FEDERAL RESERVE BANK OF BOSTON™

RISK AND POLICY ANALYSIS UNIT

Working Paper
RPA 15-2
October 22, 2015

Cross-Sectional Factor Dynamics and Momentum Returns

Doron Avramov

Satadru Hore

Cross-Sectional Factor Dynamics and Momentum Returns

Doron Avramov[1] and Satadru Hore[2]

[1]The Hebrew University of Jerusalem

[2]The Federal Reserve Bank of Boston

October 22, 2015

Abstract

This paper proposes and implements an inter-temporal model wherein aggregate consumption and asset-specific dividend growths jointly move with two mean-reverting state variables. Consumption beta varies through time and cross sectionally due to variation in half-lives and stationary volatilities of the dividend signals. Winner (Loser) stocks exhibit high (low) half-lives and stationary volatilities, and thus exhibit high (low) consumption beta commanding high (low) risk-premium. The model also rationalizes the "momentum crashes" phenomenon discussed in Daniel and Moskowitz (2014). High half-lives of dividend signals in Winners keep their consumption betas low long after recovering from a prolonged economic downturn, while low half-lives in Losers make their consumption betas grow rather quickly. Thus, coming out of a recession, the long Winner/short Loser strategy reduces in consumption beta and, hence, risk-premia.

Keywords: Momentum; Cross-Sectional Dynamics; Long-Run Risk; Bayesian Filtering.

JEL Classification Codes: G12, C32.

1 Introduction

Momentum effects in stock prices are robust. Fama and French (1996) show that momentum is the only deviation from the CAPM unexplained by the Fama and French (1993) three-factor model. Schwert (2003) shows that profit opportunities, such as the size and value effects as well as the equity premium predictability by the dividend yield, typically disappear, reverse, or attenuate following their discovery. Momentum is an exception. For instance, Jegadeesh and Titman (2001, 2002) document large momentum profitability during the period after its discovery by Jegadeesh and Titman (1993), while Geczy and Samonov (2013) document momentum effects going back to the early nineteenth century. Asness, et al. (2013) uncover robust momentum in various equity markets and across several asset classes, and Fama and French (2008) argue that momentum is among the few robust asset pricing anomalies. However, while momentum strategies, across and within asset classes, equity markets, and time-periods, have generally been successful, Daniel and Moskowitz (2014) document momentum crashes following major market downturns. For instance, in April 2009, during the recovery from the recent financial crisis, the WML portfolio returned -45.52%.

Several economic theories have attempted to explain the prominence of momentum profitability. Berk, Green and Naik (1999) and Johnson (2002) provide risk based explanations, while Barberis, Schleifer and Vishny (1998), Daniel, Hirshleifer and Subrahmanyam (1998), and Hon, Lim and Stein (2000) focus on the behavioral front. Recently, the long-run risk model of Bansal and Yaron (2004) has been in the forefront of explaining momentum (among other) effects through combining intertemporal dynamics with recursive preferences. For one, Yang (2007) and Zurek (2007) find that cash-flows that are exposed to long-run risk are significant sources of momentum profitability, while Ferson, Nallareddy and Xie (2013) show that long-run risk factors perform particularly well in explaining momentum returns out-of-sample. Bansal, Dittmar and Lundblad (henceforth BDL) (2005) provide the background for these findings, suggesting that economic risks embedded in cash-flows account for a significant portion of differences in risk-premiums across assets. However, standard asset-pricing theory has not rationalized momentum crashes, noted earlier.

