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THE BALANCE SHEET CHANNEL

FEDERAL RESERVE BANK OF BOSTON™

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The Balance Sheet Channel*

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Abstract

In this paper, we study the role of the credit channel of monetary policy in the context of a DSGE model. Through the use of a regulated banking sector subject to a regulatory capital constraint on lending, we provide an alternative interpretations that can potentially explain differences in the implementation of monetary policy without appealing to ad-hoc central bank preferences. This is accomplished through the characterization of the external finance premium as a function of bank leverage and systemic aggregate risk.

JEL Classification: E52, E58, G18, G28

KEY WORDS: Financial Accelerator, Capital Adequacy Requirements, Monetary Policy Rules

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1 Introduction

We study the role of the balance sheet channel of monetary policy in an environment in which credit plays an important role in the funding of new capital investment. Specifically, we ask whether the transmission mechanism of monetary policy is altered in an environment in which financial intermediation with agency costs, aggregate risk on the performance of loans, and banking regulations are all features that can potentially amplify the impact of shocks over the cycle. Because monetary policy has empirically been asymmetric and marked by periods of pronounced action, our approach provides an alternative plausible mechanism that generates the necessary intuition to account for these patterns. Indeed our model is consistent with current New Neoclassical Synthesis models in 'good' times. In 'bad' times (crisis periods), when systemic losses are potentially large, the model can generate sharp changes in the external finance premium and, therefore, on the patterns of investment.

To illustrate these phenomenons we posit, from first principles, a model with financial intermediation as well as aggregate risk. We articulate a simple characterization of the link between policy and the real economy that passes through leveraged and regulated financial intermediaries to leveraged borrowers, and then use the model to explore the role of monetary policy and banking regulation. Our model can provide an intuitive, simple, and micro-founded explanation of the financial accelerator. We also show that basic features of banking regulation like deposit reserve requirements or capital adequacy requirements can amplify the cycle by adding to the costs that entrepreneurs have to pay to borrow from the financial system. Hence, that lends some validity to the argument that banking regulation can help mitigate the effect of crises.

The recent crisis has highlighted the fact that first-generation New Neoclassical Synthesis models are not well-equipped to interpret the role of monetary policy under financial stress. They were based on a couple of classic imperfections, such as nominal rigidities and monopolistic competition, to allow for non-trivial relative price distortions. The goal, of course, was to illustrate how demand shifts could impact real output, and thus how monetary policy shifting the nominal demand could have real effects. These models permitted an extensive literature that could study the basic role of monetary policy. The models, however, omitted details of market imperfections that are central to the questions that we explore here. We conjecture that this omissions may be partly responsible for the fact that consensus Taylor rules cannot describe the path of monetary policy (Rudebusch, 2006).

A new round of (second-generation) New Neoclassical Synthesis models focuses on the implications of other frictions. Because of the current financial crisis, a huge number of new papers, this one included, have turned their attention to the role of financial and credit market imperfections by building on work by Bernanke *et al.* (BGG) (1999) and Carlstrom and Fuerst (1997, 2001). In particular, there is renewed interest in a real economy link that passes through the banking sector. This channel is now widely believed to play an important role in the conduct of monetary policy. Our question is how and, specifically, how to model it.

We think that a successful model should be able to accomplish a few things. One, it should be able to characterize monetary policy in both 'normal' and 'crisis' periods. Two, it should do so without relying on *ad hoc* assumptions on the goals of monetary policy. Three, if indeed there is a financial channel or a banks' balance sheet channel, the model should provide an articulation of how this mechanism operates. Of course, the model should do so without sacrificing many of the gains of research to date in characterizing the path of other aggregates, or, crucially, parsimony.

How do we accomplish this? A combination of bank regulation and systemic risk allows us flexibility in a few important ways. First, the presence of systemic risk provides the framework to motivate state-contingent monetary policy that retains the structure of targeting output and inflation explicitly. Second, a fully described regulated banking sector allows us both to maintain the costly state verification (CSV) framework of Townsend (1979), Gale and Hellwig (1985) and BGG (1999) and to introduce the bank lending channel. This provides an answer to criteria one and three directly. Indeed, we can motivate changes in the pro-inflation response in crisis periods without resorting to *ad hoc* financial stability targets.

To produce the desired parsimony, we build a variant of the model of BGG (1999) that includes a regulated (but still competitive) banking sector and frictions on the secondary market for used capital. We take this generalization and identify a parsimonious characterization of the external financing premium that incorporates, intuitively, agency costs due to costly monitoring (costly state verification) as well as the costs of bank regulation on the balance sheet of the financial intermediaries. We show that the external finance premium (EFP) can be represented as follows,

$$EFP = f\left(Aggregate\ Shock,\ Agency\ Cost\ Channel,\ Balance\ Sheet\ Channel\right).$$

We think our approach is useful for a few reasons. One, it tries to reconcile the research agendas that look at stability targeting with those that want a pure monetary policy objective function. Two, it provides a simple and tractable mechanism to explain the financial channel that is consistent both with the banking literature that finds a link between monetary policy and the real economy and with the financial stability literature on the role of capital regulation for monetary policy.[1]

Importantly, our approach differs from existing work in a few ways. In one sense, it provides a tractable model via which regulation matters. Unlike models that generate financial channel effects through exogenous spread changes, our model gives an important role to banking intermediation precisely because of the trade-offs present in banking regulation and monetary policy and maintains the view that spreads are at least partly endogenous. In another sense, the model stands on its own also because it provides a simple way to think about financial intermediation via leverage and regulatory constraints.

The remainder of the paper is structured as follows. We fully describe the foundations of our model in section 2 and present our characterization of the external finance premium in sector 3. Moreover, it articulates the intuition of the model for monetary policy and banking regulation. It also discusses a couple of areas for future research, particularly with respect to our characterization of the stance of monetary policy and the banking sector. Section 4 concludes.

2 The Building Blocks of the Model

The financial system is hampered by asymmetries of information between borrowers and lenders and costly state verification, but it is also constrained by regulatory features like capital adequacy and deposit reserve

[1] The literature on this is wide ranging. Bernanke and Lown (1991) argue that the 1992 Basel 1 deadline contributed to the early 1990s credit crunch to others that suggest capital regulation generates magnified business cycles. Some relevant papers include: Berger and Udell (1994), Blum and Hellwig (1995), Brinkmann and Horvitz (1995), Thakor (1996). More recent papers include: Goodhard *et al.* (2004), Estrella (2004), Kashyap and Stein (2004), Gordy and Howells (2006). Borio and Zhu (2007) provides a comprehensive literature review.

requirements. The economy is populated by a continuum of households and entrepreneurs, each with unit mass. In addition, we include three types of non-financial firms – capital goods producers, wholesale producers, and retailers – and one type of financial institution – the banks –. All firms, whether financial or non-financial, operate under perfect competition, except for the retailers that exploit a monopoly power in their own varieties to add a retail mark-up on their prices. Ownership of all the firms is given to the households, except for wholesale producers who are owned and operated by the entrepreneurs.

The banks originate the loans and channel the savings of the households towards the investment needs of the entrepreneurs. The central bank, in turn, has the power to set both banking regulation as well as monetary policy. Monetary policy is characterized by an interest rate feedback rule in the tradition of Taylor (1993). Banking regulation is summarized in a compulsory reserve requirement ratio on deposits and a capital adequacy requirement on bank capital (or bank equity). The fiscal authority plays a mostly passive role.

In the financial accelerator model of BGG (1999), the relevant friction arises from asymmetric information between entrepreneurs-borrowers and banks-lenders. Monitoring costs make external financing costly for entrepreneurs and, therefore, the borrowers' balance sheet conditions play out an important role over the business cycle. Otherwise, banks act as a third party inserted between the households and the entrepreneurs whose mission is to intermediate the flow of savings towards investment. In other words, the balance sheet of the lenders that originate the loans becomes passive because loan supply must be equal to the bank deposits demanded by the households.

Our benchmark extends the BGG (1999) model to enhance the role of the banking balance sheet. In particular, we explore the role that banking regulation has on the banks' lending channel and its relevance for monetary policy. We also investigate the interaction between banking regulation and monetary policy. We fit, nonetheless, in the BGG (1999) tradition since the basic structure of banking relationships, intermediation, and contract loans is taken as given, rather than arising endogenously, and since we also maintain the illusion of a perfectly competitive banking system. Our model also shares an important characteristic with the framework of Kiyotaki and Moore (1997) in that asset price movements serve to reinforce credit market imperfections.

We depart from BGG (1999) because we note that banking regulation affects the decisions of banks and, therefore, alters the transmission mechanism in the financial accelerator model. We also depart from them because we introduce systemic (or aggregate) risk on capital income to help us analyze the interest rate spreads, the borrower-lender relationship and the business cycle dynamics in response to 'rare or unusual events' of large capital income losses.

2.1 Households

There is a continuum of household of unit mass. Households are infinitely-lived agents with an identical utility function which is additively separable in consumption, C_t, and labor, H_t, i.e.

$$\sum_{\tau=0}^{+\infty} \beta^{\tau} \mathbb{E}_t \left[\frac{1}{1-\sigma^{-1}} \left(C_{t+\tau}\right)^{1-\sigma^{-1}} - \frac{1}{1+\varphi^{-1}} \left(H_{t+\tau}\right)^{1+\varphi^{-1}} \right], \tag{1}$$

where $0 < \beta < 1$ is the subjective intertemporal discount factor, $\sigma > 0$ ($\sigma \neq 1$) is the elasticity of intertemporal substitution, and $\varphi > 0$ is the Frisch elasticity of labor supply. Households' income comes from renting non-managerial labor to the wholesale producers at competitive nominal wages, W_t. It also comes from the ownership of retailers and capital producers which rebate their total nominal profits (or losses) to them in every period, Π_t^r and Π_t^k respectively. The unanticipated profits of the banking system are also fully rebated in each period, Π_t^b. Households' also obtain their income from interests on their one-period nominal deposits in the banking system, D_t, and from yields on their stake on bank capital, B_{t+1}. With this disposable income, households finance their aggregate consumption, C_t, open new deposits, D_{t+1}, buy new bank shares, B_{t+1}, and pay their nominal (lump-sum) tax bill, T_t.

Accordingly, the households' sequence of budget constraints is described by,

$$P_t C_t + T_t + D_{t+1} + B_{t+1} \leq W_t H_t + I_t D_t + \left(1 - \iota^h\right) R_t^b B_t + \Pi_t^r + \Pi_t^k + \Pi_t^b, \tag{2}$$

where I_t is the nominal short-term interest rate offered to the depositors, R_t^b is the yield on bank capital, and P_t is the consumption price index (CPI). The nominal tax on bank equity, ι^h, is a convenient simplification to capture the differential tax treatment of capital gains from equity holdings and deposits in many tax codes around the world. As a matter of convention, D_{t+1} and B_{t+1} denote nominal deposits and bank equity held from time t to $t+1$. Therefore, the interest rate I_{t+1} paid at $t+1$ is known and determined at time t, but the yield on bank equity R_{t+1}^b could potentially depend on the state of the world at time $t+1$. Household optimization yields the standard first-order conditions for consumption-savings and labor supply,

$$\frac{1}{I_{t+1}} = \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma^{-1}} \frac{P_t}{P_{t+1}} \right], \tag{3}$$

$$1 = \beta \mathbb{E}_t \left[\left(1 - \iota^h\right) R_{t+1}^b \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma^{-1}} \frac{P_t}{P_{t+1}} \right], \tag{4}$$

$$\frac{W_t}{P_t} = \left(C_t\right)^{\sigma^{-1}} \left(H_t\right)^{\varphi^{-1}}, \tag{5}$$

plus the appropriate no-Ponzi, transversality condition. It also implies that each period budget constraint holds with equality.

As we shall see later, the problem of the banks is such that the yield on bank capital is also known and determined at time t. Therefore, by simple arbitrage between (3) and (4), it follows that $\left(1 - \iota^h\right) R_{t+1}^b = I_{t+1}$ is necessary for an interior solution to exist (where households hold both bank deposits and bank equity).

