Internal Measurement Approach
(Foundation model)

1. Proposal for an IMA formula[1]

1.1 Standardised Approach

Under the Standardised Approach, the required capital for the bank as a whole is the aggregate of the required capital amounts for all the business lines. The required capital for each business line is calculated by multiplying β determined by the regulators by an indicator which is intended to represent the level and size of the activity in the business area concerned. In the "Working Paper on the Regulatory Treatment of Operational Risk (WP)", it was proposed that Gross Income (GI) be used as such an indicator for every business line. The required capital under the Standardised Approach is therefore the aggregate of the following for all the business lines.

$$\text{Required Capital} = \beta * GI \qquad (1\text{-}1)$$

As a result, under the Standardised Approach, (i) The level and size of the activity in each business line is reflected in GI, and (ii) The risk characteristic of each business line is reflected in β.

The Standardised Approach, however, has the following limitations.

(i) The result is not directly linked to the loss data.
(ii) The profile of operational risk varies to a great extent between event types even within the same business line. Formula (1-1) shown above does not reflect the risk profile of each event type because it cannot be broken down to the event type level.

1.2 Advanced Measurement Approaches (AMA)

Under the AMA, each bank can use its own measurement method for operational risk.

The features of the AMA are: (i) It is based on the collection of loss data; (ii) The characteristic of "low-frequency / high-severity" for each event type in addition to business line can be reflected. Each bank is to measure the required capital based on its own loss data using the holding period and confidence interval determined by the regulators (1 year and 99.9%, respectively, in the WP). The WP refers to three approaches, i.e. LDA, IMA and Scorecard Approach as examples of the methods under the AMA.

These methods should be verified through backtesting based on historical loss data. The capital charge calculated under the AMA, however, is initially subject to a floor set at 75% of that under the Standardised Approach at least until the development of measurement methodologies is examined.

[1] Operational risk can be discussed in comparison with credit risk. Business line / event type for operational risk corresponds to exposure / rating for credit risk. β in the Standardised Approach for operational risk is established for each business line while the risk weight in the context of credit risk under the Standardised Approach is set for each exposure type (partly combined with external credit ratings). The parameters for the operational risk IMA and the credit risk IRB corresponds to each other as follows; $\gamma \Leftrightarrow$ Coefficients in the Risk Weight Function (including LGD), $A \Leftrightarrow \rho$, EL and $n \Leftrightarrow$ PD. γ [coefficients in the Risk Weight Function] is determined for each business line [exposure type] while A, EL and n [ρ, PD] are established for each business line / event type combination [exposure type / credit rating or size]. Furthermore, λ, coefficients in the Risk Weight Function, A and ρ are determined by the regulators while EL, n and PD are estimated by banks.

1.3 Proposal for an IMA formula

The objective of this paper is to propose an explicit formula for the IMA, one alternative under the AMA.

The IMA formula we are proposing in this paper calculates the required capital for each combination of business line / event type as follows;

$$\text{Required Capital} = \gamma * EL$$

where EL denotes the average annual loss amount. EL is derived from the bank's own loss data based on the collection of internal data.

Furthermore, in order to reflect appropriately the characteristics of operational risk, i.e. low-frequency / high-severity, an adjustment factor $(1+A/\sqrt{n})$ has been incorporated as follows.

$$\text{Required Capital} = \lambda * EL * (1+A/\sqrt{n}) \qquad (1\text{-}2)$$

where λ denotes a constant determined for each business line, A is a constant for each business line / event type combination and n denotes the number of events. λ is a constant determined based on the holding period and confidence interval specified by the regulators.

1.4 IMA Foundation Model

Parameters A and λ in the IMA formula shown above implicitly incorporate holding period and confidence interval. These parameters can be estimated by each bank individually, or alternatively, determined by the regulators uniformly based on the global data. We propose an IMA formula for which A and λ are determined by the regulators based on the global data, and call it Foundation Model. (We call the other type, for which these parameters are set by banks, Generic Model.)

1.5 Floor for AMA

As for the implementation of the AMA, a floor is imposed for the following reasons; (i) The internal methods used to quantify operational risk are still in early stages of implementation. (ii) The AMA do not, as yet, contain detailed criteria for the specific quantification methods likely to be used by banks. The effect of such factors, however, varies to a great extent between different methods. The regulators should therefore examine the degree of such an effect to determine the level of the floor accordingly.