This paper proposes and implements a dynamic asset pricing model accomplishing two main goals in further explaining momentum effects. For one, we successfully match unconditional risk-premia and Sharpe ratios of the momentum trading strategy across various equity portfolios. Moreover, and more importantly, the model developed here is able to generate large drop in momentum risk-premium, across the various assets considered, during recovery from episodes of economic downturns. Methodologically, we depart from the extant literature and model aggregate consumption and asset specific dividend growths move with two mean-reverting state variables. The additional state variable serves as a signal for future dividend growth. The dividend signal displays attractive statistical and economic properties. For one, it captures a large component of the dynamics of cash-flows making it an essential ingredient in modeling dividend growth. Moreover, innovations in the signal are correlated with innovations in the consumption growth rate, making this new state variable a valuable source of long-run risk-premium. We term this state-variable the fundamental factor of an asset - “fundamental” as it captures the dynamics of an asset’s cash-flows which are correlated with consumption growth. Indeed, the lion’s share of risk-premium is attributable to exposure of the fundamental factor to expected consumption growth rate, giving rise to what we term “fundamental beta.” The fundamental beta generates long-run risk-premia across various assets, as it varies through time and cross sectionally, consistent with the seminal works of Gomes, Kogan and Zhang (2003) and Pastor and Veronesi (2003) as well as with the empirical findings of Avramov and Chordia (2006). We do not explicitly model the firm investment policy or technology. Rather, we use cross-sectional dynamics directly to model firm cash-flows. In that context, our formulation is in the spirit of Lucas (1978) and Breeden (1979); both these papers show that an asset risk-premium is determined by its ability to insure against changes in consumption growth.

In our setup, risk-premium differs across assets due to differential exposures of asset cash-flows to consumption growth. In Loser (Winner) portfolios, the fundamental factors generate low, often even negative, (high) correlation of asset cash-flows with consumption growth. Hence, Loser portfolios offer some hedge against negative aggregate shocks to consumption growth and thus command a negative, or low, risk-premium along with low fundamental beta. Winners do not provide a similar hedge and hence command higher premium and exhibit higher fundamental beta. Parameter heterogeneity of the fundamental factors across assets plays a significant role in the variation of fundamental betas. Heterogeneity in two parameters particularly stand out - half-life and stationary volatility. Winner portfolios tend to have fundamental factors with high half-lives and high stationary volatilities. Loser portfolios exhibit the opposite patterns. This heterogeneity interacts well with recursive preferences to generate high (low) fundamental beta for Winners (Losers). An agent endowed with such preferences can, under certain parameter restrictions, desire an early resolution of uncertainty. Thus, exposure to a volatile growth rate, which co-varies with consumption growth and requires longer resolution of uncertainty (due to high half-life and volatility), requires high premium. The variation in these parameters is large enough to produce economically significant momentum risk premium across Winners and Losers that largely match average returns based on real data.

A significant contribution of our fundamental factor lies in its ability to generate "momentum risk-premium crashes" similar to "momentum crashes" in Daniel and Moskowitz (2014). In order to understand the economic channel outlined here, let us briefly visit the major arguments made by Daniel and Moskowitz (2014). In particular, during episodes of market stress, high-beta firms fall in tandem with the market, while low-beta firms perform better. Following market falls, a momentum strategy is likely to hold low-market-beta stocks (the past winners) and short high-market-beta stocks (the past losers). The end result is that immediately following a market crash, the momentum strategy exhibits net negative beta and performs poorly when the market recovers. Using daily returns, Daniel and Moskowitz (2014) indeed find that, following major market downturns, market beta for loser stocks rises above three while that for winners falls below 0.5.