2.2 Retailers

There is a continuum of retail firms of unit mass. The retail sector transforms wholesale output into differentiated goods using a linear technology. For simplicity, we assume that no capital or labor is needed in the retail sector, so the wholesale good is the only input of production. Each retail variety is then sold to households, entrepreneurs and capital goods producers, and bundled up for either consumption or investment (only capital goods producers acquire these varieties for investment purposes.) The retailers add a 'brand' name to the wholesale good to introduce differentiation. Variety is valued by all potential costumers, consequently retailers gain monopolistic power to charge a retail mark-up on them.

Aggregation We denote the differentiated varieties as $Y_t(z)$, where the index $z \in [0,1]$ identifies each individual retailer. Final goods used for consumption and investment, Y_t, are bundles of these differentiated varieties, $Y_t(z)$, aggregated by means of a common CES index as follows,

$$Y_t = \left[\int_0^1 Y_t(z)^{\frac{\theta-1}{\theta}} dz\right]^{\frac{\theta}{\theta-1}}. \tag{6}$$

The elasticity of substitution across varieties is represented by $\theta > 1$. The corresponding consumption price index (CPI) is given by,

$$P_t = \left[\int_0^1 P_t(z)^{1-\theta} dz\right]^{\frac{1}{1-\theta}}, \tag{7}$$

where $P_t(z)$ is the price charged by retailer z for its variety. The optimal allocation of expenditure to each variety, i.e.

$$Y_t(z) = \left(\frac{P_t(z)}{P_t}\right)^{-\theta} Y_t, \tag{8}$$

implies that retailers face a downward-sloping demand function.

Optimal Pricing Retailers set prices to maximize profits, but their ability to re-optimize is constrained because they face nominal rigidities à la Calvo (1983). The retailer maintains its previous period price with an exogenous probability $0 < \alpha < 1$ in each period. However, with probability $(1-\alpha)$, the retailer is allowed to optimally reset its price. Whenever re-optimization is possible, a retailer z chooses its price, $\widetilde{P}_t(z)$, to maximize the expected discounted value of its net nominal profits, i.e.

$$\sum\nolimits_{\tau=0}^{+\infty} \mathbb{E}_t\left[\alpha^\tau M_{t,t+\tau} \widetilde{Y}_{t,t+\tau}(z)\left(\widetilde{P}_t(z) - (1-\iota^r) P^w_{t+\tau}\right)\right], \tag{9}$$

where $M_{t,t+\tau} \equiv \beta^\tau \left(\frac{C_{t+\tau}}{C_t}\right)^{-\sigma^{-1}} \frac{P_t}{P_{t+\tau}}$ is the household's stochastic discount factor (SDF) for τ-periods ahead nominal payoffs, $P^w_{t+\tau}$ is the nominal price of wholesale goods, and $\widetilde{Y}_{t,t+\tau}(z) = \left(\frac{\widetilde{P}_t(z)}{P_{t+\tau}}\right)^{-\theta} Y_{t+\tau}$ is the demand at time $t+\tau$ given that prices remain fixed at $\widetilde{P}_t(z)$ (see equation (8)). We also include a subsidy on inputs for retailers, ι^r, which is used by the government to eliminate the retail mark-up distortion whenever $\iota^r = \frac{1}{\theta}$.

The solution to the retailer's maximization problem satisfies the following first-order condition,

$$\sum\nolimits_{\tau=0}^{+\infty} \mathbb{E}_t\left[(\alpha\beta)^\tau \left(\frac{C_{t+\tau}}{C_t}\right)^{-\sigma^{-1}} \widetilde{Y}_{t,t+\tau}(z)\left(\frac{\widetilde{P}_t(z)}{P_{t+\tau}} - \frac{\theta(1-\iota^r)}{\theta-1}\frac{P^w_{t+\tau}}{P_{t+\tau}}\right)\right] = 0, \tag{10}$$

where $\frac{\theta}{\theta-1}$ denotes the retail mark-up, and $\frac{P^w_t}{P_t}$ denotes the price of wholesale output in units of consumption. The latter provides a measure for the real marginal costs before the government subsidy. The first-order condition in (10) is often referred to as the price-setting rule. Given that a fraction α of retailers maintains prices in period t, and that all re-optimizing retailers face a symmetric problem, the aggregate CPI in (7) can be re-written in the following terms,

$$P_t = \left[\alpha P_{t-1}^{1-\theta} + (1-\alpha)\widetilde{P}_t(z)^{1-\theta}\right]^{\frac{1}{1-\theta}}, \tag{11}$$

where $\widetilde{P}_t(z)$ is the (symmetric) optimal price implied by equation (10).

Technically, there is no 'aggregate production function' for the final output, Y_t. However, there is a simple way to account for the distribution of resources. By market clearing, the sum of the individual retailers demands of the wholesale good has to be equal to the total production of the wholesale producers, i.e.

$$\int_0^1 Y_t(z)\,dz = Y_t^w. \tag{12}$$

Using the optimal allocation of expenditure in (8) we get that,

$$Y_t = \left(\frac{P_t^*}{P_t}\right)^{\theta} Y_t^w, \tag{13}$$

where,

$$P_t^* \equiv \left[\int_0^1 P_t(z)^{-\theta}\,dz\right]^{-\frac{1}{\theta}} = \left[\alpha\left(P_{t-1}^*\right)^{-\theta} + (1-\alpha)\,\widetilde{P}_t(z)^{-\theta}\right]^{-\frac{1}{\theta}}. \tag{14}$$

The term $p_t^* \equiv \left(\frac{P_t^*}{P_t}\right)^{\theta} \leq 1$ characterizes the magnitude of the efficiency distortion due to sticky prices.

Since households own the retailers, we assume that all profits (or losses) from the retail activity are rebated lump-sum to the households in every period. After a bit of algebra, the aggregate nominal profits received by the households can be computed as,

$$\begin{aligned} \Pi_t^r &\equiv \int_0^1 \left[Y_t(z)\left(P_t(z) - (1-\iota^r)\,P_t^w\right)dz\right] \\ &= P_t\left(\frac{P_t^*}{P_t}\right)^{\theta} Y_t^w - (1-\iota^r)\,P_t^w Y_t^w, \end{aligned} \tag{15}$$

where the second equality follows from the optimal allocation of expenditure in each variety described in (8), the aggregation formulas in (6) – (7), and the relationship between final output and wholesale output implied by (12).

2.3 Capital Goods Producers

There is a continuum of capital goods producers of unit mass. At time t, these producers combine aggregate investment goods, X_t, and depreciated capital, $(1-\delta)\,K_t$, to manufacture new capital goods, K_{t+1}. The production of new capital is limited by technological constraints. We assume that the aggregate stock of new capital evolves according to the following law of motion,

$$K_{t+1} \leq (1-\delta)\,K_t + \Phi\left(X_t, X_{t-1}, K_t\right) X_t, \tag{16}$$

where X_t is real aggregate investment, K_t stands for real aggregate capital, and $0 < \delta < 1$ is the depreciation rate. The function $\Phi\left(X_t, X_{t-1}, K_t\right)$ implicitly characterizes the technology available to the capital goods producers to transform investment goods into new capital.

We explore three different specifications of the technological constraint. The neoclassical case (NAC) assumes that the transformation of investment goods into new capital can be attained at a one-to-one rate, i.e.

$$\Phi\left(X_t, X_{t-1}, K_t\right) = 1. \tag{17}$$

The so-called capital adjustment (CAC) specification, favored *inter alia* by BGG (1999), takes the following form,

$$\Phi\left(\frac{X_t}{K_t}\right) = 1 - \frac{1}{2}\chi\frac{\left(\frac{X_t}{K_t} - \delta\right)^2}{\frac{X_t}{K_t}}, \tag{18}$$

where $\frac{X_t}{K_t}$ denotes the investment-to-capital ratio. And, finally, the investment adjustment (IAC) specification, preferred by Christiano *et al.* (2005), takes the following form,

$$\Phi\left(\frac{X_t}{X_{t-1}}\right) = 1 - \frac{1}{2}\kappa\frac{\left(\frac{X_t}{X_{t-1}} - 1\right)^2}{\frac{X_t}{X_{t-1}}}, \tag{19}$$

where $\frac{X_t}{X_{t-1}}$ denotes the gross investment growth rate. The parameters $\chi > 0$ and $\kappa > 0$ regulate the degree of concavity of the technological constraint and, therefore, the sensitivity of investment in new capital. In steady state, the CAC function satisfies that $\Phi\left(\delta\right) = 1$, $\Phi'\left(\delta\right) = 0$, and $\Phi''\left(\delta\right) = -\frac{\chi}{\delta} < 0$. Similarly, the IAC function satisfies that $\Phi\left(1\right) = 1$, $\Phi'\left(1\right) = 0$, and $\Phi''\left(1\right) = -\kappa < 0$.

Capital goods producers choose their investment demand, X_t, and their output of new capital, K_{t+1}, to maximize the expected discounted value of their net profits, i.e.

$$\sum\nolimits_{\tau=0}^{+\infty} \mathbb{E}_t\left[M_{t,t+\tau}P_{t+\tau}\left(Q_{t+\tau}K_{t+\tau+1} - \left(1-\delta\right)\overline{Q}_{t+\tau}K_{t+\tau} - X_{t+\tau}\right)\right], \tag{20}$$

subject to the law of motion for capital described in (16). Here, $M_{t,t+\tau} \equiv \beta^{\tau}\left(\frac{C_{t+\tau}}{C_t}\right)^{-\sigma^{-1}}\frac{P_t}{P_{t+\tau}}$ is the household's stochastic discount factor (SDF) for τ-periods ahead nominal payoffs, since households own the capital goods producers. As a matter of convention, K_{t+1} denotes the real stock of capital built (and determined) at time t for use at time $t+1$.

The investment good is bundled in the same fashion as the consumption good and is bought at the same price, P_t. The depreciated capital is bought at a resale price $\overline{Q}_t$ in units of the consumption good. However, the new capital is sold to the entrepreneurs at a price Q_t, which determines the relative cost of investment in units of consumption and is often referred to as Tobin's Q. We assume that frictions in the secondary market for used capital prevent arbitrage between the resale value of old capital and the sale value of new capital, i.e. $\overline{Q}_t = o_t Q_t$ where $o_t \neq 1$. Those frictions are left unmodelled, however we also assume that the parties involved in the secondary market (entrepreneurs and capital goods producers) view them as entirely out of their control. Hence, they treat the wedge o_t as an exogenous and random shock.

Moreover, there is no centralized market that ensures a uniform pricing for used capital, so each individual entrepreneur and capital producer pair matched in the secondary market gets a different draw of this random wedge. In other words, o_t is modelled not as an aggregate shock, but as an idiosyncratic one. Nonetheless, we map this resale shock into BGG (1999)'s framework as closely as possible. That keeps our departure from their model to a minimum, but requires us to note that the wedge o_t has also a component that depends on other endogenous variables that have an influence on the capital returns that the entrepreneurs can generate.

