{z^{j*}_{t+1}}^{\bar{z}} \left[(1-\tau) \left(R_{t+1} - z^j_{t+1} \right) k^j_{t+1} \right. \\ \left. + V \left(k^j_{t+1}, b^j_{t+1}, \mu_{t+1} \right) + (1-\lambda) \frac{p^j_{t+1} b^j_{t+1}}{\mu_{t+1}} - \xi k^j_{t+1} \right] d\Phi(z_{t+1}) \end{array} \right\} \tag{11}$$

It is immediate to establish that this market value of corporate debt is decreasing in the restructuring losses, ξ_k, and the default probability, implied by the cutoff z^{j*}. It can also be shown that bond prices are declining in the expected rate of inflation - since equity values increase in μ_{t+1}. Furthermore, p^j_t is homogeneous of degree zero in k^j_{t+1} and b^j_{t+1}.

All together, our assumptions ensure that when the restructuring process is complete a defaulting firm is indistinguishable from a non-defaulting firm. All losses take place in the current period and are absorbed by the creditors. Going onward default has no consequences because all idiosyncratic shocks are i.i.d. and there are no adjustment costs. As a result, both defaulting and non-defaulting firms adopt the same optimal policies and look identical at the beginning of the next period.

2.2 Households

To complete our general equilibrium model we now need to describe the household sector. This is made of a single representative family that owns all securities and collects all income in the economy, including a rebate on corporate income tax revenues. Household have time-

[5]We can think of these costs as including legal fees, but also other efficiency losses and frictions associated with the bankruptcy and restructuring processes. These costs represent a collective loss for bond and equity holders, and also imply a loss of resources for the economy as a whole.

[6]Note that creditors discount the future using the same discount factor as shareholders, $M_{t,t+1}$. This is consistent with our assumption that they belong to the same risk-sharing household.

separable preferences over consumption C and hours worked, N which obey:

$$U = \mathrm{E}\left\{\sum_{t=0}^{\infty} \beta^t \frac{\left[u\left(C_t, N_t\right)\right]^{1-\sigma} - 1}{1-\sigma}\right\} \tag{12}$$

where the parameters $\beta \in (0,1)$ and $\sigma > 0$ are tied to the rate of inter temporal preference and household risk aversion. We further assume that momentary utility is described by the Cobb-Douglas function:

$$u\left(C_t, N_t\right) = C_t^{1-\theta}\left(3 - N_t\right)^{\theta} \tag{13}$$

where the value of θ will be linked to the elasticity of labor supply.

As is common in the literature, we find it useful to assume that each member of the family works or invests independently in equities and bonds, and all household income is then shared when making consumption and savings decisions.

2.3 Equilibrium and Aggregation

Given the optimal decisions of firms and households implied by the problems above we can now characterize the general dynamic competitive equilibrium in this economy. As stated above, the nature of the problem means that, outside default this equilibrium is symmetric, in the sense that all firms make identical decisions at all times. The only meaningful cross-sectional difference concerns the realization of the shocks z_t^j which induce default for a subgroup of firms with mass $1 - \Phi(z^*)$. Defaulting implies one time restructuring charges for firms, but these temporary losses have no further impact on the choices concerning future capital and debt. Thus all firms remain ex-ante identical in all periods. This means that we can drop all subscripts j for firm specific variables.

It follows from our discussion that aggregate output in the economy, Y_t can be expressed as:

$$Y_t = y_t \; - \left[1 - \Phi\left(z^*\right)\right] \xi \xi^r K_t \tag{14}$$

As discussed above, $\xi \in [0,1]$ captures the loss, relative to the size of the firm's assets, that creditors suffer in bankruptcy. Some of these losses may be in the form of legal fees and might be recouped by to other members of the representative family. But some may represent a genuine destruction of resources. The relative balance between these two alternatives is governed by the parameter $\xi^r \in [0,1]$. In the special case where $\xi^r = 0$ default entails no loss of resources at the aggregate level.

Since all firms make identical choices, the aggregate capital stock is equal to $K_t = k_t^j$ and its law of motion is simply:

$$K_{t+1} = (1-\delta)\, K_t + I_t \tag{15}$$

where again aggregate investment is simply $I_t = i_t k_t$.

To complete the description of the economy we require that both goods and labor market clear. This is accomplished by imposing the aggregate resource constraint:

$$Y_t = C_t + I_t \tag{16}$$

and the labor market consistency condition:

$$N_t = n_t \tag{17}$$

3 Characterization

Before describing our quantitative findings it is helpful to examine our model in some detail to gain some intuition about its key properties.