Our suggested equilibrium model emphasizes the role of consumption beta, hence, we focus on stressful periods of consumption growth, e.g., economic recessions. The large time-variation in the fundamental factor creates a large time-variation in risk premia. Since market price of risk is assumed to be constant, the entire time-variation in risk premia is attributable to time variation in the fundamental beta. We show that the fundamental beta for long winner and short loser strategy decreases, and momentum risk-premium falls at the end of the recession. Our explanation lies entirely on the variation in half-lives of the fundamental factors across the extreme portfolios. When a recession emerges and poor economic shocks start building in the economy, the fundamental factors of the Loser portfolios react rather promptly since they have low half-lives, i.e. low persistence with past shocks. They fall with the oncoming economic shocks, and rise back up when the economy starts recovering almost immediately after exiting a recession. The fundamental factors of the Winners, on the other hand, have high half-lives. They react slowly, as they are still under the influence of the pre-recession shocks, starting to respond to the adverse economic shocks only after several quarters. These negative shocks eventually force the fundamental factors in Winners to start falling, however, their effects last longer - long after the economy has exited the recession. Thus, for some time after exiting a recession, Loser fundamental factors rise sharply while Winner fundamental factors keep on dropping. That is the mechanism that makes fundamental betas of Loser (Winner) portfolios getting higher (lower). The net effect is a large decrease in fundamental beta, and risk-premia, of long Winners/short Loser strategy after exiting a recession. Perhaps to emphasize again, the key cross-sectional variation that is responsible for the momentum crash phenomenon is differential half-lives of the fundamental factors. Notice that one cannot obtain such dynamics of Winner and Loser risk premia using a model in which all assets are exposed to the same long-run risk variable, and hence, mean-revert simultaneously. That is indeed an important innovation of our setup.

While rationalizing "momentum crashes" is an attractive inference from our theory, we also provide a more extensive coverage of the momentum phenomenon in US equities. We expand the set of momentum assets from the traditional set of portfolios sorted based on past returns to examine momentum interactions with firm size and book-to-market ratio. The data show that there is significant momentum profitability in these groups. In the lowest size decile (Small stocks), the strategy that invests long Winner and short Loser stocks yields 2.69%, while the momentum payoff in low book-to-market and small stocks (Low/Small) is 4.29% and in High/Small it is 3.47%. Our fundamental beta is able to rationalize such dispersions in momentum returns across these equity portfolios. Half-lives of the fundamental factors continue to play an important role in the cross-section, as it is a major determinant for variation in cross-sectional fundamental betas which explain average returns. In the time-series dimension, stochastic model risk-premia generates significant return predictability. We directly test how our model-implied risk-premia predicts future returns, relative to the standard Campbell and Shiller (1988) predictive regression. Our model performs significantly better in the momentum portfolios constructed in the size and book-to-market interactions, but it does not do as well in the high return deciles of the traditional past return sorted portfolios.

Econometrically, the fundamental factor and its parameters are estimated from the time-series of asset-specific cash flows and aggregate consumption growth. The joint dynamics of aggregate consumption and asset-specific cash-flow growths creates a state-space framework. Hence, we are able to implement Bayesian econometric techniques which have gained popularity in the finance literature (Johannes and Polson (2009), Johannes, Polson and Stroud (2009)) to draw from the posterior distributions of the parameters and state-variables. In particular, we use a Bayesian Kalman filter, Forward Filtering Backward Sampling (Carter and Kohn (1994), Frühwirth-Schnatter (1994)), to draw from the posterior distribution of the fundamental factor implied by the joint state-space.

The rest of the paper proceeds as follows. Section 2 provides a summary of the data. Section 3 provides the intuition underlying our model. The description and derivation of the model follow in Section 4. Section 5 describes the estimation methodology using Bayesian Markov Chain Monte Carlo. Section 6 describes the empirical findings. Second 7 concludes. The technical part of our work is explained in a greater detail in the Appendix.

2 Data and Summary Statistics

We propose and implement a dynamic inter-temporal model to explain the source of momentum profitability as well as the observed crashes of momentum following episodes of recovery from economic downturns. Our empirical analysis examines momentum among several equity portfolios. Portfolio returns are value weighted and are based on individual stock return data from CRSP throughout the sample period 1963-2013. The book value of stocks is from COMPUSTAT. Dividend data are constructed following BDL (2005). Our dividends include share repurchases and are deseasonalized to account for strong seasonal patterns in payouts. Further details on the construction of these portfolios and their cash-flows are provided in the Appendix. Next, consumption growth is constructed from Personal Consumption Expenditures (PCE) in Non-Durables and Services collected by the Bureau of Labor Statistics, and converted into real consumption growth by the PCE implicit price deflator. All nominal asset return and dividend growth figures are adjusted to real terms using the PCE deflator.