The optimization of the capital goods producers yields a standard first-order condition that determines the linkage between Tobin's Q, Q_t, and investment, X_t, i.e.

$$Q_t \left[\Phi \left(X_t, X_{t-1}, K_t \right) + \frac{\partial \Phi \left(X_t, X_{t-1}, K_t \right)}{\partial X_t} X_t \right] + \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma^{-1}} Q_{t+1} \frac{\partial \Phi \left(X_{t+1}, X_t, K_{t+1} \right)}{\partial X_t} X_{t+1} \right] = 1, \tag{21}$$

which does not depend on the wedge o_t. The law of motion for capital is binding in each period. Given our alternative specifications of the technological constraint, we could re-write the first-order condition in (21) more compactly as,

$$\begin{cases} Q_t = 1, \text{ if NAC}, \\ Q_t \left[\Phi \left(\frac{X_t}{K_t} \right) + \Phi' \left(\frac{X_t}{K_t} \right) \frac{X_t}{K_t} \right] = 1, \text{ if CAC}, \\ Q_t \left[\Phi \left(\frac{X_t}{X_{t-1}} \right) + \Phi' \left(\frac{X_t}{X_{t-1}} \right) \frac{X_t}{X_{t-1}} \right] = 1 + \beta \mathbb{E}_t \left[\left(\frac{C_{t+1}}{C_t} \right)^{-\sigma^{-1}} Q_{t+1} \Phi' \left(\frac{X_{t+1}}{X_t} \right) \left(\frac{X_{t+1}}{X_t} \right)^2 \right], \text{ if IAC}. \end{cases} \tag{22}$$

The neoclassical (NAC) case is of particular interest because without the asset price fluctuations captured by Tobin's Q, the BGG (1999) framework loses the characteristic that asset price movements serve to reinforce credit market imperfections. For more details on the derivations of the Tobin's Q equations, see Martínez-García and Søndergaard (2008).

Profits (or losses) may arise since X_{t-1} and K_t are pre-determined at time t and cannot be adjusted freely. The aggregate profits at each point in time for the capital goods producers, i.e.

$$\begin{aligned} \Pi_t^k &\equiv P_t Q_t K_{t+1} - (1-\delta) P_t \left[\int_0^1 \overline{Q}_t \mu_t^o (o_t)\, do_t \right] K_t - P_t X_t \\ &= P_t Q_t \Phi \left(X_t, X_{t-1}, K_t \right) X_t - \left(\int_0^1 o_t \mu_t^o (o_t)\, do_t - 1 \right) (1-\delta) P_t Q_t K_t - P_t X_t, \end{aligned} \tag{23}$$

must be added to the budget constraint of the households (since households are their only shareholders.) Here, $\mu_t^o (o_t)$ denotes the mass of capital goods producers receiving a given realization of the idiosyncratic shock o_t.

2.4 Wholesale Producers

There is a continuum of mass one of wholesale producers. Wholesale producers combine the non-managerial labor provided by the households with the managerial labor supplied and the capital rented from the entrepreneurs to produce wholesale goods according to the following Cobb-Douglas technology, i.e.

$$Y_t^w \leq e^{a_t} (K_t)^{1-\psi-\varrho} (H_t)^{\psi} (H_t^e)^{\varrho}, \tag{24}$$

where Y_t^w is the output of wholesale goods, K_t is the aggregate capital rented, and H_t and H_t^e are the demands for non-managerial and managerial labor respectively.

With a constant returns-to-scale technology, the non-managerial and managerial labor shares in the production function are determined by the coefficients $0 < \psi < 1$ and $0 < \varrho < 1$. In keeping with BGG (1999), the managerial share is often assumed to be very small, i.e. ϱ would be close to zero. The productivity shock, a_t, follows an $AR(1)$ process of the following form,

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a, \tag{25}$$

where ε_t^a is a zero mean, uncorrelated and normally-distributed innovation. The parameter $-1 < \rho_a < 1$ determines the persistence of the productivity shock, and $\sigma_a^2 > 0$ the volatility of its innovation.

Wholesale producers maximize their static profit, i.e.

$$\Pi_t^w \equiv P_t^w Y_t^w - R_t^w K_t - W_t H_t - W_t^e H_t^e, \tag{26}$$

subject to the technological constraint implied by (24). Wholesale producers rent labor from households and entrepreneurs at competitive nominal wages W_t and W_t^e, respectively, and compensate the entrepreneurs with a nominal return per unit of capital rented, R_t^w. The optimization of the wholesale producers results in the following well-known rules to compensate the factors of production, i.e.

$$R_t^w = (1 - \psi - \varrho) \frac{P_t^w Y_t^w}{K_t}, \tag{27}$$

$$W_t = \psi \frac{P_t^w Y_t^w}{H_t}, \tag{28}$$

$$W_t^e = \varrho \frac{P_t^w Y_t^w}{H_t^e}. \tag{29}$$

The optimization of the wholesale producer can be summarized in these first-order conditions plus the technological constraint in (24) holding with equality. Wholesale producers make zero profits in every period (i.e., $\Pi_t^w = 0$), therefore the entrepreneurs who own them do not receive any dividends. All the income entrepreneurs extract comes from their supply of two key inputs in the production function, managerial labor and specially capital. Wholesale producers rent the capital they use from the entrepreneurs and return the depreciated capital after production has taken place.

As we shall see shortly, uncertainty about the resale value of depreciated capital is the underlying risk that distorts the relationship between borrowers (the entrepreneurs) and lenders (the banks). In fact, asymmetries of information on this type of risk and costly state verification lead to a distorted allocation of households' savings towards the productive capital investments operated by the entrepreneurs.

2.5 Entrepreneurs

There is a continuum of entrepreneurs of unit mass. Entrepreneurs are infinitely-lived agents with identical preferences which are linear in consumption, C_t^e, i.e.

$$\sum\nolimits_{\tau=0}^{\infty} (\beta\eta)^{\tau} \mathbb{E}_t \left[C_{t+\tau}^e \right], \tag{30}$$

where $0 < \beta\eta < 1$ is the subjective intertemporal discount factor. Entrepreneurs inelastically supply one unit of managerial labor, i.e.

$$H_t^e = 1, \ \forall t. \tag{31}$$

The entrepreneurs' utility function also differs from that of the households because they are risk-neutral (linear utility), and they discount utility at a higher rate (i.e., $0 < \eta < 1$). The relative impatience is intended to insure that entrepreneurs never save enough resources to overcome their financing constraints. The assumption of risk-neutrality implies that entrepreneurs care only about expected returns and, therefore, considerably simplifies the financial contract which we discuss in the Appendix.

At the end of period t, the entrepreneur receives a competitive nominal wage, W_t^e, and earns income from the capital rented at the beginning of the period for the production of wholesale goods, $R_t^w K_t$, as well as from the resale value on the depreciated capital bought by the capital goods producers, $(1-\delta) P_t \overline{Q}_t K_t$.[2] After repaying the outstanding loans to the banking system, L_t, entrepreneurs can appropriate a fraction of the aggregate capital income, i.e. a share of $R_t^w K_t + (1-\delta) P_t \overline{Q}_t K_t$. The entrepreneurs own the wholesale producers, but these firms generate zero profits after paying for the factors of production and, therefore, produce no dividends for the entrepreneurs.

Using the resources coming from managerial wages and capital rental rates, the entrepreneurs must buy the new capital, K_{t+1}, and decide how much to consume, C_t^e. New capital is needed for the production of wholesale goods at time $t+1$. Net of consumption, the entrepreneurs set aside a portion of their income in the form of entrepreneurial net worth, N_{t+1}. Entrepreneurial net worth is, in effect, a form of savings for the entrepreneur that can be applied partly to acquire new capital. The entrepreneurs use these savings, N_{t+1}, as well as external loans from the banking system, L_{t+1}, to fund the acquisition of the entire stock of new capital, $P_t Q_t K_{t+1}$, i.e.

$$P_t Q_t K_{t+1} = N_{t+1} + L_{t+1}. \tag{32}$$

Equation (32) also tells us that new capital is the only asset in which entrepreneurs can invest their savings. As in BGG (1999), we rule out a more complex portfolio setting for entrepreneurs.

Idiosyncratic and Anticipated Systematic Risk. We define the returns on capital relative to its acquisition cost whenever the resale value of capital and the cost of new capital are equalized as $R_t^e \equiv \frac{R_t^w K_t + (1-\delta) P_t Q_t K_t}{P_{t-1} Q_{t-1} K_t}$. For an individual entrepreneur, we define the returns on the capital that was acquired at time $t-1$, $\omega_t R_t^e$, as the total income generated by a unit of capital at time t after accounting for the effects of the distortion in the secondary market,[3]

$$\omega_t R_t^e \equiv \frac{R_t^w K_t + (1-\delta) P_t \overline{Q}_t K_t}{P_{t-1} Q_{t-1} K_t} = \left(\frac{\frac{R_t^w}{P_t} + (1-\delta) o_t Q_t}{Q_{t-1}} \right) \frac{P_t}{P_{t-1}}, \tag{33}$$

where the rental rate on capital, R_t^w, is defined in equation (27). Returns on capital are subject to idiosyncratic shocks, ω_t, which reflect the impact of the random resale distortion, $o_t \equiv O\left(\omega_t; \frac{R_t^w}{P_t Q_t}\right)$. The function

[2] Distortions in the secondary market create a random wedge between the acquisition cost of new capital and the resale value of old capital in each period.

[3] To be more precise, we define the rate of return on capital, R_t^e, as the rate that would prevail if the secondary market for used or depreciated capital led to arbitrage between the resale value of capital and the cost of acquiring new capital, i.e. $\overline{Q}_t = Q_t$. The returns of capital are realized under distortions in the secondary market, so the actual rate of return on capital is $\omega_t R_t^e$ as defined in equation (33). For convenience, we implicitly capture the randomness of the wedge in the resale value, o_t, by positing that ω_t is the purely exogenous random variable.

that links the wedge on the secondary market, o_t, to the idiosyncratic shock, ω_t, can be expressed as,

$$\begin{aligned} o_t &\equiv O\left(\omega_t; \frac{R_t^w}{P_t Q_t}\right) = \frac{\omega_t R_t^e P_{t-1} Q_{t-1} K_t - R_t^w K_t}{(1-\delta) P_t Q_t K_t} \\ &= \omega_t + (\omega_t - 1) \frac{R_t^w}{(1-\delta) P_t Q_t}, \end{aligned} \tag{34}$$

where the second equality follows from the definition of R_t^e.

We interpret the shock $\omega_{t+1} \in (0, +\infty)$ as a reduced form representation for the exogenous losses on the resale value of the depreciated capital due to frictions in the secondary market. Those frictions, that are left unmodelled, imply a wedge between the resale value of capital and the acquisition cost of new capital (or Tobin's Q) within the period. We denote $\phi(\omega_{t+1} \mid s_{t+1})$ the density and $\Phi(\omega_{t+1} \mid s_{t+1})$ the cumulative distribution of ω_{t+1} conditional on a given realization of the aggregate shock s_{t+1}.

We assume that the expected capital return of each entrepreneur is a function of the aggregate shock s_{t+1} (e.g., Faia and Monacelli, 2007). The aggregate shock s_{t+1} captures our notion of systemic risk on the resale value of depreciated capital, which has the effect of shifting the mean of the distribution of the risky capital returns. The systemic risk shock, s_t, follows an $AR(1)$ process of the following form,

$$s_t = \rho_s s_{t-1} + \varepsilon_t^s, \tag{35}$$

where ε_t^s is a zero mean, uncorrelated and normally-distributed innovation. The parameter $-1 < \rho_s < 1$ determines the persistence of the systemic shock, and $\sigma_s^2 > 0$ the volatility of its innovation. We assume that the realization of the time $t+1$ shock is publicly observed at time t. Therefore, these systemic shocks are interpreted as anticipated (rather than unanticipated) losses.