3.1 Normalized Equity and Debt Values

The constant returns to scale nature of the technology and costs allows us to rewrite the entire model in terms of ratios to the capital stock. Specifically we can rewrite the expression for the value of equity (6) as:

$$E/k = e(\omega, z, \mu) = \max\left\{0, (1-\tau)(R-z) - ((1-\tau)c + \lambda)\frac{\omega}{\mu} + v(\omega,\mu)\right\} \tag{18}$$

where $\omega = b/k$ is a measure of the leverage ratio. Similarly, we scale equation (7) to express the continuation function, $v(\cdot) = V/k$, as:

$$v(\omega,\mu) = \max_{\omega', i}\left\{\begin{array}{c} p\left(\omega' g(i) - (1-\lambda)\frac{\omega}{\mu}\right) - i + \tau\delta + g(i)\mathrm{E}M' \int_{\underline{z}}^{z^{*\prime}} \left[(1-\tau)(R'-z')\right. \\ \left. -((1-\tau)c+\lambda)\frac{\omega'}{\mu'} + v(\omega',\mu')\right] d\Phi(z') \end{array}\right\} \tag{19}$$

where we use primes to denote future values, and the definition $g(i) = (1-\delta+i)k$.

The market value of the outstanding debt (11) can be expressed as:

$$\omega' p = \mathrm{E}M'\left\{\begin{array}{c} \Phi(z^{*\prime})\left[c+\lambda\right]\frac{\omega'}{\mu'} + (1-\lambda)\frac{p'\omega'}{\mu'} \\ +(1-\Phi(z^{*\prime}))\left[(1-\tau)R' - \xi + v(\omega',\mu')\right] - (1-\tau)\int_{z^{*\prime}}^{\bar{z}} z' d\Phi(z) \end{array}\right\} \tag{20}$$

3.2 Optimal Default

The response of the default rate is an important ingredient in our results below. To understand its behavior we can look at the response of the optimal default cutoff level, z^*, implied by the limited liability condition on equity holders. Expressed as a function of the leverage ratio, ω, the optimal default threshold is

$$z^* = R - c\frac{\omega}{\mu} - \frac{\lambda}{(1-\tau)}\frac{\omega}{\mu} + \frac{1}{(1-\tau)}v(\omega,\mu) \tag{21}$$

Differentiating this expression with respect to outstanding leverage ω we get:

$$\frac{\partial z^*}{\partial \omega} = -\left(c + \frac{\lambda}{1-\tau}\right)\frac{1}{\mu} + \frac{1}{(1-\tau)}\frac{\partial v(\omega,\mu)}{\partial \omega} < 0 \tag{22}$$

Intuitively, an increase in outstanding debt increases the required principal and coupon payments, and by reducing the cut-off z^*, makes default more likely.

In addition, the envelope condition implies that:

$$\frac{\partial v(\omega,\mu)}{\partial \omega} = -p\frac{1-\lambda}{\mu} \leq 0 \tag{23}$$

so that when debt maturity exceeds one period ($\lambda < 1$) an increase in outstanding debt also decreases the (expected) future payments to equity holders and thus the value of equity itself, again encouraging default.

3.3 Debt Overhang and the Impact of Inflation

To understand the impact of changes in the inflation rate it is useful to examine the behavior of firm leverage and investment to these shocks.

The optimal leverage ratio follows from the first order condition for the normalized value function (19):

$$pg(i) + \frac{\partial p}{\partial \omega'}\left(\omega' g(i) - (1-\lambda)\frac{\omega}{\mu}\right) = -(1-\tau)\, g(i)\, \mathrm{E}M'\Phi(z^{*\prime})\frac{\partial z^{*\prime}}{\partial \omega} \tag{24}$$

Similarly the optimal investment policy obeys:

$$\begin{aligned} 1 &= p\omega' + \mathrm{E}M' \int_{\underline{z}}^{z^{*\prime}} \left[(1-\tau)(R'-z') - ((1-\tau)c+\lambda)\frac{\omega'}{\mu'} + v(\omega',\mu')\right] d\Phi(z') \\ &= p\omega' + \mathrm{E}M' \int_{\underline{z}}^{z^{*\prime}} (1-\tau)(z^{*\prime}-z')\, d\Phi(z') \end{aligned} \tag{25}$$

These conditions are more informative versions of the more general versions (9) and (10).

Before proceeding, it is important to note that the constant returns scale nature of the problem implies that the first order condition (25) does not pin down the optimal level of the capital stock. This is determined by the competitive equilibrium. Instead, this optimality condition describes the required equilibrium value of the rental rate, R'.