We first examine ten momentum portfolios sorted on past returns. For any month t, stock returns are cumulated from month $t-12$ till month $t-2$, skipping month $t-1$ to avoid the price reversal effect. The holding period is one month following BDL (2005). It should be noted that we also considered quarterly holding periods and the results are unchanged. Table 1 provides the summary statistics of the ten portfolios. The evidence shows that these portfolios are fairly diversified with average number of stocks ranging between 247 and 588. A momentum strategy that buys past winner (highest decile portfolio) and sells short past loser (lowest decile return portfolio) stocks earns a real risk-premium of 2.40% quarterly, which is both economically and statistically significant.

We also examine the interaction of momentum with the size and book-to-market effects. Focusing on size, we sort stocks first on market capitalization and then on past returns. We consider three size groups - Big, Medium and Small. Then, within each size category, we sort stocks based on past returns into three momentum portfolios - Winner, Middle and Loser. Table 2 provides summary statistics for these nine (three times three) portfolios. Momentum profits are the highest within Small stocks. A strategy of buying Small/Winner and selling short Small/Loser generates a risk-premium of 2.69% quarterly. Momentum strategies within the Medium and Big portfolios are economically small and statistically insignificant.

Next, we triple sort, first by book-to-market (BM), then by size, and finally by past returns. We consider three BM, three size, and three momentum categories, thereby constructing 27 portfolios. Table 3 provides summary statistics of these portfolios. Momentum profits are significant in Low BM/Small and High BM/Small stocks. Buying Low BM/Small/Winner and selling Low BM/Small/Loser generates a risk-premium of 4.29% quarterly. Similarly, buying High BM/Small/Winner and shorting High BM/Small/Loser generates a risk-premium of 3.47%. Momentum returns are more modest among the other portfolios. Whereas the long-short strategy delivers high returns, the portfolios in the middle vary considerably across the portfolios we construct. In the data, there is a slight Size premium and a considerable Value premium. Consequently, the Middle portfolio under High/Small has an average return of 3.49% and under Low/Small has an average return of 1.42%.

For the rest of the paper, we focus on the following portfolios. For momentum portfolios based on past returns, we look at Winner, Loser and portfolios 3, 5, and 8. For the double sorted portfolios, we focus on the Winner, Middle and Loser portfolios in Small stocks. For the triple sorted portfolios, we focus on the Winner, Middle and Loser portfolios interacted with the Low/Small and High/Small. For these portfolios, we attempt to link the dynamics of their cash-flows to their risk-premiums. In particular, we investigate how the cross-sectional differences in the dynamics of cash-flows create cross-sectional differences in risk-premia. Moreover, we study how cross-sectional variation in dynamics of fundamental factors in cash-flows affects the risk-premium of a momentum strategy during episodes of economic recovery. The next section builds the intuition behind our model.

3 Intuition

In this section, we build an intuition on how incorporating asset-specific information in cash-flow dynamics help deliver the asset-pricing results. We start off with a simple state-space model of consumption growth and dividend growth of assets described in the previous section. Let $Z_t = \begin{bmatrix}\Delta c_t & \Delta d_t^i\end{bmatrix}'$, where $\Delta c_t = \ln C_t/C_{t-1}$ is aggregate consumption growth and $\Delta d_t^i = \ln D_t^i/D_{t-1}^i$ is the dividend growth of asset i. Assume Z_t follows a state-space system determined by

$$Z_{t+1} = AS_t + \Sigma^1 U_{t+1}^1 \tag{1}$$

$$S_{t+1} = \Lambda S_t + \Sigma^2 U_{t+1}^2 \tag{2}$$

The "observation" equation (1) describes the joint dynamics of aggregate consumption and dividend growth rate of asset i. In this equation, A is a 2X2 matrix of coefficients which loads on the latent state-variables S_t and Σ^1 is a diagonal 2X2 volatility matrix controlling the noise of the system. The "state-transition" equation (2) describes the transition density of the latent states, S_t, which is a 2X1 vector and follows an autoregressive process. Its dynamics are controlled by a diagonal 2X2 matrix of autoregressive parameters, Λ, and a 2X2 volatility matrix, Σ^2, whose off-diagonal term makes innovation in the state-variables correlated with each other. Also, U^1 and U^2 are iid $N(0, I_{2X2})$.