The expected idiosyncratic shock on capital income, ω_{t+1}, conditional on the realization of the aggregate shock, s_{t+1}, is given by,

$$\mathbb{E}[\omega_{t+1} \mid s_{t+1}] = 1 - J(s_{t+1}), \tag{36}$$

where $0 \leq \lambda \equiv J(0) < 1$ determines the level of the expected losses in steady state, and $-\infty < \xi \equiv J'(0) < +\infty$ characterizes the sensitivity of the expected losses. This specification is flexible enough to allow for catastrophic losses due to a sizable systemic risk shock, s_{t+1}. By choosing λ sufficiently close to zero, we ensure that the expected idiosyncratic shock remains relatively close to one most of the time, i.e. $\mathbb{E}[\omega_{t+1} \mid s_{t+1}] \simeq 1$. That means entrepreneurs get on average a capital return that is approximately equal to R_t^e, which is what is expected whenever the acquisition cost and the resale value of capital are equalized within each period.[4]

[4] Given the characterization of the idiosyncratic shock ω_t in (33) and the definition of the capital return under equalization between the resale value of capital and the acquisition cost, R_t^e, we can argue that the expected or average value of depreciated capital is equal to,

$$\begin{aligned} (1-\delta) P_t Q_t K_t \left[\int_0^1 o_t \mu_t^o(o_t)\, do_t\right] &= \left[\int_0^{+\infty} \omega_t \phi(\omega_t \mid s_t)\, d\omega_t - 1\right] R_t^w K_t + \left[\int_0^{+\infty} \omega_t \phi(\omega_t \mid s_t)\, d\omega_t\right] (1-\delta) P_t Q_t K_t \\ &= (1-\delta) P_t Q_t K_t - J(s_t)\left[R_t^w K_t + (1-\delta) P_t Q_t K_t\right] \\ &= (1-\delta) P_t Q_t K_t - P_{t-1} Q_{t-1} K_t R_t^e J(s_t), \end{aligned}$$

where we use the fact $1 - J(s_t)$ is the expectation of ω_t. Given this, we can rewrite the aggregate profits for the capital goods

The Loan Contract. At time t, the entrepreneurs-borrowers and the banks-lenders must agree on a contract that facilitates the acquisition of new capital, K_{t+1}, and that has to be repaid at time $t+1$. The entrepreneurs operate in a legal environment that ensures them limited liability. Hence, in case of default at time $t+1$, the banks can only appropriate the total capital returns of the entrepreneur at that time, i.e. $\omega_{t+1} R^e_{t+1} P_t Q_t K_{t+1}$. The loan is restricted to take the standard form of a one-period risky debt contract as in Townsend (1979), Gale and Hellwig (1985) and BGG (1999).[5]

It is assumed that the idiosyncratic shock ω_{t+1} is not known at time t when the loan contract is signed, and that the realization of the idiosyncratic shock can only be observed privately by the entrepreneur himself at time $t+1$. Banks, however, observe the systemic shock s_{t+1} at time t and have access to a costly monitoring technology that permits them to uncover the true realization of the idiosyncratic shock ω_{t+1} at a cost, i.e. at a cost of $\mu\omega_{t+1} R^e_{t+1} P_t Q_t K_{t+1}$ where $0 < \mu < 1$.

Default on a loan signed at time t occurs whenever the capital returns obtained by the entrepreneur at time $t+1$ after the realization of the idiosyncratic shock ω_{t+1}, i.e. $\omega_{t+1} R^e_{t+1} P_t Q_t K_{t+1}$, fall short of the amount that needs to be repaid. Hence the default space is implicitly characterized by,

$$\omega_{t+1} R^e_{t+1} P_t Q_t K_{t+1} \leq I^l_{t+1} L_{t+1}, \tag{37}$$

where I^l_{t+1} is short-hand notation for the repayment amount agreed at time t per unit of loan, and L_{t+1} represents the loan size. A risky one-period loan contract at time t can be defined in terms of a threshold on the idiosyncratic shock, $\overline{\omega}_t$, and a measure of capital returns, $R^e_{t+1} P_t Q_t K_{t+1}$, such that the repayment is equal to,

$$I^l_{t+1} L_{t+1} = \overline{\omega}_{t+1} R^e_{t+1} P_t Q_t K_{t+1}. \tag{38}$$

Given the terms of the loan contract, the lenders will commit to supply as much external funding as the entrepreneurs choose to demand under those conditions. Another way to interpret the implication of equations (37) and (38) is that making a loan to the entrepreneurs entitles the lenders to share on their capital returns.

When default occurs, i.e. when $\omega_t < \overline{\omega}_t$, is because the entrepreneur cannot repay the amount it owns based on the capital returns that he has derived from his investment. To avoid misreporting on the part of the defaulting entrepreneur, the lender must verify the individual entrepreneur's income statement. That requires the lender to expend resources by an amount of $\mu\omega_{t+1} R^e_{t+1} P_t Q_t K_{t+1}$ in monitoring costs. In case of default, the lender always chooses to monitor and the entrepreneur gets nothing, while the bank appropriates $(1-\mu)\,\omega_{t+1} R^e_{t+1} P_t Q_t K_{t+1}$ for itself. If the entrepreneur does not default, i.e. if $\omega_t \geq \overline{\omega}_t$, then the entrepreneur pays $\overline{\omega}_{t+1} R^e_{t+1} P_t Q_t K_{t+1}$ back to the lender and keeps the rest for himself. In other words, the entrepreneur gets to keep $(\omega_{t+1} - \overline{\omega}_{t+1})\, R^e_{t+1} P_t Q_t K_{t+1}$.

producers in equation (23) as,

$$\begin{aligned} \Pi^k_t &\equiv P_t Q_t K_{t+1} - (1-\delta)\, P_t Q_t K_t - P_t X_t + (1-\delta)\, P_t Q_t K_t \left[1 - \int_0^1 o_t \mu^o_t\,(o_t)\, do_t\right] \\ &= P_t Q_t K_{t+1} - (1-\delta)\, P_t Q_t K_t - P_t X_t + P_{t-1} Q_{t-1} K_t R^e_t J\,(s_t)\,. \end{aligned}$$

We can see from this aggregate profit function that what we call systemic losses for the entrepreneur are additional profits for the capital goods producers.

[5] For a discussion of optimal contracts in a dynamic costly state verification framework, see Monnet and Quintin (2005).

We take this defaulting rule and the implied sharing agreement of capital returns between the entrepreneur-borrower and the bank-lender as given. At time $t+1$, the capital returns net of borrowing costs expected by the entrepreneur after observing all aggregate shocks,[6] but before the realization of its own idiosyncratic shock ω_{t+1}, can be computed as,

$$\begin{aligned}
& \int_{\overline{\omega}_{t+1}}^{+\infty} \left[\omega_{t+1} R^e_{t+1} P_t Q_t K_{t+1} - I^l_{t+1} L_{t+1}\right] \phi\left(\omega_{t+1} \mid s_{t+1}\right) d\omega_{t+1} \\
= \; & R^e_{t+1} P_t Q_t K_{t+1} \left[\int_{\overline{\omega}_{t+1}}^{+\infty} \left(\omega_{t+1} - \overline{\omega}_{t+1}\right) \phi\left(\omega_{t+1} \mid s_{t+1}\right) d\omega_{t+1}\right] \\
= \; & R^e_{t+1} P_t Q_t K_{t+1} f\left(\overline{\omega}_{t+1}, s_{t+1}\right), \qquad (39)
\end{aligned}$$

where,

$$f\left(\overline{\omega}_{t+1}, s_{t+1}\right) \equiv \int_{\overline{\omega}_{t+1}}^{+\infty} \omega_{t+1} \phi\left(\omega_{t+1} \mid s_{t+1}\right) d\omega_{t+1} - \overline{\omega}_{t+1}\left(1 - \Phi\left(\overline{\omega}_{t+1} \mid s_{t+1}\right)\right). \qquad (40)$$

By the law of large numbers, (40) can be interpreted also as the fraction of the expected capital return obtained by the average entrepreneur. In a similar fashion, the capital returns net of monitoring costs expected by the lenders after observing all aggregate shocks at time $t+1$ would be equal to,

$$\begin{aligned}
& (1-\mu) \int_0^{\overline{\omega}_{t+1}} \left[\omega_{t+1} R^e_{t+1} P_t Q_t K_{t+1}\right] \phi\left(\omega_{t+1} \mid s_{t+1}\right) d\omega_{t+1} + \int_{\overline{\omega}_{t+1}}^{+\infty} \left[I^l_{t+1} L_{t+1}\right] \phi\left(\omega_{t+1} \mid s_{t+1}\right) d\omega_{t+1} \\
= \; & R^e_{t+1} P_t Q_t K_{t+1} \left[(1-\mu) \int_0^{\overline{\omega}_{t+1}} \omega_{t+1} \phi\left(\omega_{t+1} \mid s_{t+1}\right) d\omega_{t+1} + \overline{\omega}_{t+1} \int_{\overline{\omega}_{t+1}}^{+\infty} \phi\left(\omega_{t+1} \mid s_{t+1}\right) d\omega_{t+1}\right] \\
= \; & R^e_{t+1} P_t Q_t K_{t+1} g\left(\overline{\omega}_{t+1}, s_{t+1}\right), \qquad (41)
\end{aligned}$$

where,

$$g\left(\overline{\omega}_{t+1}, s_{t+1}\right) \equiv (1-\mu) \int_0^{\overline{\omega}_{t+1}} \omega_{t+1} \phi\left(\omega_{t+1} \mid s_{t+1}\right) d\omega_{t+1} + \overline{\omega}_{t+1}\left(1 - \Phi\left(\overline{\omega}_{t+1} \mid s_{t+1}\right)\right). \qquad (42)$$

By the law of large numbers, (42) can be interpreted as the fraction of the expected capital returns that accrues to the average lender.

As explained in the Appendix, the formal contracting problem reduces to choosing the quantity of physical capital, K_{t+1}, and the threshold, $\overline{\omega}_{t+1}$, that maximize the entrepreneurs' expected nominal return on capital net of the loan costs (see equation (39)), i.e.

$$P_t Q_t K_{t+1} \mathbb{E}_t \left[R^e_{t+1}\right] \left(1 - J\left(s_{t+1}\right) - \Gamma\left(\overline{\omega}_{t+1}, s_{t+1}\right)\right), \qquad (43)$$

subject to the participation constraint for the lenders (see equation (41)), i.e.

$$P_t Q_t K_{t+1} \mathbb{E}_t \left[R^e_{t+1}\right] \left(\Gamma\left(\overline{\omega}_{t+1}, s_{t+1}\right) - \mu G\left(\overline{\omega}_{t+1}, s_{t+1}\right)\right) \geq I^b_{t+1} L_{t+1} = I^b_{t+1}\left[P_t Q_t K_{t+1} - N_{t+1}\right]. \qquad (44)$$

[6] Here, aggregate shocks includes the productivity shock, a_{t+1}, the monetary shock, m_{t+1}, and the systemic risk shock, s_{t+1}.

We write the share of capital returns going to the entrepreneurs as,

$$f\left(\overline{\omega}_{t+1}, s_{t+1}\right) = 1 - J\left(s_{t+1}\right) - \Gamma\left(\overline{\omega}_{t+1}, s_{t+1}\right). \tag{45}$$

and the share going to the lenders as,

$$g\left(\overline{\omega}_{t+1}, s_{t+1}\right) = \Gamma\left(\overline{\omega}_{t+1}, s_{t+1}\right) - \mu G\left(\overline{\omega}_{t+1}, s_{t+1}\right). \tag{46}$$

For more details on the characterization of the functions $\Gamma\left(\overline{\omega}_{t+1}, s_{t+1}\right)$ and $G\left(\overline{\omega}_{t+1}, s_{t+1}\right)$, see the Appendix.