We are now ready to study the effect of inflation shocks on the optimal behavior of firms. Because we want to fully isolate the effects of inflation movements in our model, we will, for the moment, focus on the case where μ is exogenous and follows a simple i.i.d. process. Our quantitative section generalizes these results.

Proposition 1 discusses how the effect of an unanticipated inflation shock on leverage depends on the maturity of debt.

Proposition 1. *Consider an economy where:*

- *There are no aggregate resource costs associated with bankruptcy, i.e., $\xi^r = 0$.*
- *all realizations of exogenous shocks have been zero for a long time so that at time $t-1$ $\mu_{t-1} = \bar{\mu}$ and $\omega_t = \bar{\omega}$*

Suppose that at time t the economy experiences a temporary decline in the inflation rate so that $\mu_t < \mu_{t-1}$. Then $\omega_{t+1} \geq \omega_t$, with equality if and only if $\lambda = 1$.

Proof. With $\lambda = 1$ the current inflation rate, μ_t, has no direct effect on the choice of ω' in (24). Since

- μ is i.i.d., there is no effect on the debt pricing equation (20) and the equilibrium value of p;
- $\xi^r = 0$, there are no resource costs and neither aggregate consumption, C, nor the stochastic discount factor, M', are affected;

there are no indirect general equilibrium effects either. It follows that the optimal choice of leverage, ω' is unaffected by the shock.

When $\lambda < 1$ a decline in μ raises the marginal benefit of issuing new debt (since $\frac{\partial p}{\partial \omega'} \leq 0$) and ω' will increase accordingly. ∎

Proposition 1 shows that temporary movements in inflation will be propagated over time as long as debt maturity is not instantaneous. Intuitively, this is because a change in the existing leverage has a direct impact on the marginal cost of issuing new debt. An unanticipated decline in the rate of inflation μ_t increases the (real) value of currently outstanding liabilities, $(1-\lambda)b_t^j/\mu_t$, so that it is in effect relatively cheaper to issue new debt.[7] This encourages the firm to maintain a persistently elevated level of ω - so that leverage is "sticky".

In a sense, this response of leverage, reflects the fact that prices act like an (endogenous) convex adjustment cost that slows down the response to shocks. In practice this means that leverage is state variable not just due to its obvious impact on equity values, but because it imparts the essential dynamics into optimal firms' decisions.

The extreme assumptions in Proposition 1 also highlight how persistent movements in leverage can occur even when $\lambda = 1$. This occurs if either the underlying inflation movements are persistent or if there are some resource costs associated with default ($\xi^r \neq 0$).

Proposition 2 discusses how the effect of an unanticipated inflation shock affects the rental rate.

Proposition 2. *Consider the same economy of Proposition 1. Suppose that at time t the economy experiences a temporary decline in the inflation rate so that $\mu_t < \mu_{t-1}$. Then $R_{t+1} \leq R_t$, with equality if and only if $\lambda = 1$.*

Proof. Proposition 1 implies that $\omega_{t+1} > \omega_t$ if and only if $\lambda > 1$ It follows from (22) that there is an increase in the default probability $1 - \Phi(z^*)$ and, from the optimality condition

[7] Alternatively, it is more expensive to retire the outstanding debt.

for investment that $p\omega'$ must rise. Equation (20) then implies that the expected rental rate R_{t+1} must rise. ∎

Because of constant returns to scale we cannot establish a direct effect on capital accumulation. However, Proposition 2 shows that the equilibrium required return on capital must rise following declines in expected inflation, a result similar to that of tightening financial constraints. This is in effect a type of debt overhang, by which firm's investment is adversely impacted when corporate debt rises.

Together these two propositions establish the key real effects of the movements on the inflation rate in our model. Under all but the most extreme set of assumptions about debt maturities and restructuring costs, inflation will generate long-lived movements in corporate leverage which in turn change firm investment and, in general equilibrium, aggregate consumption and welfare.

In the language of wedges, changes in the inflation rate distort consumption-investment allocations and, when default has aggregate costs, also impact aggregate productivity and output. Note that this last effect is present even if inflation is i.i.d. and $\lambda = 1$, as long as $\xi^r > 0$. In a general model it adds both persistence and amplification to inflation shocks.

4 Parameterization

The model is calibrated at quarterly frequency. While we choose parameters to match steady state targets whenever feasible, we use model simulations to pin down parameters that determine the stochastic properties of the model economy. Whenever possible, the steady state targets correspond to empirical moments computed over the $1955.I - 2012.IV$ period.