The state-variables in this system are signals for future growth path of consumption and dividends of asset i. The first state-variable contains information about the macro economy, and the second state-variable pulls information from both cross-sectional dividend growth and the macro economy. As such, it contains information about firm fundamentals which covaries with aggregate consumption. This has the potential of generating significant risk-premia over and above what is generated by the first state-variable. We empirically test whether the second state-variable has meaningful statistical and economic content.

We explore two nested models, Base and Alternate, within this state-space framework. The models are nested through the coefficient matrix A, where

$$A^{\text{Base}} = \begin{bmatrix} 1 & 0 \\ a^i & 0 \end{bmatrix} \qquad\qquad A^{\text{Alternate}} = \begin{bmatrix} 1 & 0 \\ b_1^i & b_2^i \end{bmatrix}$$

The Base model reduces the system to a univariate model where the effect of the second state-variable is shut off. The only relevant state-variable in this model is the one associated with consumption growth. This is similar to the one-factor model used in BDL (2005) and Zurek (2007) where they estimate the state-variable using several lags of observed consumption growth. The Alternate is a two-factor model which brings in an additional state-variable associated with dividend dynamics of asset i and is the focus of our paper. For both models, the loading on expected consumption growth (the (1,1) term in matrix A) is set to 1, and a_1^i (Base) and $[b_1^i \quad b_2^i]$ (Alternate) become the "leverage" parameters. The whole system - parameters and state-variables, is estimated using a Bayesian methodology described in detail in Section 5 and in the Appendix. Specifically, the state variables are obtained by a Bayesian version of Kalman Filter. First, we draw the consumption states, and then draw the dividend states conditional on the consumption states. This way, the dividend states are estimated from cash-flow data after controlling for the impact of the macro states on dividend growth. At the end, we obtain posterior distribution of the parameters of the state-space system as well as posterior distributions of filtered and smoothed state-variables.

Here we provide a preview of the results leaving the details for Section 6. We find that the inclusion of the second state-variable is both statistically and economically important. First of all, the second state variable provides a better statistical fit with an asset's dividend dynamics. Bayes

Factors (BF)[1] for the Alternate model is very high suggesting that the dividend dynamics under the Alternate model is statistically sound. Secondly, we find that the correlation between the innovations in the two state-variables is sizeable, especially for the extreme momentum portfolios. This makes the second state-variable a source of risk-premia under recursive preferences.

While both signals are sources of risk-premia, the variation in risk-premia across portfolios is largely borne by parameter heterogeneity of the asset-specific state-variable. Two main sources of cross-sectional variation in risk-premia are stationary volatilities and half-lives of the dividend-specific signals. The asset-specific state-variable in Winners are highly persistent and have high stationary volatilities, whereas the Losers have low persistence and low stationary volatilities. We show that this makes the Winner portfolio a high consumption beta asset and the Loser portfolio a low consumption beta asset. Thus, a momentum strategy of going long on the Winner and shorting the Loser is really going long on a high beta asset and going short on a low beta asset. The "leverage" parameters further exacerbate the difference in risk-premia across assets, but their main role is in determining the sign of the risk-premia.