Solving this optimization problem results into two additional equilibrium conditions. On the one hand, the participation constraint for the lenders becomes,

$$\frac{P_t Q_t K_{t+1}}{N_{t+1}} = \frac{1}{1 - \left(\frac{\Psi\left(\overline{\omega}_{t+1}, s_{t+1}\right) + \Gamma\left(\overline{\omega}_{t+1}, s_{t+1}\right) - \left(1 - J\left(s_{t+1}\right)\right)}{\Psi\left(\overline{\omega}_{t+1}, s_{t+1}\right)}\right)}, \tag{47}$$

which implies that the threshold $\overline{\omega}_{t+1}$ can be viewed as a function of variables that are either known or observed at time t, i.e. $\overline{\omega}_{t+1} \equiv \overline{\omega}\left(\frac{P_t Q_t K_{t+1}}{N_{t+1}}, s_{t+1}\right)$. The expression $\Psi\left(\overline{\omega}_{t+1}, s_{t+1}\right)$ is defined in the Appendix as,

$$\Psi\left(\overline{\omega}_{t+1}, s_{t+1}\right) \equiv 1 - J\left(s_{t+1}\right) - \Gamma\left(\overline{\omega}_{t+1}, s_{t+1}\right) + \lambda\left(\overline{\omega}_{t+1}, s_{t+1}\right)\left(\Gamma\left(\overline{\omega}_{t+1}, s_{t+1}\right) - \mu G\left(\overline{\omega}_{t+1}, s_{t+1}\right)\right), \tag{48}$$

where $\lambda\left(\overline{\omega}_{t+1}, s_{t+1}\right)$ is the Lagrange multiplier on the lenders' participation constraint in (44) (and represents the shadow cost of enticing the participation of the lenders.) The threshold depends on the anticipated systemic risk shock, s_{t+1}, but it also depends on the asset-to-net worth ratio of the entrepreneur-borrower, $\frac{P_t Q_t K_{t+1}}{N_{t+1}}$. Given the relationship in equation (32), the asset-to-net worth ratio can be related to the leverage of the borrower as,

$$\frac{P_t Q_t K_{t+1}}{N_{t+1}} = 1 + \frac{L_{t+1}}{N_{t+1}}, \tag{49}$$

where $\frac{L_{t+1}}{N_{t+1}}$ is a conventional measure of the debt-to-net worth of the entrepreneur. Moreover, it can be argued that a formulation for the external financing premium arises in the following terms,

$$\mathbb{E}_t\left[R^e_{t+1}\right] = s\left(\frac{P_t Q_t K_{t+1}}{N_{t+1}}, s_{t+1}\right) I^b_{t+1}. \tag{50}$$

This characterization of the external financing premium expands the BGG (1999) framework by adding the explicit possibility that the spread itself be affected by the impact of an anticipated aggregate shock, s_{t+1}. However, we preserve the key feature of the financial accelerator model which is the linkage between the spread on capital returns and the leverage of the entrepreneurs-borrowers. Moreover, the costly-state verification theory implies that external funding (loans) is more expensive than internal funding (savings of the entrepreneur).

The Optimal Capital Investment for the Entrepreneurs. As we noted before, entrepreneurs obtain income from managerial labor at a competitive nominal wage, W^e_t, and from renting capital to wholesale firms and reselling the depreciated capital to the capital goods producers, $\omega_{t+1} R^e_{t+1} P_t Q_t K_{t+1}$. With these resources at hand, each entrepreneur must repay the previous period loans at the agreed rate, i.e. must repay $I_t^l L_t \equiv \overline{\omega}_{t+1} R_{t+1}^e P_t Q_t K_{t+1}$, or choose to default. The entrepreneur must also finance his own consumption, C_t^e, acquire new capital from the capital producers, $P_t Q_t K_{t+1}$, and borrow again, L_{t+1}. In this environment, the budget constraint of a representative entrepreneur can be described in the following terms,

$$P_t C_t^e + P_t Q_t K_{t+1} \leq W_t^e H_t^e + \int_{\overline{\omega}_t}^{+\infty} \left[\omega_t R_t^e P_{t-1} Q_{t-1} K_t - I_t^l L_t \right] \phi\left(\omega_t \mid s_t\right) d\omega_t + L_{t+1}, \tag{51}$$

which accounts for the uses of all those resources. After observing the aggregate shocks at time t, all uncertainty is resolved regarding the aggregate split of capital income between the entrepreneurs-borrowers and the bank-lenders under the terms of the loan contract signed at time $t-1$. Using the sharing rule for the capital returns described in equations (45) and (46) we can rewrite the budget constraint as,

$$P_t C_t^e + P_t Q_t K_{t+1} \leq W_t^e H_t^e + \left[1 - J\left(s_t\right) - \Gamma\left(\overline{\omega}_t, s_t\right)\right] R_t^e P_{t-1} Q_{t-1} K_t + L_{t+1}. \tag{52}$$

The objective of a representative entrepreneur that internalizes the risks associated with default would be to maximize his lifetime utility in (30) subject to the sequence of budget constraints described in (52) and the constraint on the financing of capital investment already noted in equation (32).

Since the constraint in (32) holds with equality, the budget constraint in (52) defines an upper bound on entrepreneurial net worth or savings as follows,

$$N_{t+1} \leq W_t^e H_t^e + \left[1 - J\left(s_t\right) - \Gamma\left(\overline{\omega}_t, s_t\right)\right] R_t^e P_{t-1} Q_{t-1} K_t - P_t C_t^e. \tag{53}$$

Moreover, using the equilibrium participation constraint as expressed in equation (47) to replace $P_{t-1} Q_{t-1} K_t$ out, it immediately follows that,

$$N_{t+1} \leq W_t^e H_t^e + \Psi\left(\overline{\omega}_t, s_t\right) R_t^e N_t - P_t C_t^e. \tag{54}$$

Based on this characterization of the budget constraint of the representative entrepreneur, we can infer that an interior solution of his optimization problem in which (54) holds with equality can be obtained as the solution to an equivalent maximization problem according to which the entrepreneur chooses his real net worth, $\frac{N_{t+1}}{P_t}$, to maximize,

$$\sum\nolimits_{\tau=0}^{+\infty} \left(\beta\eta\right)^{\tau} \mathbb{E}_t \left[\frac{W_{t+\tau}^e}{P_{t+\tau}} + \Psi\left(\overline{\omega}_{t+\tau}, s_{t+\tau}\right) R_{t+\tau}^e \frac{P_{t+\tau-1}}{P_{t+\tau}} \frac{N_{t+\tau}}{P_{t+\tau-1}} - \frac{N_{t+\tau+1}}{P_{t+\tau}} \right], \tag{55}$$

where we implicitly use the fact that managerial labor is inelastically supplied and normalized to one (as pointed out in (31)).

This intertemporal optimization must satisfy the following Euler equation,

$$1 = \beta\eta \mathbb{E}_t \left[\Psi\left(\overline{\omega}_{t+1}, s_{t+1}\right) R_{t+1}^e \frac{P_t}{P_{t+1}} \right], \tag{56}$$

which determines the consumption-savings margin for the representative entrepreneur. The left-hand side of (56) is the marginal utility of entrepreneurs' consumption. The right-hand side is the expected discounted real rate of return of acquiring a unit of capital after taking into account the costs associated with the need for external funding. The latter term has two components. The first term, $\Psi\left(\overline{\omega}_{t+1}, s_{t+1}\right)$, captures the effect of default on external borrowing costs and it also accounts for the role of anticipated systemic losses. The second component, $R^e_{t+1}\frac{P_t}{P_{t+1}}$, is the real rate of return on capital whenever the resale value of depreciated capital and the acquisition cost of new capital are equalized.

2.6 Banks

There is a continuum of banks of unit mass. All banks are symmetric and perfectly competitive, so they take all prices as given. The bank offers the households two types of assets for investment purposes: one type which we call bank equity and another type which we call one-period deposits. Deposits offer a nominal risk-free rate, while equity is rewarded with a riskless return in every period that induces households-shareholders to hold bank capital as well. All households who own bank equity must be indifferent between investing in equity or simply making a deposit.

For convenience, we define the safe return promised to the equity-holders in terms of a yield, R^b_{t+1}, over the value of the banks equity, B_{t+1}. Household deposits are perfectly insured, and pay a risk-free rate, I_{t+1}. Banks use all the resources they attract (deposits and bank capital) to offer one-period loans to the entrepreneurs with the conditions described above. At the end of each loan contract, all unanticipated profits accrued by the bank are rebated (lump-sum) to the households independently of their portfolio allocation between the bank's liabilities (deposits) and equity.

At the end of period t, the balance sheet of the banking system can be summarized as follows,

$$L_{t+1} + \varpi D_{t+1} = B_{t+1} + D_{t+1}, \tag{57}$$

where the right-hand side describes the liabilities (that is, the deposits) taken at time t, D_{t+1}, and the equity offered at the same time, B_{t+1}. The left-hand side shows the assets, $L_{t+1} + \varpi D_{t+1}$. Among the assets, we count the reserves on deposits maintained at the central bank, i.e. ϖD_{t+1}, where $0 \leq \varpi < 1$ represents the compulsory reserve requirement on nominal deposits set by the regulator, and the loans offered at time t, L_{t+1}. As a matter of convention, D_{t+1} denotes nominal deposits and L_{t+1} nominal loans held from time t to $t+1$. Similarly, B_{t+1} is the bank capital outstanding between time t and time $t+1$.

We can rewrite more conveniently the balance sheet as,

$$L_{t+1} = \left(\frac{1-\varpi}{1-\upsilon_{t+1}}\right) D_{t+1}, \tag{58}$$

where we define the leverage ratio on bank capital as $\upsilon_{t+1} \equiv \frac{B_{t+1}}{L_{t+1}}$. In other words, the rate at which deposits are transformed into loans is affected by the compulsory reserve requirement as well as by the bank's capital leverage policy. In BGG (1999), with $\varpi = 0$ and no bank equity (i.e., $\upsilon_{t+1} = 0$), the transformation rate is one-to-one. In other words, it holds that $L_{t+1} = D_{t+1}$. Although the model preserves the basic underlying structure of the bank's balance sheet in BGG (1999), equation (58) already points out that the regulatory features should play a significant role on the cost structure of loan supply.

The banks profits on a given one-period loan contract are realized at time $t+1$. We can express the profits of the banking system as,

$$\Pi^b_{t+1} \equiv R^e_{t+1} P_t Q_t K_{t+1} \left(\Gamma\left(\overline{\omega}_{t+1}, s_{t+1} \right) - \mu G\left(\overline{\omega}_{t+1}, s_{t+1} \right) \right) + \varpi \overline{I}_{t+1} D_{t+1} - R^b_{t+1} B_{t+1} - I_{t+1} D_{t+1}, \tag{59}$$

while the expected profits at the time the loan is agreed upon should be,

$$\mathbb{E}_t \left(\Pi^b_{t+1} \right) \equiv I^b_{t+1} L_{t+1} + \varpi \overline{I}_{t+1} D_{t+1} - R^b_{t+1} B_{t+1} - I_{t+1} D_{t+1}. \tag{60}$$

The required nominal participation returns on loans, I^b_{t+1}, are determined at time t when the loans are signed between the banks-lenders and the entrepreneurs-borrowers (see the participation constraint in (44)). Deposits held at the central bank in the form of reserves are also returned to the banks. We assume that they earn an interest on reserves, $\overline{I}_{t+1}$, which is known at time t and designed as a two-part rate, i.e.

$$\overline{I}_{t+1} \equiv (1 - c) + \zeta \left(I_{t+1} - 1 \right), \tag{61}$$

whereby banks pay a fixed fee as a management cost per unit of reserve held at the central bank, $0 < c < 1$, and get back the principal (minus the management fee) and a net rate of return that is proportional to the net risk-free rate. The parameter $0 < \zeta < 1$ denotes the discount rate relative to the monetary net short-term rate at which reserves are compensated. Although in most instances the practice is to set this rate of return to zero (i.e., $c = \zeta = 0$), there are precedents for paying interest on reserves.[7] We also make the simplifying assumption that there is full deposit insurance, so that deposits are riskless and the gross interest rate paid on deposits is equal to the risk-free nominal rate, I_{t+1}, which is known at time t.