4.1 Aggregate Shocks

In order to assess the dynamic effects of inflation shocks we need estimates of the joint behavior of innovations to productivity and inflation. We begin by assuming the following general VAR(1) representation for the stationary component of productivity and inflation:

$$\begin{bmatrix} a_t \\ \pi_t \end{bmatrix} = \Gamma \begin{bmatrix} a_{t-1} \\ \pi_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_t^a \\ \varepsilon_t^\pi \end{bmatrix}.$$

where $a_t = \ln A_t$, $\pi_t = \ln \mu_t - \ln \bar{\mu}$, and $\bar{\mu}$ is the average (gross) rate of inflation during this period. To do this, we first construct series for Solow residuals and inflation using data on GDP, hours, capital stock and the GDP deflator from the BEA and the BLS. Estimating this autoregressive system yields empirical measures of the standard deviations σ_a and σ_π of the productivity and inflation shocks, as well as their cross-correlation, $\rho_{\pi a}$.

For our sample period, we find that:

$$\Gamma = \begin{bmatrix} 0.98 & -0.094 \\ 0.012 & 0.85 \end{bmatrix}$$

and $\sigma_a = 0.0074$ and $\sigma_\pi = 0.0085$, and $\rho_{\pi a} = -0.19$. We call this the VAR specification of our shocks. We also consider a more restrictive AR(1) specification where $\rho_{\pi a} = 0$and $\Gamma_{12} = \Gamma_{21} = 0$. This version of the model allows us to examine on the case where the exogenous inflation shocks have no real effects, other than those on firm leverage and offers more reliable variance decomposition results.

4.2 Idiosyncratic Profit Shocks

Instead of adopting a specific distribution for the p.d.f. $\phi(z)$ we use a general quadratic approximation of the form:

$$\phi(z) = \eta_1 + \eta_2 z + \eta_3 z^2 \tag{26}$$

The distribution is assumed symmetric with $\bar{z} = -\underline{z} = 1$. Our other assumptions about this distribution's mean imply that $\eta_2 = 0$, and η_3 is tied to our choice of the only free parameter, η_1. The value for η_1 is picked to ensure that our model matches the unconditional volatility of the leverage ratio, a key variable in our model.

4.3 Technology and Preferences

Our choices for the capital share α, depreciation rate δ and the subjective discount factor β correspond to fairly common values and pin down the capital-output, investment-output and the average rate of return on capital in our economy. As long as they remain in a plausible range, the quantitative properties of the model are not very sensitive to these parameter values. The preference parameter θ is chosen so that in steady state working hours make up one third of the total time endowment, while σ is set to deliver a plausible level of risk aversion.

4.4 Institutional Parameters

As we have seen the parameter λ pins down average debt maturity. This is a very important parameter to determine the propagation of inflation shocks. Our benchmark calibration implies an average maturity of about 4 years, and an actual duration of 3 years. These values are similar to those of industrial and commercial loans, but significantly shorter than those for corporate bonds. Given the importance of this parameter, we prefer to err on the save side and focus on the quantitative results when debt maturity is conservatively

calibrated. We will document how the results change with alternative average maturities.

The tax wedge τ is chosen so that average firm leverage matches the observed leverage ratio for the U.S. non-financial business sector. This average about 0.42 in the period since 1955.[8]

Finally, the default loss parameter ξ is chosen to match the average credit spread, defined as the difference between the yield on corporate debt and a that of a riskless security of identical maturity. Because our preferences do not allow for much risk aversion from investors, matching the credit spread requires that actual losses upon default are quite large. [9]. For the benchmark calibration, we set the aggregate loss $\xi^r = 1$, and also document the impact of this parameter on the behavior of the model.

Table 1 summarizes our parameter choices for the benchmark calibration. The model is quite parsimonious, and requires only 10 structural parameters, in addition to the stochastic process for the shocks. Table 2 shows the implications of these choices for the first and second (unconditional) moments of a number of key variables. As discussed above our calibration ensures that the model matches the key financial variables associated with the optimal capital structure of the firm. These include, a leverage ratio of 42%, and a quarterly default rate and credit spread of around 40 basis points.

The second panel in Table 2 shows that our quantitative model shares many of the properties of other variations of the stochastic growth model. All the main aggregates have plausible volatilities, except for labor. The model is calibrated to match the leverage ratio, a crucial ingredient in the transmission of shocks.

[8]Alternatively we could fix the tax parameter by attempting to match the (statutory) wedged implied by the interaction between corporate income rates and the tax treatment individual interest and equity income. Depending on the time period, this method would imply a value of τ around 25%.

[9]It is important that credit prices and quantities are matched. We prefer not to introduce more complex specifications for the utility $U(\cdot)$ because they could mask our key insights.