Half-lives and stationary volatilities also play crucial roles in the time-series results. The large time-variation in risk-premia is primarily due to high volatility of the dividend signals. However, the "momentum crash" phenomenon is primarily due to the cross-sectional variation in half-lives of the dividend signals. The dividend signals in the Loser portfolios have low half-lives, while the Winner portfolios have high. This makes Loser portfolios rapidly increase in consumption beta after a recession, while Winners experience a decrease for quite some time. The net effect is a

[1]Bayes Factor (BF) is a convenient Bayesian tool for hypothesis testing. It is very similar to a likelihood-ratio test, but the main difference is that the likelihood is evaluated at the entire posterior distribution and not just at the maximum likelihood estimate. In comparing two models A (Alternate) and B (Base), the Bayes factor is computed as $BF = \frac{P(D|A)}{P(D|B)}$, where D is the data. In other words, it measures the relative probability of observing the data from model A relative to model B. If BF>1, then it gives us some indication that model A is preferable to model B. We compute each of the conditional probabilities by integrating out the parameter space, i.e. $P(D|A) = \int_{\Theta_A} P(D|A, \Theta_A)P(\Theta_A|A)d\Theta_A$. Similarly, we compute $P(D|B)$. In essence, the probability of observing the data from Model A (or B) is a weighted average of the likelihood evaluated at each point of the parameter space, where the weights are provided by the density of the model parameters. We compute BF using a Monte Carlo method. We take the entire set of draws from the posterior distribution of parameters and state-variables, evaluate the likelihood at each of those draws and take an average. Kass and Raftery (1995) provide some guidance on how to interpret BFs. They claim any BF value over 6 is a strong evidence against model B in favor of model A.

large reduction in momentum risk-premia after a recession.

4 The Model

4.1 The Aggregate Economy

In this section we build a dynamic model of asset-pricing which exploits the empirical evidence described above. Going forward we call the state-variable associated with consumption growth the macro risk-factor, and the state-variable associated with each asset's dividend growth the fundamental factor.

Let the representative agent be endowed with Duffie-Epstein preferences with unit elasticity of intertemporal substitution which is given by

$$f(C,J) = \beta(1-\gamma)J\left[\log C - \frac{\log(1-\gamma)J}{1-\gamma}\right] \tag{3}$$

where C is aggregate consumption, J is the value function, β is the time-discount parameter, and γ is risk-aversion. Let aggregate consumption C_t follow

$$\frac{dC_t}{C_t} = (\mu_C + X_t)dt + \sigma_C dW_C \tag{4}$$

$$dX_t = -\kappa_X X_t dt + \sigma_X dW_X \tag{5}$$

where the Brownian innovations, dW_C and dW_X, are independent. Aggregate consumption growth contains a macro risk-factor X_t which follows a mean-reverting process and generates long-run risks in this economy. This model for aggregate consumption growth is exactly the same as the single channel economy in Bansal and Yaron (2004).

The Appendix shows that the solution of the value function takes the form

$$J(C_t, X_t, v_t) = \frac{C_t^{1-\gamma}}{1-\gamma} e^{\bar{A}+\bar{B}X_t}$$

where $\bar{A}$, and $\bar{B}$ are constants defined in the Appendix. Duffie and Epstein (1992) shows that the pricing kernel takes the form $\Lambda_t = e^{\int_0^t f_J ds} f_c$. For our model, the pricing kernel follows the dynamics

$$\frac{d\Lambda}{\Lambda} = -r_t^f dt - \gamma\sigma_C dW_C + \bar{B}\sigma_X dW_X \tag{6}$$

where $r_t^f = \beta + \mu_C + X_t - \gamma\sigma_C^2$. There are two sources of risk in this model - one is due to a transient shock, dW_C, to aggregate consumption, and the other, dW_X, is due to inter-temporal shocks from innovations in growth rates giving rise to long-run risks in the economy. The risk-price due to the inter-temporal shocks in X_t is purely due to Duffie-Epstein preferences, and vanishes for time-separable preferences. The long-run risk price is given by $-\bar{B}\sigma_X = \frac{(\gamma-1)\sigma_X}{\kappa_X+\beta}$ which is increasing in risk-aversion, volatility and autocorrelation of the macro risk-factor. For a full treatment of recursive preferences and asset pricing, refer to Weil (1990), Duffie and Epstein (1992) and Bansal and Yaron (2004).