1.6 Floor for IMA Foundation Model

As for the implementation of IMA being in its early stages as referred to in (i) above, it is necessary in the case of the Generic Model based on banks' own estimates to verify the level of implementation through backtesting etc. using historical loss data to check the quantification result as in the case with the LDA based on banks' own estimates and the Scorecard Approach. The stage of implementation, however, does not matter to the Foundation Model based on the parameters uniformly determined by the regulators for which such a verification of methods employed by individual banks is not required.

Furthermore, the detailed criteria for quantification methods can be uniformly established because all the parameters are fixed under the IMA Foundation Model as in the case with the Standardised Approach.

Therefore, if IMA Foundation Model in a rigorous form is developed, it should be able to enjoy a floor set at a lower level in light of the very reasons for imposition of the floor articulated in the WP. Eventually, such a floor could be dropped. (This is equivalent to the rule that a floor is imposed on the Advanced IRB for credit risk in relation to the Foundation IRB until the measurement result is verified.)

2. Relationship with the basic structure proposed in Consultative Paper 2

Basel Committee proposed the following basic structure of the IMA formula in CP2 (January 2001).

$$\text{Required Capital} = \gamma * EI * PE * LGE$$
$$= \lambda * EI * PE * LGE * RPI$$

Formula (1-2) proposed in this paper can be related to this basic structure in CP2 as follows.

EI	PE	LGE	RPI
EL			$1+A/\sqrt{n}$

Each factor is explained in more detail in the following sections.

2.1 EL

The formula for IMA is simpler than that for LDA, which facilitates its application to regulations and business practice. In the case where the size of the bank's business operation is changed due to merger / demerger on a large scale or acquisition / divestiture of important new businesses, the bank can modify the internal loss data based on the EI (scaling adjustment).
The following issues, however, would be raised as to the actual implementation of;
"Required Capital = γ * EI * PE * LGE" proposed in CP2.

- Definition of EI can be difficult depending on the event type.
- Even if such a definition is possible, it is difficult to actually collect data on the EI. The calculation of PE is therefore difficult.

When, for example, total transaction amount (= $N\mu$) is selected as EI, which would be the most appropriate for commercial banking, actual calculation of EI * PE * LGE shows that EI and PE, which are difficult to measure, cancel out each other and the result equals the annual loss amount as follows.

$$EI * PE * LGE = N\mu * n/N * \mu_L/\mu = n\,\mu_L = EL \text{ (annual loss amount)}$$

where; N: Total number of transactions, μ: Average transaction amount, n: Number of events, μ_L : Average of loss amount.

Thus, by incorporating EL, Formula (1-2) enables calculation of required capital without directly measuring EI and PE, which resolves the issues shown above.

2.2 λ

λ is a factor related to the required capital / EL ratio. It is a constant determined for each business line by the confidence interval and holding period for the calculation of required capital.

2.3 $1+A/\sqrt{n}$

According to 2-1, EL is equal to EI * PE * LGE. Therefore, $1+A/\sqrt{n}$ equals RPI. RPI reflects the "low-frequency / high-severity" characteristic of operational risk, and can be divided into the adjustment factor for severity and that for frequency.

2.3.1 Adjustment factor for frequency

The adjustment factor for frequency incorporates the profile of each bank as to the level of low-frequency so that required capital / EL becomes greater when n becomes smaller. This feature can be reflected in the IMA formula by introducing a non-linear factor $1 / \sqrt{n}$. This adjustment factor can be easily calculated based on internal data

2.3.2 Adjustment factor for severity

The greater the dispersion of the loss distribution (mean μ_L; standard deviation σ_L), the greater becomes the adjustment factor for severity. It incorporates the profile of each bank as to the level of high-severity. In consideration of the following, this factor is determined for each business line / event type combination as a constant A.

- The profile of loss distribution varies to a great extent between business line / event type combinations. Such a difference can be well explained by the difference between business line / event type combinations. By establishing A for each business line / event type combination, therefore, it is possible to reflect different characteristics of different loss distribution in the formula.

2.4 Common determination of the parameters A and λ based on the global data (Foundation Model)

A and λ can be different between banks. We propose, however, the Foundation Model for which these parameters are determined by the regulators based on the global data on the ground that λ and A depend mainly on business line and business line / event type combination, respectively, as explained above.

2.5 Characteristics of the IMA formula (1-2)

The characteristics of the IMA formula (1-2) discussed so far are summarised below.