Table 1: **Calibration**

Parameter	Description	Value
β	Subjective Discount Factor	0.99
γ	Risk Aversion	1
θ	Elasticity of Labor	0.63
α	Capital Share	0.36
δ	Depreciation Rate	0.025
λ	Debt Amortization Rate	0.06
ξ^r	Fraction of Resource Cost	1
ξ	Default Loss	0.38
τ	Tax Wedge	0.40
η_1	Distribution Parameter	0.6617

5 Quantitative Analysis

In this section, we discuss the properties of calibrated model. In particular, we are interested in determining how the model responds to inflation shocks and how much endogenous propagation can plausibly be generated by the combination of our sticky leverage and debt overhang mechanisms.

5.1 Model with Exogenous Inflation

5.1.1 Impulse Responses

As we have seen above, our model predicts that under very general conditions, even exogenous i.i.d. movements in inflation will induce prolonged movements in corporate leverage and investment, and in equilibrium in aggregate output and consumption. Figure 1 shows how these responses would look like in a plausible quantitative version of the model, when inflation follows an exogenous AR(1) process that is uncorrelated with productivity.

We can see that following a lower than expected inflation shock, the default rate in-

Table 2: **Aggregate Moments**

	Data	Model	Model
		AR(1)	VAR(1)
First Moments			
Investment/Output, I/Y	0.21	0.24	0.24
Leverage, ω	0.42	0.42	0.42
Default Rate, $\Phi(z^*)$	0.42%	0.42%	0.42%
Credit Spread	0.39%	0.39%	0.39%
Second Moments			
σ_Y	1.66%	1.58%	1.67%
σ_I/σ_Y	4.12	4.22	4.48
σ_C/σ_Y	0.54	0.41	0.43
σ_N/σ_Y	1.07	0.49	0.54
σ_ω	1.7%	1.5%	1.7%

creases as the real value of outstanding corporate liabilities increases. This increase in the default rate immediately produces output losses since restructuring costs are not rebated to households and represent real deadweight losses.

As Proposition 1 implies, leverage - we report its market value, $p\omega$ - rises and remains elevated for a long time even though inflation quickly returns to its long run mean. This stickiness in leverage contributes to a prolonged, and significant contraction in investment spending as firms are forced to allocate more of their profits to service the outstanding liabilities - the debt overhang result.

Initially at least there is an important change in the inter temporal allocation of resources, as consumption actually rises, reflecting the fact that the required rental rate on capital has increased. Soon however, the decline in the capital stock and output begins to tell, and consumption declines as well. Labor mostly, but not entirely, mirrors the behavior of consumption, as households seek to smooth their leisure decisions as well.[10]

[10] With GHH preferences, consumption and labor do not move at all on impact and then immediately

5.1.2 Variance Decompositions

Tables 3 and 4 perform a variance decomposition of movements in the key macroeconomics aggregates in the baseline model driven by independent AR(1) shocks to inflation and productivity. This ensures that the measured contributions of the inflation shocks are entirely due to their effects on the endogenous variables.

Table 3 investigates the importance of the average leverage ratio to our results. For the baseline case, where leverage matches the data for the period since 1955, inflation movements account for a little over 1/3 of the variation in output and hours and nearly 2/3 of that in consumption and investment. Even when the leverage ratio is only 32%, inflation still accounts for 35% of the movements in investment. Conversely, when the leverage ratio matches the value observed in the period since 2005, inflation shocks dominate business cycle fluctuations and account for 60% of the variance of output.

Table 4 examines the role of debt maturity. The table confirms that it is indeed crucial that debt lasts for more than a single quarter. However the results are not monotone in average maturity.

Overall, these results appear quite striking to us. We have taken a model with exogenous inflation shocks that have no direct impact on real quantities. However, despite the fact that the price level is entirely flexible, inflation has potentially very large real effects.

5.2 Model with Endogenous Inflation

A common way endogenize inflation is to introduce a monetary policy rule of the form:

$$\ln(r_t/\bar{r}) = \rho_r \ln(r_{t-1}/\bar{r}) + (1-\rho_r)v_\mu(\ln(\mu_t/\bar{\mu}) + (1-\rho_r)v_y(\ln\left(Y_t/\bar{Y}\right) + \zeta_t \tag{27}$$

decline.