Bank capital shareholders, the households, have to be compensated with a certain nominal yield determined at time t, R^b_{t+1}. Since at time t expected profits depend exclusively on variables that are chosen and known at that time by the banks and the households, then competitive banks must end up offering a yield to the shareholders that is also known at time t. By arbitrage implied in equations (3) and (4), hence, it must be the case that,

$$\left(1 - \iota^h \right) R^b_{t+1} = I_{t+1}, \tag{62}$$

which insures that households remain indifferent between holding bank capital or deposits. For a competitive banking sector, the expected profit function in (60) must satisfy a zero-expected profit condition (i.e., $\Pi^b_{t+1} = 0$) in the following terms,

$$\mathbb{E}_t \left(\Pi^b_{t+1} \right) \equiv \left[I^b_{t+1} - \upsilon_{t+1} R^b_{t+1} - (1 - \upsilon_{t+1}) \left(\frac{I_{t+1} - \varpi \overline{I}_{t+1}}{1 - \varpi} \right) \right] L_{t+1} = 0, \tag{63}$$

after using the balance sheet equation in (58). The problem of the banks is to optimize their capital structure, their trade-off between bank equity and deposits, subject to the constraint that banks must offer a yield on bank capital that makes households indifferent given the existing option of a risk-free rate on deposits as given by equation (62). Of course, this problem is also subject to the features of the policy of paying

[7] Until very recently reserve requirements held at the Federal Reserve did not pay interest. The Federal Reserve announced changes to reserve management after winning the power to pay interest on required and excess reserves on October 3, 2008. The Federal Reserve has argued that paying interest would deter banks from lending out excess reserves and as such would make it easier for the Fed to attain its target rate. We do not attempt to model this feature explicitly.

reserves followed by the central bank as given by equation (61) and subject to a regulatory constraint on capital adequacy that implies banks must satisfy,

$$1 \geq \upsilon_{t+1} \equiv \frac{B_{t+1}}{L_{t+1}} \geq \upsilon, \tag{64}$$

where $0 \leq \upsilon < 1$ is equal to the minimum mandatory capital adequacy requirement set by the regulator.[8] The lower bound υ may also reflect a buffer above the minimum regulatory requirement implied by the statutory requirements of the banks themselves, and could even be time-varying over the cycle.

We shall make two key parametric assumptions to simplify the problem of the banks, and we leave the exploration of more complex banking cost structures for future research. Our goal, at this stage, is to make only the smallest possible departure from the original BGG (1999) framework. We assume that $\zeta = 1 - c$ and, furthermore, that taxes on bank equity are bounded by $0 < 1 - \iota^h < \frac{1-\varpi}{1-\zeta\varpi}$. Whenever $\xi = 0$, this bound implies that $\iota^h > \varpi$; whenever $\xi = 1$, it merely requires that $\iota^h > 0$. Given the fact that tax rates are quite often much higher than the minimum reserve ratios, these bounds are likely not excessively restrictive. Both assumptions put together imply that,

$$R^b_{t+1} > \left(\frac{I_{t+1} - \varpi \overline{I}_{t+1}}{1 - \varpi} \right). \tag{65}$$

In other words, it is costlier for banks to finance themselves with bank equity than with deposits. Therefore, the lower bound on the leverage ratio must be binding at all times.

In turn, these assumptions imply that the participation rate of return required by the banks to fund the entrepreneurs is fully determined by the cost structure of the banks themselves as follows,

$$\begin{aligned} I^b_{t+1} &= \upsilon R^b_{t+1} + (1-\upsilon) \left(\frac{I_{t+1} - \varpi \overline{I}_{t+1}}{1 - \varpi} \right) \\ &= \left[\upsilon \left(\frac{1}{1-\iota^h} \right) + (1-\upsilon) \left(\frac{1 - \varpi\zeta}{1-\varpi} \right) \right] I_{t+1}. \end{aligned} \tag{66}$$

This is what we call the balance sheet channel of banking regulation. It can be easily seen that without capital adequacy requirements, i.e. $\upsilon = 0$, and without reserve requirements, i.e. $\varpi = 0$, we would be back to the world of BGG (1999) where $I^b_{t+1} = I_{t+1}$. Our equation (66) is a heavily parameterized version of the following expression for returns on the portfolio of loans under constant returns to scale,

$$\frac{I^b_{t+1}}{I_{t+1}} \equiv \upsilon_{t+1} \times \frac{cost(bank\ equity_{t+1})}{I_{t+1}} + (1-\upsilon_{t+1}) \times \frac{cost(deposits_{t+1})}{I_{t+1}}, \tag{67}$$

[8] The current regulatory regime was shaped primarily by the 1988 international Basle Accord and the 1991 Federal Deposit Insurance Corporation Improvement Act (FDICIA). The Basle Accord established minimum capital requirements as ratios of two aggregates of accounting capital to risk weighted assets (and certain off-balance sheet activities.) The risk weights are supposed to reflect credit risk. For example, commercial and industrial loans have weight one, while U.S. government bonds have zero weight, and consequently do not require any regulatory capital. Primary or tier 1 (core) capital (= book value of its stock plus retained earnings) is required to exceed 4% of risk weighted assets, while total (tier 1 plus tier 2) capital must be at least 8%. In calculating the risk weighted capital asset ratio all loans are assumed to be in the highest risk category in the sense of the Basle Accord, with a risk weight of 100%. This category includes all claims to the non-bank private sector, except for mortgages on residential property, which receive a risk weight of 50%. The riskless securities are in the lowest risk category, with weight zero. Typical examples are Treasury bills and short loans to other depository institutions.

where υ_{t+1} represents the leverage ratio as before. The realized profits at the time the loan contract expires in (59) can, alternatively, be represented as,

$$\Pi^b_{t+1} \equiv \left(R^e_{t+1} - \mathbb{E}_t\left[R^e_{t+1}\right]\right) P_t Q_t K_{t+1} \left(\Gamma\left(\overline{\omega}_{t+1}, s_{t+1}\right) - \mu G\left(\overline{\omega}_{t+1}, s_{t+1}\right)\right), \tag{68}$$

where we have used the participation constraint in (44) appropriately. Hence, we can see that the realized profits reflect the intertemporal aggregate risks associated with the portfolio of loans supplied to the entrepreneurs (which is captured by the margin $R^e_{t+1} - \mathbb{E}_t\left[R^e_{t+1}\right]$) in the asset side of the banks' balance sheet. The assumption that all realized profits are rebated to the households (no profits are retained by the banks) transfers the consequences of the aggregate risks to the households, who cannot avoid them by adjusting their portfolio between bank equity and bank deposits. We leave for future research the exploration of a more complex environment in which banks' dividends are related to equity holdings and, more interestingly, in which retained profits can affect the evolution of bank equity and expose the bank capital to aggregate risks.

2.7 Government

We close our description of the model with the specification of a consolidated (and balanced) budget constraint and an interest rate rule for monetary policy. We assume that government expenditures and the subsidy on inputs for the retailers are financed through lump-sum taxes on households, taxes on bank equity and seigniorage, i.e.

$$\begin{aligned} P_t G_t + T_t + \iota^h R^b_t B_t + M_{t+1} &= \iota^r P^w_t \left[\int_0^1 Y_t(z)\, dz\right] + \overline{I}_t M_t \\ &= \iota^r P^w_t Y^w_t + \overline{I}_t M_t, \end{aligned} \tag{69}$$

where G_t denotes the real government expenditure. We do assume for simplicity that government consumption is equal to zero in every period, i.e. $G_t = 0$. The characteristics and bounds on the tax subsidy for retailers, ι^r, and the tax rate on dividends, ι^h, as well as the nature of the non-distortionary (lump-sum) tax or transfer to the households, T_t, have already been discussed elsewhere. The government also funds its operations by issuing at time t high-powered money (the monetary base), M_{t+1}.

For the purpose of defining the monetary base, money consists only of the total reserves of the banking sector on their accounts at the central bank. Therefore, given the compulsory requirement on reserves, the equilibrium in the money market requires that,

$$M_{t+1} = \varpi D_{t+1}. \tag{70}$$

As it was noted before, those reserves deposited at time t accrue a rate of return, $\overline{I}_t$, which is characterized by the formula in (61). For simplicity, money plays exclusively the role of a unit of account and acts as the counterpart for deposit reserves on the balance sheet of the central bank.

The central bank policy is modelled by means of an interest rate reaction function. In the spirit of Taylor (1993), the policy rule targets the short-term nominal interest rate, I_{t+1}, and is linear in the logs of the relevant arguments,

$$i_{t+1} = \rho_i i_t + (1-\rho_i)\left[\psi_\pi \ln\left(\frac{P_t}{P_{t-1}}\right) + \psi_q \ln(Q_t) + \psi_y \ln(Y_t)\right] + m_t, \tag{71}$$

where $i_t \equiv \ln(I_t)$ is the logarithm of the risk-free rate. In line with most of the literature, we assume that the monetary authority is willing to smooth changes in the actual short-term nominal interest rate, i.e. $0 \leq \rho_i \leq 1$, where ρ_i is the smoothing parameter. The other parameters in the reaction function satisfy that $\psi_\pi \geq 1$, $-\infty < \psi_q < +\infty$, and $\psi_y \geq 0$. The monetary shock in logs, m_t, follows an $AR(1)$ process of the following form,

$$m_t = \rho_m m_{t-1} + \varepsilon_t^m, \tag{72}$$

where ε_t^m is a zero mean, uncorrelated and normally-distributed innovation. The parameter $-1 < \rho_m < 1$ determines the persistence of the monetary shock, and $\sigma_m^2 > 0$ the volatility of its innovation.

A few observations on the specification of (71) are in order. First, we model monetary policy in terms of an implementable rule, whereby the central bank sets the short-term nominal interest rate in response to observable variables only. Second, this general specification allows for a reaction of the monetary policy instrument to deviations of the relative price of capital goods Q_t from its long-run value of one. This is the channel through which we allow asset price fluctuations to feed into the setting of monetary policy.

Third, equation (71) can always be rewritten in terms of a pure trade-off between inflation and output as,

$$i_{t+1} = \left[\rho_i + \psi_q (1-\rho_i)\left(\frac{\ln(Q_t)}{i_t - \ln\left(\frac{P_t}{P_{t-1}}\right)}\right)\right] i_t + (1-\rho_i)\left[\left(\psi_\pi - \psi_q \frac{\ln(Q_t)}{i_t - \ln\left(\frac{P_t}{P_{t-1}}\right)}\right)\ln\left(\frac{P_t}{P_{t-1}}\right) + \psi_y \ln(Y_t)\right] + m_t, \tag{73}$$

where the coefficient on inflation and the inertia parameter vary depending on whether the Tobin's Q is growing faster than the *ex post* real interest rate or not. This is obviously one out of many observationally equivalent rules that we could write that are consistent with the structure of equation (71). In more general terms what would be rather appealing is to fix monetary policy in terms of a well-known trade-off between inflation and output, but at the same time allowing for flexibility in the rule in order to respond differently to systemic risk which is a critical source of uncertainty in our framework.

The specification of the Taylor rule that we have in mind would take the form of,

$$i_{t+1} = \rho_i(s_t - \overline{s})\, i_t + (1-\rho_i(s_t - \overline{s}))\left[\psi_\pi (s_t - \overline{s}) \ln\left(\frac{P_t}{P_{t-1}}\right) + \psi_y (s_t - \overline{s}) \ln(Y_t)\right] + m_t, \tag{74}$$

where the inertia and the weights on inflation and output are a function of the perceived riskiness of the current environment as determined by the distance of the actual systemic risk shock realization, s_t, relative to the breaking point after which losses in the secondary market for used capital become 'catastrophic'.

Resource Constraint. Equilibrium in the final goods market requires that the production of the final good be allocated to total private consumption by households and entrepreneurs (and possibly the government), to investment by capital goods producers, and to cover the costs that originate from the monitoring technology required to enforce the loan contract described before (and in the Appendix), i.e.

$$Y_t = C_t + C_t^e + G_t + X_t + \underbrace{\mu G\left(\overline{\omega}_t, s_t\right) R_t^e \frac{P_{t-1}}{P_t} Q_{t-1} K_t}_{\text{Loss from monitoring costs}}, \tag{75}$$

where final output and wholesale output are related as $Y_t = \left(\frac{P_t^*}{P_t}\right)^{\theta} Y_t^w$. In the above equation, the impact of government consumption is trivial since we have assumed for simplicity that $G_t = 0$. In the model of BGG (1999), government consumption evolves exogenously and is assumed to be financed by means of lump-sum taxes. A similar extension can be implemented in our setting too.