- It is based on the linear formula EI * PE * LGE (= EL).
- Non-linearity is incorporated through multiplication by the inverse of the square root of the number of events --- in order to incorporate the characteristic that risk is higher for the same EL in the case of low-frequency.
- The level of severity is differentiated between event types --- in order to incorporate the characteristic that risk is higher for the same EL in the case of high-severity.
- Exposure Indicator is not explicitly shown

– Under the Foundation Model, it is possible to set the floor at a different level from other methods under the AMA because the parameters A and λ can be commonly determined on a global basis, which would negate the necessity for model validation for each bank in the actual implementation of the regulatory framework.

3. Determination of the parameters for the IMA formula

Under the AMA, the required capital amount is measured based on the actual loss data using the holding period and confidence interval specified by the regulators. IMA implicitly incorporates holding period and confidence interval in the parameters for the formula as described below. Therefore, IMA is one methodology under the AMA. The Foundation Model of IMA is different from other methods under the AMA in that parameters λ and A are uniformly determined by the regulators, not estimated by banks.

In this section, we propose a method for calibrating λ and A using either internal or global data and show a sample calculation.

3.1 Method for calibration

The required capital under the IMA formula (1-2) is expressed as;

$$\lambda * EL * (1+A/\sqrt{n})$$

where λ and A are determined for each business line and each combination of event type / business line, respectively, and EL and n are observed for each combination of event type / business line. The required capital for event type i / business line j under the IMA is therefore calculated with the following formula.

$$\lambda_j * EL_{ij} * (1+A_{ij}/\sqrt{n_{ij}}) \text{ where; i: event type / j: business line}$$

In this formula, λ_j and A_{ij} are constant. EL_{ij} and n_{ij} are observed directly in the loss data.

As IMA is an alternative under the AMA, the required capital for event type i / business line j calculated under the IMA is equal to the unexpected loss U□$_{ij}$ (tail of the distribution) measured with the holding period and the confidence interval determined by the regulators. U□$_{ij}$ is determined either on the basis of actual distribution or theoretically.

Calibrating the IMA formula is equivalent to approximating the UL with the IMA. That is, in order to determine an IMA formula, it is required to find λ_j and A_{ij} for each event type i and each business line j which satisfy the following;

$$UL_{ij} = \lambda_j * EL_{ij} * (1+A_{ij}/\sqrt{n_{ij}}) \qquad (3\text{-}1)$$

where

UL_{ij}: Measured either on the basis of actual distribution or theoretically (LDA) using the holding period and confidence interval specified by the regulators.
$\lambda_j * EL_{ij} * (1+A_{ij}/\sqrt{n_{ij}})$, : Required capital measured with the IMA formula.
i: Event type / j: Business line

As UL_{ij}, EL_{ij} and n_{ij} are factors that can be observed (directly or theoretically) based on the loss data, λ_j and A_{ij} can be obtained through regression analysis with multi-variables.

It is, however, difficult to find the optimal solution (λ_j, A_{1j}, A_{2j},----) in this form. In order to reduce the number of variables, the following empirical rule is introduced.