Table 3: **Variance Decomposition and Leverage**

	Y	Inv	Cons	Hrs	Lev	Default
Bench., $\bar{\omega} = 0.42$						
TFP shock a	0.63	0.37	0.39	0.60	0.10	0.05
Inflation shock μ	0.37	0.63	0.61	0.40	0.90	0.95
Low Lev, $\bar{\omega} = 0.32$						
TFP shock a	0.84	0.65	0.83	0.89	0.10	0.01
Inflation shock μ	0.16	0.35	0.17	0.11	0.90	0.99
High Lev, $\bar{\omega} = 0.52$						
TFP shock a	0.40	0.21	0.29	0.55	0.03	0.03
Inflation shock μ	0.60	0.79	0.71	0.45	0.97	0.97

where r is the nominal one period interest rate which must satisfy the Euler equation:

$$r_t = \mathrm{E}\frac{1}{M_{t,t+1}\mu_{t+1}}, \tag{28}$$

and the bars denote the steady-state values of the relevant variables. We follow the literature and set the monetary policy responses $\rho_y = 0.5$ and $\rho_\mu = 1.5$, the smoothing parameter is $\rho_r = 0.6$.

In this version of the model, inflation is driven either by endogenous monetary policy responses to movements in output, or through exogenous shocks to monetary policy. We now investigate these two possibilities in turn.

5.2.1 Monetary Policy Shocks

Figure 2 considers the effects of an exogenous shock to the policy rule, ζ_t. The shock is set so as to produce an inflation response in the second panel that resembles the exogenous inflation shock considered above. This is accomplished by using an interest smoothing parameter, ρ_r, of 0.3, and with ζ_t following an AR(1) with a persistence parameter of 0.99.

Table 4: **Variance Decomposition and Maturity**

	Y	Inv	Cons	Hrs	Lev	Default
Bench., $\lambda = 0.06$						
TFP shock a	0.63	0.37	0.39	0.60	0.10	0.05
Inflation shock μ	0.37	0.63	0.61	0.40	0.90	0.95
Low Lev, $\lambda = 0.03$						
TFP shock a	0.45	0.26	0.65	0.80	0.83	0.01
Inflation shock μ	0.55	0.74	0.35	0.20	0.17	0.99
High Lev, $\lambda = 1$						
TFP shock a	1	1	1	1	0.01	0.02
Inflation shock μ	0.00	0.00	0.00	0.00	0.99	0.98

As can be seen from this figure, the responses of the key variables are similar regardless of whether the inflation movements are exogenous or induced by the monetary policy rule. All that matters for the response of the real economy is the actual behavior of the inflation rate itself.

5.2.2 Productivity Shocks

With endogenous monetary policy, the inflation rate will change when the economy is hit by productivity shocks. Figure 3 documents the different impact of shocks to total factor productivity, A_t, with and without a monetary policy response.

The green lines show responses to the productivity shock without the monetary policy rule, where inflation is unaffected by the shock. In this case, we observe the patterns common to RBC models, except that our model produces more persistence. In particular, output now displays a hump shaped response. This is due to the amplification induced by our financial friction.

The blue lines represent the responses when the nominal interest rate follows the monetary police rule (27). Here, output, investment, labor, and to some extent consumption, move

much less. This is because the monetary policy rule lowers the inflation rate and increases the real burden of outstanding debt. As discussed above, this reduction in inflation then raises corporate defaults, and negatively impacts investment and the other real variables.

Interestingly, the higher output in the policy rule, everything else equal, would lead to a higher interest rate. However, in this model, the nominal interest rate actually declines because inflation falls too.

5.2.3 Wealth Shocks

Finally, Figure 4 examines the important case of a shock to the stock of wealth in the economy. We believe this experiment captures some aspects of the contraction seen since 2007/08.

Formally, this is implemented by a decrease in the value of the capital stock k of 5% through a one time increase in the depreciation rate, δ. On impact, this destruction of the capital stock lowers both overall firm and equity values. This leads to a mechanical increase in the leverage ratio and an immediate spike in corporate defaults.

When the inflation rate remains constant (green lines), stickiness in leverage and debt overhang produce long lasting real declines in investment and output. However, when monetary policy responds according to the rule (27) (blue lines), the inflation rate increases immediately, which reduces the burden of outstanding liabilities and significantly mitigates the effects of this shock on the real aggregates. With a 5% reduction in the capital stock, the model's implied inflation increases by a total of 3.6% above its steady state level over the first year after the shock.

A priori, this policy is consistent with a popular policy prescription summarized in the following quote:

"I'm advocating 6 percent inflation for at least a couple of years," says Rogoff, 56, who's now a professor at Harvard University. "It would ameliorate the debt bomb and help us work

through the deleveraging process." (Bloomberg May 19, 2009 00:01 EDT).

6 Conclusion

In this paper we have presented a general equilibrium model with nominal debt contracts that can help us better understand the ongoing financial crisis and the observed monetary policy responses. Unlike other popular models of monetary non-neutralities, we eschewed price rigidities. Yet our model is capable of generating very large and persistent movements in output and investment.