3 Discussion and Interpretation

The relationship in (66) clearly ties down the participation return, I_{t+1}^b, to the risk-free rate, I_{t+1}, which happens to be also the relevant instrument for monetary policy. The regulatory restriction on capital adequacy in (64) does not prevent 'bad outcomes' from happening. Instead, the purpose of this regulatory constraint is to effectively give the monetary authority a way to 'regulate' the supply of loans without having to manipulate the interest rate directly. In that sense, we can visualize the banks' 'balance sheet' channel in this framework by combining (50) and (66) as follows,

$$\mathbb{E}_t\left[R_{t+1}^e\right] = \underbrace{s\left(\frac{P_t Q_t K_{t+1}}{N_{t+1}}, s_{t+1}\right)}_{\text{"Agency costs" channel - as in BGG (1999) -}} \underbrace{\left[\upsilon\left(\frac{1}{1-\iota^h}\right) + (1-\upsilon)\left(\frac{1-\varpi\zeta}{1-\varpi}\right)\right]}_{\text{"Balance sheet" channel } \geq 1} I_{t+1}. \tag{76}$$

This equation shows that the balance sheet channel has the potential to amplify the external financing premium spread. However, because this channel is regulated by the central bank, the monetary authority can potentially 'manipulate' the requirements in order to reduce the amplification effect at times when the agency cost component is rising.

We have a fairly standard setting that quite closely follows the derivation of the equilibrium conditions in BGG (1999), and therefore our linearization shows obvious similarities with theirs. The main differences arise because we have introduced frictions on the secondary market for used capital that have the potential to alter the conditions under which borrowers and lenders operate in this economy, and because we have expanded the balance sheet of the banks-lenders to give banking regulation a role on loan pricing decisions.

Entrepreneurs cannot borrow at the riskless rate as revealed in equation (76). The cost of external financing differs from the risk-free rate because the idiosyncratic component to their returns on capital is unobservable from the point of view of the banks. In order to infer the realized return of the entrepreneur, the bank has to pay a monitoring cost. The banks monitor the entrepreneurs that default, pay the verification cost and seize the remaining capital income. In equilibrium, entrepreneurs borrow up to the point where the expected return on capital equals the cost of external financing,

$$\mathbb{E}_t\left[\widehat{r}_{t+1}^e\right] \approx \widehat{i}_{t+1} + \vartheta\left(\widehat{p}_t + \widehat{q}_t + \widehat{k}_{t+1} - \widehat{n}_{t+1}\right) + \Lambda\widehat{\upsilon}_{t+1} + \Theta\widehat{s}_{t+1}, \tag{77}$$

where $\widehat{k}_{t+1}$ denotes capital, $\widehat{n}_{t+1}$ is the entrepreneur's net worth, $\widehat{q}_t$ is Tobin's Q, $\widehat{p}_t$ is the CPI, $\widehat{i}_{t+1}$ is the risk-free rate, $\widehat{\upsilon}_{t+1}$ determines changes in banking regulation (capital adequacy) or the bank's leverage policy, and $\widehat{s}_{t+1}$ stands for the systemic risk shock which captures the distortions on the secondary market for used capital. The composite parameters ϑ, Λ and Θ can be expressed as functions of the structural parameters of the model, and all variables in lower case letters with an upper hat represent log deviations from the steady state.

The right-hand side of the external financing premium equation in equation (77) can be decomposed in two terms: the nominal risk-free rate itself on one hand, and the external financing premium on the other hand.[9] The parameter ϑ measures the elasticity of the external financing premium to variations in leverage of the entrepreneurs, measured by its capital expenditures relative to net worth. The larger the share of the capital purchase financed with the entrepreneurs' net worth, the closer the spread is to zero and the lower the associated moral hazard. In case entrepreneurs have sufficient savings to finance the entire capital stock, agency problems vanish, so the risk-free rate and the expected return to capital income must coincide unless the leverage of the banks, $\widehat{\upsilon}_{t+1}$, or the systemic risk, $\widehat{s}_{t+1}$, do vary. So far, this is exactly the same result found in BGG (1999). Our model, however, illustrates that changes in the banking regulation on capital adequacy and systemic risk add a new dimension to the external financing premium that cannot be discounted.

Among others, two points are sufficiently salient to warrant further discussion here. One, our specification of a Taylor rule in (74) depends on exogenous shocks that are potentially unobservable to policy-makers. Two, our characterization of banks, while more complete than BGG (1999) is nonetheless a simple one.

3.1 Taylor Rules

A potential disadvantage of our specification of the Taylor rule in (74), i.e.

$$i_{t+1} = \rho_i \left(s_t - \overline{s}\right) i_t + \left(1 - \rho_i \left(s_t - \overline{s}\right)\right) \left[\psi_\pi \left(s_t - \overline{s}\right) \ln\left(\frac{P_t}{P_{t-1}}\right) + \psi_y \left(s_t - \overline{s}\right) \ln\left(Y_t\right)\right] + m_t, \tag{78}$$

is that monetary policy depends on an exogenous shock which is not necessarily observable to the policy-maker, the systemic shock s_t. An alternative would be to explore a policy rule reflecting the assumption that monetary authorities re-adjust the weights on inflation and output in response to other observable variables every period, reacting to asset prices, Q_t, as in our conjecture in (71), i.e.

$$i_{t+1} = \rho_i i_t + \left(1 - \rho_i\right) \left[\psi_\pi \ln\left(\frac{P_t}{P_{t-1}}\right) + \psi_q \ln\left(Q_t\right) + \psi_y \ln\left(Y_t\right)\right] + m_t. \tag{79}$$

We could even explore alternative rules in which the response of the central bank depends on the size of the spreads between the risk-free rate and the implied returns on capital along the lines of Curdia and Woodford (2008). A potential specification that fits our environment would be,

$$i_{t+1} = \rho_i i_t + \left(1 - \rho_i\right) \left[\psi_\pi \ln\left(\frac{P_t}{P_{t-1}}\right) + \psi_a \ln\left(\frac{P_t Q_t K_{t+1}}{N_{t+1}}\right) + \psi_y \ln\left(Y_t\right)\right] + m_t. \tag{80}$$

[9] The key mechanism involves the link between the "external financing premium" (the difference between the cost of funds raised externally and the opportunity cost of internal funds) and the net worth of the entrepreneurs-borrowers.

This rule targets the leverage ratio of the borrowers since theory tells us in (76) that this is the unobservable component of the external financial premium.

As noted before, specification (79) is comparable with the Taylor rule presented in equation (78) and they would produce similar results when implemented as long as Tobin's Q is a sufficient statistic for the unobservable systemic shock. The same can be said of the specification in (80). Whether such Taylor rules are optimal relative to a rule with constant coefficients will likely depend on whether the observable variables, Tobin's Q or the spreads, are a good proxy to signal trouble in the secondary market for used capital or not. Monetary policy is likely to improve its performance if it can react to strong signals, but it will likely not do better than under an old-fashioned Taylor rule with constant coefficients if the signal is weak or does give the wrong message depending on the nature of the shock that hits the economy.

The transmission mechanism that affects the dynamics of the economy over the business cycle is also quite important here. Monetary policy has no direct effect on the systemic shock in equation (77), since this shock is assumed to be purely exogenous. But the central bank can either alter the bank regulatory requirements, $\widehat{v}_{t+1}$, or the short-term interest rate, $\widehat{i}_{t+1}$, in order to offset fluctuations of the spread that tend to increase the volatility of the cost of external borrowing for the entrepreneurs and potentially lead to periods of excessive investment or under-investment. Monetary policy, whether implemented conventionally through interest rate movements or by changes in banking regulation, would nonetheless have an indirect effect on the equilibrium spreads that can limit the effectivity of those actions.

3.2 Banking Sector

Arguably, our model remains a very naïve characterization of the behavior of banks. We are far from having an integrated model of the business cycle where banks operate in multiple periods, with a portfolio of loans of different maturity and where banks confront simultaneously frictions in their lending operations and nontrivial distortions on the way in which they raise capital or attract depositors. However, with this characterization of the economy we are putting the emphasis on the regulatory power to alter the operational costs of the banking system. Even in this simplified framework, it immediately transpires that the regulator is able to alter the terms of the banks' operating costs. Hence, the regulator has in its hands a tool to either amplify or reduce the loan supply without directly changing the short-term interest rate. This framework offers us a way to explore how the model responds to monetary policy and regulatory features like those.

We have already noted that regulatory features can be modified with the intention of offsetting fluctuations in the spread faced by borrowers on external funding. In principle, given the fact that in most developed countries the reserve requirements and capital adequacy requirements are not excessively punitive, one might expect that changes in banking regulation would have small effects on the cost structure of banks and, therefore, would have less of an impact on the cost of borrowing for entrepreneurs. However, in the extreme case in which $\widehat{v}_{t+1} = -\frac{\Theta}{\Lambda}\widehat{s}_{t+1}$, it might be possible to entirely eliminate the effect of systemic risk on shocks without altering the interest rate. And, therefore, it might be possible to limit the impact of the systemic risk shock on the economy without having to alter the incentives to invest for the entrepreneurs and the incentives to save for the households.

Even though the potential for banking regulation to play a counter-cyclical role is present in the model, and noted in our comments, the fact remains that being able to obtain a clear signal of the risks confronted is essential but not easy. In most instances, the systemic risks $\widehat{s}_{t+1}$ are simply not observable and relying on observables to define the cyclical patterns of banking regulation is as difficult as it was for setting interest rate rules. In practice, however, the banking leverage ratios tend to be pro-cyclical and contribute to amplify the cycle further, so the policy debate is more oriented towards policies that would reduce those tendencies rather than to turn banking regulation into a cyclical counter-balance.

4 Concluding Remarks

Our paper has offered a model of the economy that generalizes the BGG (1999) to include a compact characterization of both the financial accelerator and the role of the financial sector in propagating monetary policy to the real economy. We have identified the output costs of systemic risk and "agency costs" of costly state verification as well as their role in determining the external finance premium. Equation (76) neatly summarizes this relationship and makes clear how the financial sector can amplify the cycle as discussed in BGG (1999). Such a characterization is important as it provides a parsimonious explanation that can be compared with existing research on the interaction between monetary policy and bank regulation. This result arises as part and parcel of a model designed to explain the transmission and amplification of monetary actions.

In particular, we believe that a model that includes this type of lending channel can go some length towards explaining some of the monetary policy asymmetries that Taylor rules have been unable to account for in the last few years. As well, we think that since our model is built around the existence of a regulatory capital constraint, it provides the basis for discussions of the implications of joint determination of monetary policy and regulation. Indeed, the presence of differences in monetary policy discussed in this model implies a strong incentive for the joint monetary/regulatory authority to ensure that financial institutions remain above the capital constraint. In times of falling asset values, banks will approach or fall below capital requirements, rendering monetary policy ineffective at stimulating lending. At this point, the monetary/regulatory authority has a stronger incentive to lower capital requirements in order to facilitate monetary intervention. If falling asset values were due to a realization of inaccurate risk measurements, reduced capital levels may simply encourage reckless lending.

With this framework in place, there are potentially more open questions ahead of us (and, unfortunately, beyond the scope of this paper.) For example, while the model appears to do a reasonably good job in describing the stylized patterns of the U.S. monetary authority during the recent crisis, at least in suggesting that reductions of interest rates are a plausible policy response to systemic shocks (and bank lending constraints), it is nonetheless potentially rejected by the European case. The European Central Bank held interest rates constant until late in 2008. Though there are many possible reasons for this, we speculate that this emerges, in part, from differences in mandate. The Federal Reserve has responsibility both for monetary policy and bank regulation of parts of the financial system. This produces well-known conflicts between the goals of monetary policy and banking regulation. It also produces an incentive to keep banks above regulatory thresholds through the use of monetary policy (see Cechetti and Li, 2008, on neutralization of the capital constraint).

Why does this matter here? Two avenues are worth pursuing in future research. One, did the ECB keep rates constant as it saw no direct role within its mandate for financial sector debt deflation? Did the Fed use alternate methods of liquidity provision as a way to provide *ad hoc* regulatory tolerance – effectively removing the concern that near-term liquidity problems would decrease asset values sufficiently to lead to a binding capital constraint? By doing so, did the Fed attempt to re-open the accelerator for monetary policy?