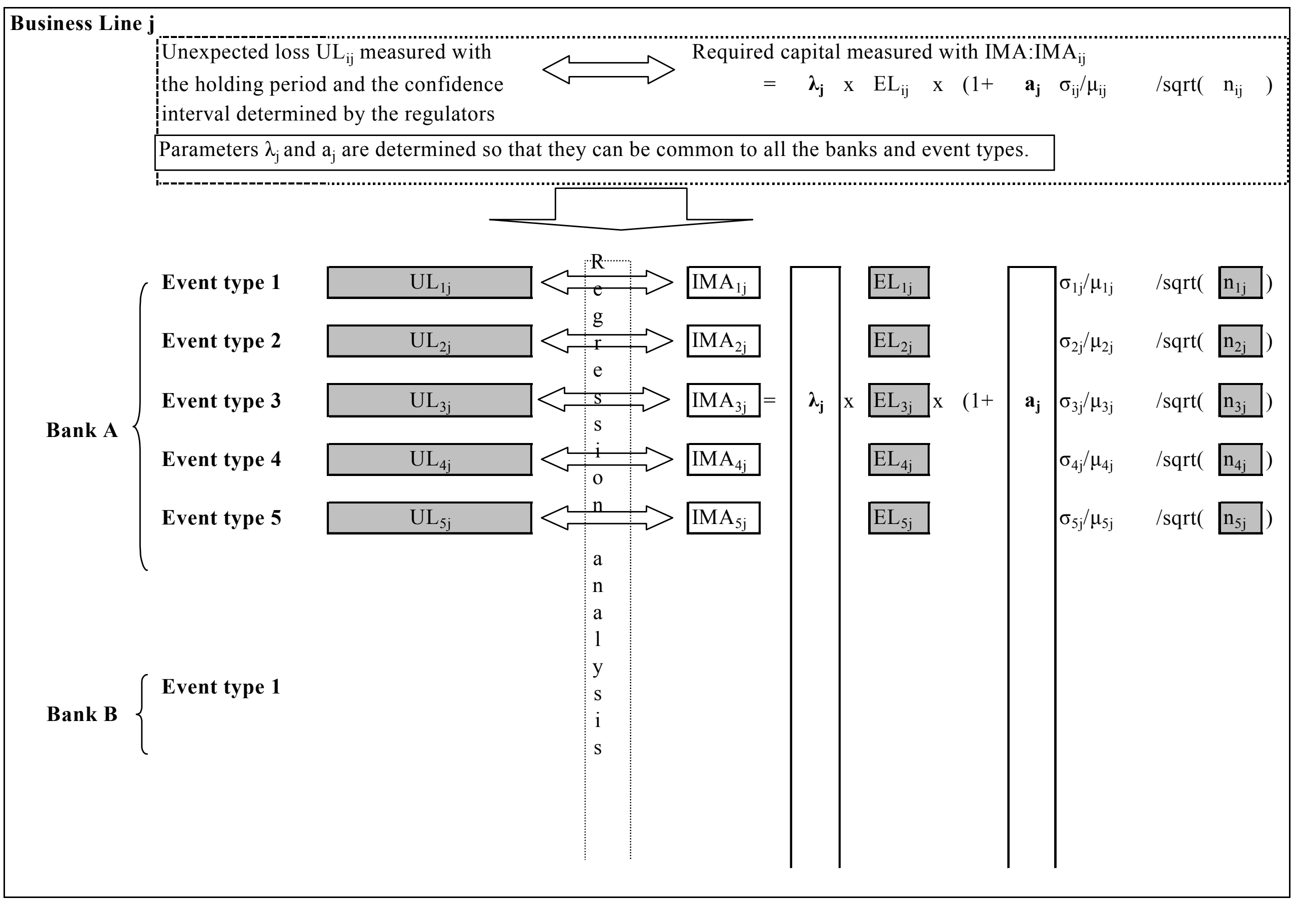
Empirical rule: $A_{ij} = a_j * \sigma_{ij} / \mu_{ij}$ (a_j is a constant)

where σ_{ij} / μ_{ij} is estimated from the loss distribution based on either internal or global data. Thus, the formula (3-1) can be rewritten as follows

$$UL_{ij} = \lambda_j * EL_{ij} * (1 + a_j * \sigma_{ij} / \mu_{ij} / \sqrt{n_{ij}})$$

Now, it is required only to estimate λ_j and a_j based on σ_{ij} / μ_{ij} estimated from the data, UL_{ij} observed for each bank (only in the case of global data) / each business line for each event type, EL_{ij} and n_{ij}. (Regression analysis with two variables is performed for each business line.)

$$UL_{ij} / \underline{EL_{ij}} = \lambda_j + \lambda_j * a_j * \underline{\sigma_{ij} / \mu_{ij} / \sqrt{n_{ij}}}$$



3.2 Sample calibration

The result of the process shown above for the business line of commercial banking (business line 1) is as follows. The unexpected loss UL has been measured with the boot-strap method using the actual loss data. Although it is difficult to determine single λ and A due to the data constraint, the λ and A shown below give a coefficient of determination of 0.93.

	UL_{i1} (1y: 99.9%)	n_{i1}	EL_{i1}		λ_1	A_{i1}
Event type 1	4,468	16	365	Regression analysis	19.46	2.11
Event type 2	*****	**	****		19.46	6.02
Event type 3	*****	**	****		19.46	0.90
Event typ	123,688	76	1,440		19.46	15.31
Event type 5	*****	**	****		19.46	1.96
Event type 6	*****	**	****		19.46	11.95
Event type 7	5,240	2,428	864		19.46	23.84

In the above table, the value of A_{i1} ranges from 0.90 to 23.84 depending on the event type, which reflects the distribution of the loss amount. The fatter the tail of the loss distribution, the larger becomes the value of A_{i1}. Conversely, the shorter the tail, the smaller the value of A_{i1}.