Almost unavoidably, our attempt to write a parsimonious and tractable model leaves out many important features. Perhaps, first and foremost, we ignore nominal debt contracts other than those held by firms, even though household debt is roughly equal in magnitude and subject to similarly large restructuring costs.

We also abstract from the role of movements in credit risk premia and the behavior of asset prices in general. In addition, while convenient, the assumption of constant returns to scale, which nearly eliminates firm heterogeneity and renders the model so tractable, also limits our ability to study firm behavior more comprehensively.

But although these and other simplifying assumptions can – and should – be explored in later work, we believe none is essential to the core ideas of the paper.

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Figure 1: An exogenous inflation shock. This figure shows the effect of an unanticipated decline in the inflation rate μ_t on the key variables of the model.

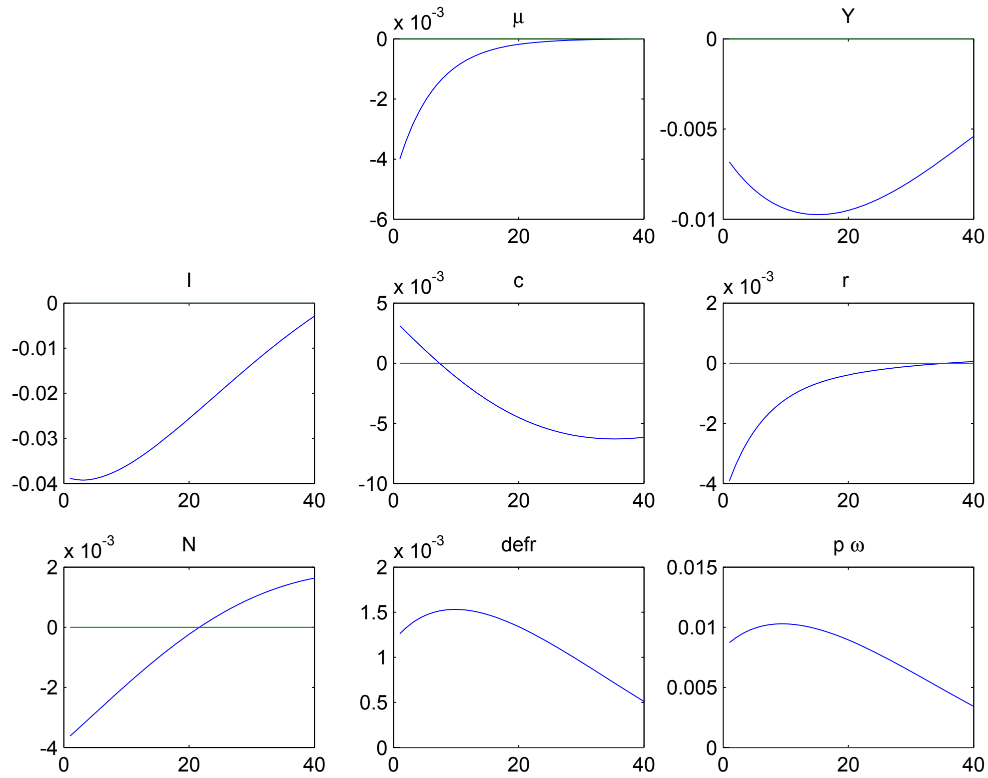

Figure 2: An endogenous inflation shock. This figure shows the effect of an exogenous shock to the monetary policy rule on the key variables of the model. The blue line illustrates the response to an exogenous change in monetary policy, ζ, while the green line shows the response in the baseline case with exogenous inflation.