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Appendix

A The Loan Contract

On the Aggregate Sharing of Capital Income. We define the following two variables for simplicity of notation,

$$\Gamma\left(\overline{\omega}_{t+1}, s_{t+1}\right) \equiv \int_{0}^{\overline{\omega}_{t+1}} \omega_{t+1}\phi\left(\omega_{t+1} \mid s_{t+1}\right) d\omega_{t+1} + \overline{\omega}_{t+1}\left(1 - \Phi\left(\overline{\omega}_{t+1} \mid s_{t+1}\right)\right), \tag{81}$$

$$\mu G\left(\overline{\omega}_{t+1}, s_{t+1}\right) \equiv \mu \int_{0}^{\overline{\omega}_{t+1}} \omega_{t+1}\phi\left(\omega_{t+1} \mid s_{t+1}\right) d\omega_{t+1}. \tag{82}$$

Then, we can rewrite the share on capital returns going to the lenders in (42) more compactly as,

$$g\left(\overline{\omega}_{t+1}, s_{t+1}\right) = \Gamma\left(\overline{\omega}_{t+1}, s_{t+1}\right) - \mu G\left(\overline{\omega}_{t+1}, s_{t+1}\right). \tag{83}$$

Given the definition of the capital returns share going to the entrepreneurs in (40), it also follows that,

$$\begin{aligned} f\left(\overline{\omega}_{t+1}, s_{t+1}\right) &= \int_{0}^{+\infty} \omega_{t+1}\phi\left(\omega_{t+1} \mid s_{t+1}\right) d\omega_{t+1} - \Gamma\left(\overline{\omega}_{t+1}, s_{t+1}\right) \\ &= 1 - J\left(s_{t+1}\right) - \Gamma\left(\overline{\omega}_{t+1}, s_{t+1}\right), \end{aligned} \tag{84}$$

where the second equality follows from our characterization of the expectation of the idiosyncratic shock in (36). Based on these definitions, we can infer that the capital income sharing rule resulting from this financial contract satisfies that,

$$f\left(\overline{\omega}_{t+1}, s_{t+1}\right) + g\left(\overline{\omega}_{t+1}, s_{t+1}\right) = 1 - J\left(s_{t+1}\right) - \mu G\left(\overline{\omega}_{t+1}, s_{t+1}\right), \tag{85}$$

where $J\left(s_{t+1}\right) \equiv 1 - \mathbb{E}\left[\omega_{t+1} \mid s_{t+1}\right]$ accounts for the expected systemic losses on the resale value of capital and $\mu G\left(\overline{\omega}_{t+1}, s_{t+1}\right)$ characterizes the conventional monitoring costs and probability of default associated with the costly-state verification framework.

The functions $f\left(\overline{\omega}_{t}, s_{t}\right)$ and $g\left(\overline{\omega}_{t}, s_{t}\right)$ represent the sharing rule between entrepreneurs-borrowers and bank-lenders on the capital returns required by the entrepreneur's partial use of one-period external loans to fund its risky capital investment. Both of them depend on the realization of the systemic risk shock, s_{t+1}. However, as can be inferred from equation (85), they do not add up to one. A fraction of the capital income, $J\left(s_{t+1}\right)$, is transferred to the capital goods producers due to inefficiencies in the secondary market for used capital, while another fraction, $\mu G\left(\overline{\omega}_{t+1}, s_{t+1}\right)$, is lost due to the burden of monitoring. It is worth pointing out that only monitoring costs result in a direct loss of capital income that detracts resources, as can be noted from the resource constraint in (75). But the fact that resources are siphoned out of the hands of borrowers and lenders due to market imperfections somewhere else still has the potential to substantially distort the incentives of both parties involved in the loan contract and, therefore, to affect the funding of investment in new capital.

The Optimization Problem. We conjecture that the threshold, $\overline{\omega}_{t+1}$, would be defined as a function of the systemic risk shock, s_{t+1}, as well as the assets-to-net worth ratio at time t, $\frac{P_t Q_t K_{t+1}}{N_{t+1}}$. Both of which, given our conventions, are either observed or determined by all parties at time t. Therefore, with the information available at time t, entrepreneurs expected capital return implied by equation (39) should be equal to,

$$P_t Q_t K_{t+1} \mathbb{E}_t \left[R^e_{t+1}\right] f\left(\overline{\omega}_{t+1}, s_{t+1}\right). \tag{86}$$

Similarly, with the information available at time t, lenders expected income given by equation (41) should be equal to,

$$P_t Q_t K_{t+1} \mathbb{E}_t \left[R^e_{t+1}\right] g\left(\overline{\omega}_{t+1}, s_{t+1}\right). \tag{87}$$

The formal contracting problem reduces to choosing the quantity of physical capital, K_{t+1}, and the threshold, $\overline{\omega}_{t+1}$, that maximizes the entrepreneurs' expected nominal return on capital net of the loan costs (see equations (86) and (84)), i.e.

$$P_t Q_t K_{t+1} \mathbb{E}_t \left[R^e_{t+1}\right] \left[1 - J\left(s_{t+1}\right) - \Gamma\left(\overline{\omega}_{t+1}, s_{t+1}\right)\right], \tag{88}$$

subject to the participation constraint for the lenders (see equations (87) and (83)), i.e.

$$P_t Q_t K_{t+1} \mathbb{E}_t \left[R^e_{t+1}\right] \left[\Gamma\left(\overline{\omega}_{t+1}, s_{t+1}\right) - \mu G\left(\overline{\omega}_{t+1}, s_{t+1}\right)\right] \geq I^b_{t+1} L_{t+1} = I^b_{t+1} \left[P_t Q_t K_{t+1} - N_{t+1}\right], \tag{89}$$

where the equality on the right-hand side follows from (32). Implicitly it is agreed that if lenders participate in this contract, they always supply enough loans, L_{t+1}, as long as an uncontingent participation rate, I^b_{t+1}, is guaranteed to them in expectation. In other words, we do not explicitly consider the possibility of credit rationing while we view the (risk-neutral) banks as bearing part of the aggregate risk. All banks share equally on the aggregate size of the loan.

The first-order condition with respect to $\overline{\omega}_{t+1}$ defines the function $\lambda_{t+1} \equiv \lambda\left(\overline{\omega}_{t+1}, s_{t+1}\right)$ in the following terms,

$$\Gamma_1\left(\overline{\omega}_{t+1}, s_{t+1}\right) - \lambda\left(\overline{\omega}_{t+1}, s_{t+1}\right)\left[\Gamma_1\left(\overline{\omega}_{t+1}, s_{t+1}\right) - \mu G_1\left(\overline{\omega}_{t+1}, s_{t+1}\right)\right] = 0, \tag{90}$$

where λ_{t+1} is the Lagrange multiplier on the lenders' participation constraint. By virtue of this optimality condition, we say that the shadow cost of enticing the participation of the lenders in this contract is given by,

$$\lambda\left(\overline{\omega}_{t+1}, s_{t+1}\right) = \frac{\Gamma_1\left(\overline{\omega}_{t+1}, s_{t+1}\right)}{\Gamma_1\left(\overline{\omega}_{t+1}, s_{t+1}\right) - \mu G_1\left(\overline{\omega}_{t+1}, s_{t+1}\right)}, \tag{91}$$

which, in turn, implies that the participation constraint must be binding since the multiplier is non-zero. The binding participation constraint can be re-written as,

$$\frac{P_t Q_t K_{t+1}}{N_{t+1}} \mathbb{E}_t \left(\frac{R^e_{t+1}}{I^b_{t+1}}\right) \left(\Gamma\left(\overline{\omega}_{t+1}, s_{t+1}\right) - \mu G\left(\overline{\omega}_{t+1}, s_{t+1}\right)\right) = \left[\frac{P_t Q_t K_{t+1}}{N_{t+1}} - 1\right], \tag{92}$$

or, more compactly,

$$\frac{P_t Q_t K_{t+1}}{N_{t+1}} = \frac{1}{1 - \mathbb{E}_t \left(\frac{R^e_{t+1}}{I^b_{t+1}}\right) \left(\frac{\Psi\left(\overline{\omega}_{t+1}, s_{t+1}\right) + J\left(s_{t+1}\right) + \Gamma\left(\overline{\omega}_{t+1}, s_{t+1}\right) - 1}{\lambda\left(\overline{\omega}_{t+1}, s_{t+1}\right)}\right)}, \tag{93}$$

where we define $\Psi(\overline{\omega}_{t+1}, s_{t+1})$ as,

$$\Psi(\overline{\omega}_{t+1}, s_{t+1}) \equiv 1 - J(s_{t+1}) - \Gamma(\overline{\omega}_{t+1}, s_{t+1}) + \lambda(\overline{\omega}_{t+1}, s_{t+1})(\Gamma(\overline{\omega}_{t+1}, s_{t+1}) - \mu G(\overline{\omega}_{t+1}, s_{t+1})). \quad (94)$$

The optimization also requires the following first-order condition with respect to capital, K_{t+1}, to hold,

$$\mathbb{E}_t\left(\frac{R^e_{t+1}}{I^b_{t+1}}\right)\Psi(\overline{\omega}_{t+1}, s_{t+1}) - \lambda(\overline{\omega}_{t+1}, s_{t+1}) = 0, \quad (95)$$

where we implicitly use the conjecture that $\overline{\omega}_{t+1}$ is conditioned on variables known at time t. Simply re-arranging gives us the following expression,

$$\mathbb{E}_t\left(\frac{R^e_{t+1}}{I^b_{t+1}}\right) = \frac{\lambda(\overline{\omega}_{t+1}, s_{t+1})}{\Psi(\overline{\omega}_{t+1}, s_{t+1})}, \quad (96)$$

which determines the excess returns per unit of capital above the participation returns on bank loans that would be required to make the financial contract worthwhile to both entrepreneurs-borrowers and bank-lenders.

If we combine equations (96) and (93), then it immediately follows that,

$$\begin{aligned}
\frac{P_t Q_t K_{t+1}}{N_{t+1}} &= \frac{1}{1 - \frac{\lambda(\overline{\omega}_{t+1}, s_{t+1})}{\Psi(\overline{\omega}_{t+1}, s_{t+1})}\left(\frac{\Psi(\overline{\omega}_{t+1}, s_{t+1}) + J(s_{t+1}) + \Gamma(\overline{\omega}_{t+1}, s_{t+1}) - 1}{\lambda(\overline{\omega}_{t+1}, s_{t+1})}\right)} \\
&= \frac{1}{1 - \left(\frac{\Psi(\overline{\omega}_{t+1}, s_{t+1}) + \Gamma(\overline{\omega}_{t+1}, s_{t+1}) - (1 - J(s_{t+1}))}{\Psi(\overline{\omega}_{t+1}, s_{t+1})}\right)},
\end{aligned} \quad (97)$$

which validates our conjecture on the threshold implying that $\overline{\omega}_{t+1} \equiv \overline{\omega}\left(\frac{P_t Q_t K_{t+1}}{N_{t+1}}, s_{t+1}\right)$. Given the relationships in (96) and (97), it can be argued that a formulation for the external financing premium arises in the following terms,

$$\mathbb{E}_t\left[\frac{R^e_{t+1}}{I^b_{t+1}}\right] = s\left(\frac{P_t Q_t K_{t+1}}{N_{t+1}}, s_{t+1}\right). \quad (98)$$

This characterization of the external financing premium expands the BGG (1999) framework by adding the explicit possibility that the spread itself be affected by the impact of an anticipated aggregate shock, s_{t+1}. The participation return on loans is set at the time the contract is signed, therefore I^b_{t+1} is known at time t and can be taken out of the expectation, i.e.

$$\mathbb{E}_t\left[R^e_{t+1}\right] = s\left(\frac{P_t Q_t K_{t+1}}{N_{t+1}}, s_{t+1}\right) I^b_{t+1}. \quad (99)$$

This relationship is the key feature of the financial accelerator model.