4. Sample calculation of required capital with IMA

Following are sample calculations based on the following assumption.

– $IMA = \lambda * EL * (1 + A/\sqrt{n})$

Parameters for IMA λ and A are as follows.

	Commercial banking		Trading & Sales	
	λ	A	λ	A
Event type 1	19.46	2.11	25.12	2.54
Event type 2	19.46	6.02	25.12	5.95
Event type 3	19.46	0.90	25.12	2.31
Event type 4	19.46	15.31	25.12	16.34
Event type 5	19.46	1.96	25.12	2.04
Event type 6	19.46	11.95	25.12	14.32
Event type 7	19.46	23.84	25.12	18.54

– β under the Standardised Approach = 12% (commercial banking), 20% (trading & sales).

The observed actual loss data are as follows.

	Commercial banking		Trading & Sales	
	EL (JPY Tho.)	n	EL (JPY Tho.)	n
Event type 1	301,287	5	54,528	5
Event type 2	8,666	200	32	20
Event type 3	60	3	0	0
Event type 4	1,880,360	30	32,497	11
Event type 5	8,920	15	0	0
Event type 6	200	5	3,421	4
Event type 7	912,204	920	5,124	56
	3,111,697	1,178	95,602	96

– GI = JPY1,500,000 mil. (commercial banking), JPY200,000mil. (trading & sales)

4.1 Sample for commercial banking

(i) Required capital measured under the IMA = JPY182,501 mil.

$$IMA=\lambda * EL * (1+A/\sqrt{n})$$

	Parameters		Observed loss data		
	λ	A	EL (JPY Tho.)	n	IMA (JPY Tho.) (=UL)
Event type 1	19.46	2.11	301,287	5	11,395,536
Event type 2	19.46	6.02	8,666	200	240,427
Event type 3	19.46	0.90	60	3	1,774
Event type 4	19.46	15.31	1,880,360	30	138,873,615
Event type 5	19.46	1.96	8,920	15	261,428
Event type 6	19.46	11.95	200	5	24,692
Event type 7	19.46	23.84	912,204	920	31,703,833
Total			3,111,697	1,178	182,501,305

UL/EL=58.6

(ii) Required capital measured under the Standardised Approach
= 1,500,000 x 12% = JPY 180,000 mil.

4.2 Sample for trading & sales

(i) Required capital measured under the IMA = JPY8,914 mil.

$$IMA=\lambda * EL * (1+A/\sqrt{n})$$

	Parameters		Observed loss data		
	λ	A	EL (JPY Tho.)	n	IMA (JPY Tho.) (=UL)
Event type 1	25.12	2.54	54,528	5	2,925,666
Event type 2	25.12	5.95	32	20	1,873
Event type 3	25.12	2.31	0	0	0
Event type 4	25.12	16.34	32,497	11	4,838,107
Event type 5	25.12	2.04	0	0	0
Event type 6	25.12	14.32	3,421	4	701,234
Event type 7	25.12	18.54	5,124	56	447,608
Total			95,602	96	8,914,488