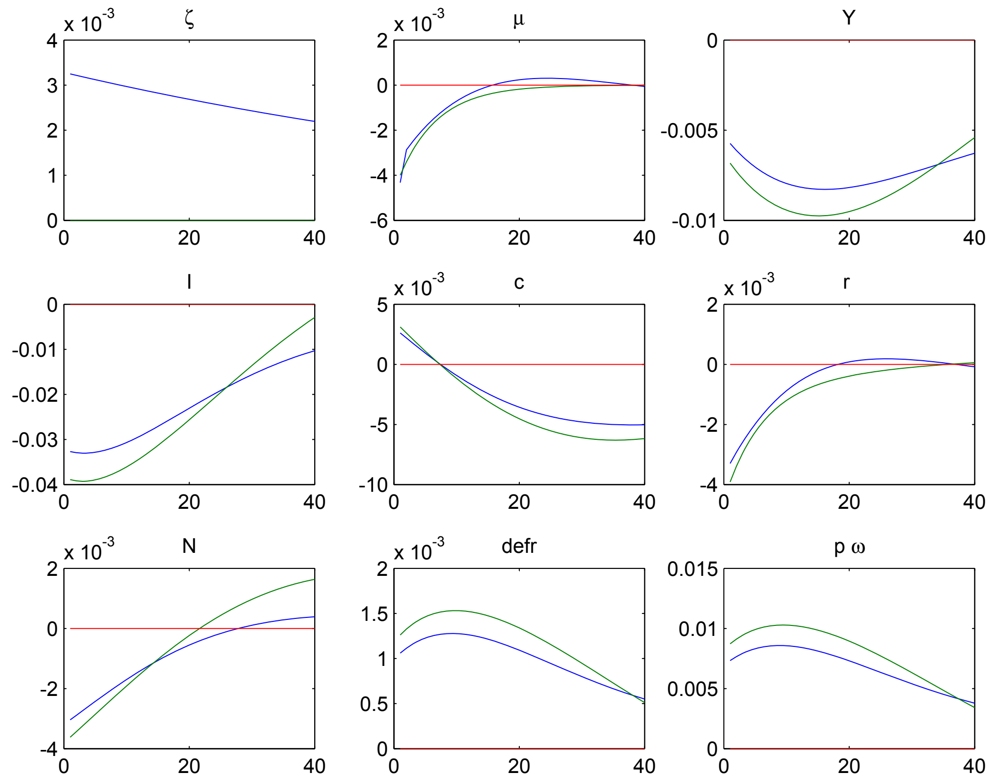

Figure 3: A productivity shock. This figure shows the effect of an exogenous shock to productivity, A. It compares the effects when inflation is exogenous (green line) and when it adjusts endogenous as a consequence of a monetary policy rule (blue line).

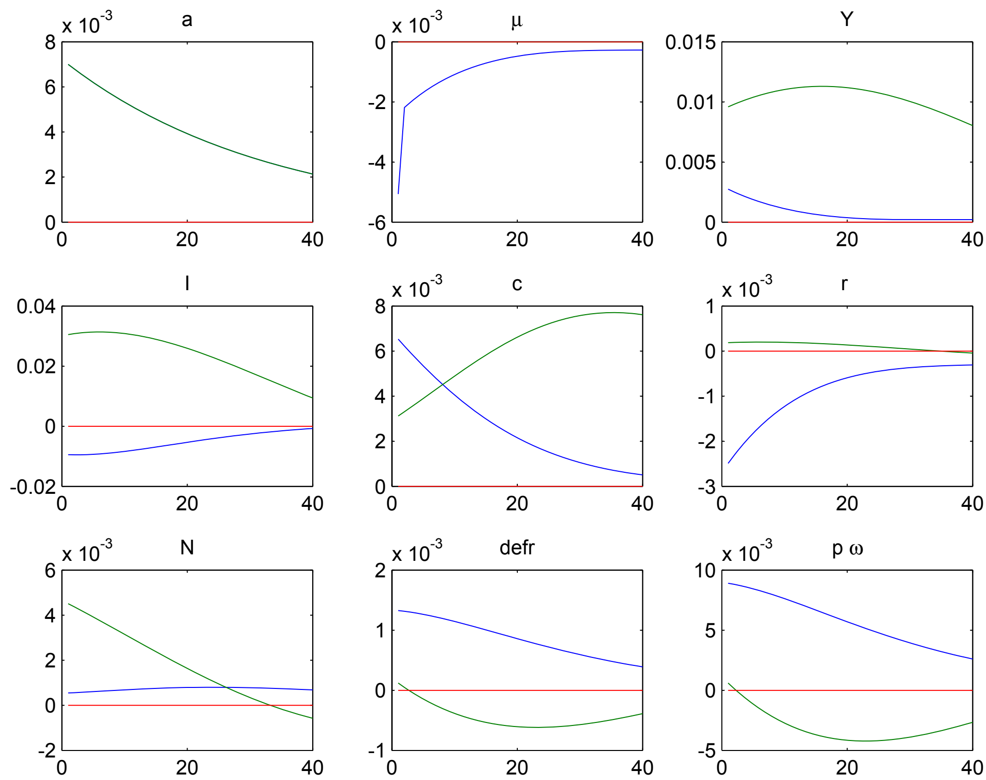

Figure 4: A wealth shock. This figure shows the effect of an exogenous destruction of the capital stock, k, through a temporary increase in the depreciation rate, δ. It compares the effects when inflation is exogenous (green line) and when it adjusts endogenous as a consequence of a monetary policy rule (blue line).