UL/EL=93.2

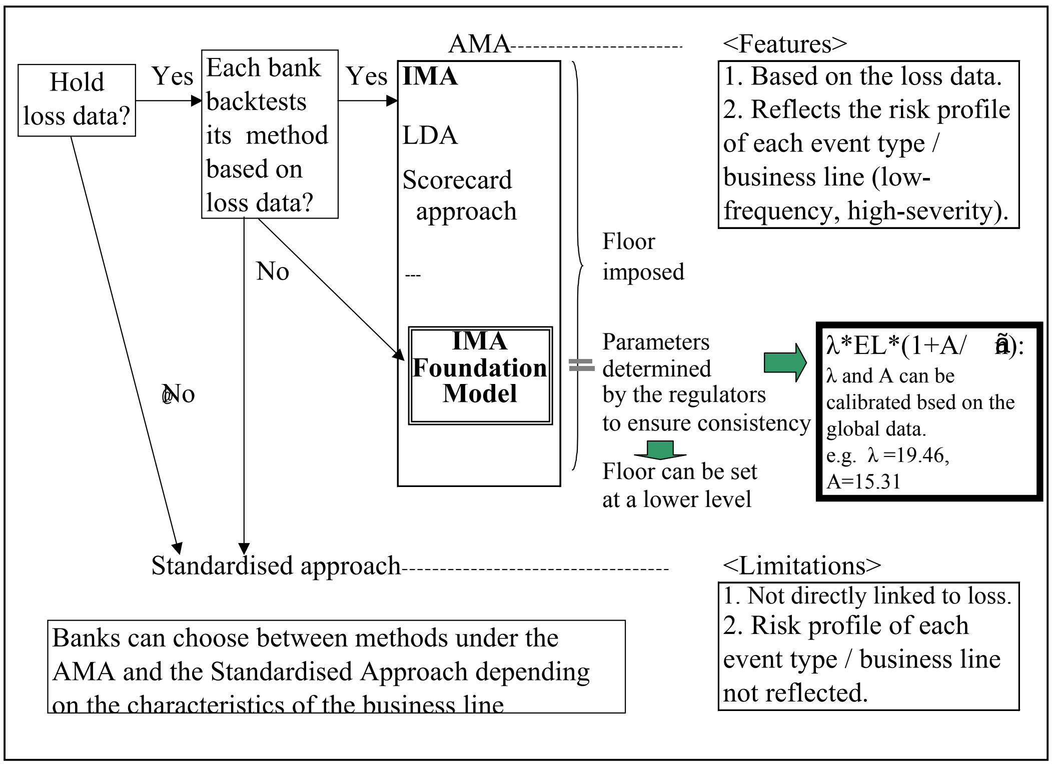
(ii) Required capital measured under the Standardised Approach
= 200,000 x 20% = JPY 40,000 mil.

4.3 Bank as a whole

If the bank has only two business lines shown above, i.e. commercial banking and trading & sales, the required capital for the bank as a whole is the sum of the above.

(i) Required capital measured under the IMA = 182,501+8,914 = JPY 191,415 mil.
(ii) Required capital measured under the Standardised Approach
= 180,000+40,000 = JPY 220,000 mil.

5. Conclusion



6. [For reference purposes only] Application criteria for the IMA formula

In this section, we discuss the sufficiency and stability of the EL, which is the key to actual application of the IMA formula (1-2) and show the methods for calculating the required capital amount under the IMA modified according to such characteristics.

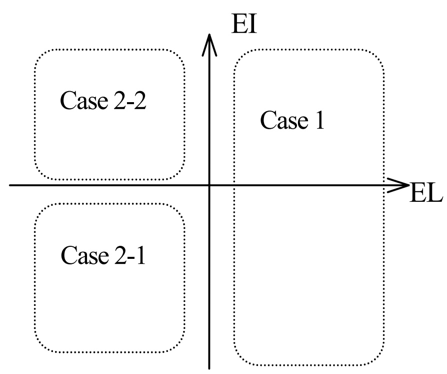
6.1 Sufficiency of EL

The IMA formula (1-2) is based on EL. In applying this formula, it is crucial whether the observed amount of EL is sufficiently large. When the observed EL is large enough, the Formula (1-2) can be applied as it is. If it is not sufficiently large, the reliability of the calculation with this formula in its original form might be low. There are two cases where EL is not adequate depending on the size of EI as follows.

Observed EL is deemed insufficient in the following instances.

(i) EI is small, i.e. no event causing EL has occurred because the number of transactions in the past is very small (Case 2-1).
(ii) EI is large, i.e. the frequency of events is limited to a very low level due to the high control capabilities etc. although the number of transactions is reasonably large (Case 2-2).

In other words, these two cases correspond to the second quadrant (Case 2-2) and the third quadrant (Case 2-1) among the three types of combinations of the size of EL and EI shown below.



In Cases 2-1 and 2-2, EL is not significant. Therefore, the required capital amount calculated using the IMA formula (1-2) is not very reliable. In order to ensure that the measurement is conservative, a floor is established for the IMA formula (1-2). The actual steps towards required capital calculation are summarised below.

– Steps towards required capital calculation

[Step 1: Collect internal data]

Banks collect internal data on loss and exposure indicators.

[Step 2: Check the significance / meaningfulness of the collected data using the exposure indicator concerned]

(Case 1) If the data collected proves statistically significant, the bank can calculate the capital charge using only the loss data. [The observed EL is sufficient.]

Formula (1-2): Required Capital = λ * EL * (1+A/$\sqrt{n}$)

(Case 2) If the data collected proves statistically not significant or the data is not available in the first place, the bank must use external data on the exposure indicator concerned to calculate the capital charge. [The observed EL is not sufficient.]

In Case 2-1, EI is small, i.e. EL is not sufficient because the number of transactions in the past is not large enough or for other reasons. In this instance, neither PE nor LGE is significant. The capital charge should be set at the required capital amount calculated with the Formula (1-2), or the required capital amount based on the PE and the LGE both set at the average level of the global data, whichever is larger, being conservative.

The composition of the required capital based on the PE and the LGE both set at the average level of the global data is described as follows.

EI	PE	LGE	γ
EI	$PE_{(G)}$	$\mu_{L(G)}$	λ* (1+A)
EI	β_1		

(Suffix G denotes global data.)

Accordingly, the capital charge in such a case is written as β_1 * EI.

The general expression for the capital charge is therefore;

$$\text{Required capital} = \max [\lambda * EL * (1+A/\sqrt{n}), \beta_1 * EI] \qquad (6\text{-}1)$$

In Case 2-2, on the other hand, EI is large, i.e. the observed EL is not sufficient because PE is low although the number of transactions is reasonably large. In this instance, LGE is not significant. PE, which is close to zero, is not significant either. The capital charge should be set at the required capital amount calculated with the Formula (1-2), or the required capital amount based on the floor PE, i.e. the fixed minimum PE, and the LGE set at the average level of the global data, whichever is larger, being conservative.

The composition of the required capital amount based on the floor PE, i.e. the fixed minimum PE, and the LGE set at the average level of the global data is described as follows.

EI	PE	LGE	γ
EI	Floor $PE_{(G)}$	$\mu_{L(G)}$	λ* (1+A)
EI	β_2		

Accordingly, the capital charge in such a case is written as β_2 * EI.

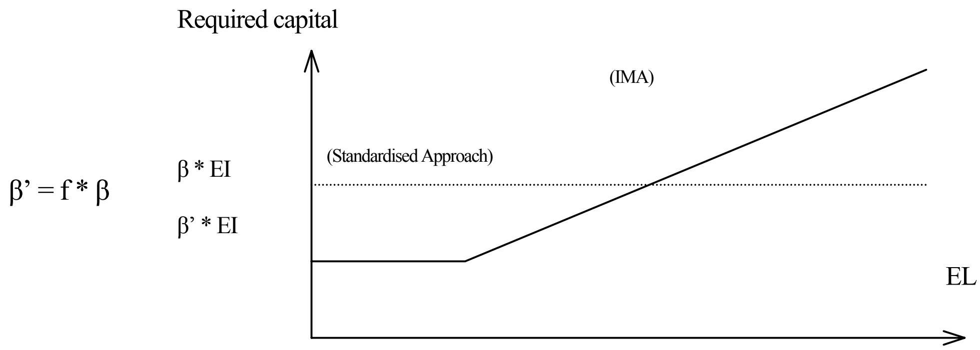
The general expression for the capital charge is therefore;

$$\text{Required capital} = \max [\lambda * EL * (1+A/\sqrt{n}), \beta_2 * EI] \qquad (6\text{-}2)$$

β_1 * EI and β_2 * EI can be interpreted in relation to the Standardised Approach under which EI is multiplied by certain factors. For the purpose of further simplification, formulae (6-1) and (6-2) can be combined by using a certain β'.

$$\text{Required capital} = \max [\lambda * EL * (1+A/\sqrt{n}), \beta' * GI]$$

In this formula, GI, the indicator under the Standardised Approach, is selected as EI. When β' = f * β is assumed (β is the multiplication factor in the Standardised Approach), f can be regarded as the floor for the IMA (in relation to the Standardised Approach).



6.2 Stability of EL

The IMA formula (1-2) is based on the EL. In actual application, it should be ensured that the observed EL does not fluctuate from year to year. Especially, when a loss is experienced, which is extremely large compared to the EL observed in the past, the EL will increase substantially, hence fluctuation of the required capital amount.

Mean is vulnerable to outliers. The method for calculating the average EL should therefore be robust or resistant enough to limit the influence of outliers. An example of easy solution is "trimmed mean".

"Trimmed mean" is a method for calculating a mean based on the data consisting only of the data points within a [1 – 2α]% range around the centre of the distribution. There are the following variations.

"Metric Trimming": Influence of outliers is removed by setting them at zero.
"(Metric) Winsorising": All the outliers are replaced with data points at [α]% or [1 –α]%.

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