4.2 Cash-Flow Dynamics of the Cross-Section

Firms in our economy are exposed to a fundamental factor Y_t^i for each firm i. This state variable can be interpreted as a productivity factor as in Gomes, Kogan and Zhang (2003), or the state of profitability of the firm as in Pastor and Veronesi (2003). We do not model the investment policy or technology of the firm, but assume that the financial health of the firm and its future cash-flow decisions are reflected through Y_t^i. We further assume that the fundamental factor is not completely idiosyncratic in nature, but its shocks are imperfectly correlated with changes in the macro risk-factor. It follows an autoregressive process

$$dY_t^i = -\kappa_Y^i Y_t^i dt + \sigma_Y^i dW_{Y^i} \tag{7}$$

where $\mathrm{Corr}(dW_X, dW_{Y^i}) = \rho_{XY^i}^i > 0$. Modeling the fundamental factor as a mean reverting process is consistent with Gomes, Kogan and Zhang (2003) and Pastor and Veronesi (2003). We embed this factor directly into the cash-flow process of the firm in line with the results presented earlier. For firm i, let dividends D_t^i follow

$$\frac{dD_t^i}{D_t^i} = \left[\mu_D^i + \phi_1^i X_t + \phi_2^i Y_t^i\right] dt + \sigma_D^i dW_D^i \tag{8}$$

where dW_D^i is a Brownian shock that is uncorrelated with any other shock in the economy. Since the correlation between the shocks in X_t and Y_t^i is assumed to be positive, the impact of the fundamental factor on cash-flows is controlled by ϕ_2^i. The inclusion of the fundamental factor distributes dividend's total exposure on consumption growth to two state-variables. It generates more time-variation in expected dividend growth, and as has been shown in Figures 1-2 and Table 4, enhances the statistical fit relative to the one-factor model. Notice, our assumption of positive correlation in the innovation is somewhat innocuous. It is only intended to make the impact of aggregate shocks to cross-sectional dividends determined through the "leverage" parameters.

In reality, when we take the model to the data, we do not focus on individual firms, but on portfolios. Our assumption is that firms with similar fundamental exposures Y_t^i are grouped together. For example, the Winner portfolio is comprised of firms with similar fundamental factor Y_t^W, and the value-weighted cash-flow growth of Winner firms is determined by "leverage" parameters ϕ_1^W and ϕ_2^W. Similarly, the Loser portfolio has "leverage" parameters ϕ_1^L and ϕ_2^L and fundamental factor Y_t^L common across firms in the Loser portfolio.

4.3 Asset-Pricing

Using the pricing kernel in (6), and the dynamics of the state variables (D_t^i, X_t, Y_t^i), we solve for the cross-sectional return dynamics.

Proposition 4.1 (Asset-Pricing Quantities)**.** *The Price-Dividend Ratio of portfolio i is given by $\frac{P_t^i}{D_t^i} = M_t^i = \int_t^\infty m(X_t, Y_t^i, \tau) ds$, where $\tau = s - t$, $m(X_t, Y_t^i, \tau) = e^{P_1^i(\tau) + P_2^i(\tau) X_t + P_3^i(\tau) Y_t^i}$ and $\{P_1^i, P_2^i, P_3^i\}$ follow a system of ODEs given in the Appendix. Let cumulative excess return be given* *by* $dR_t^i = \frac{dP^i + D_t^i dt}{P_t^i} - r_t^f dt$*. The return dynamics follow*

$$\begin{aligned}
dR_t^i &= \mu_t^i dt + \sigma_V^i \cdot dW & (9) \\
\mu_t^i &= -Cov\left(\frac{d\Lambda}{\Lambda}, \frac{dP^i}{P^i}\right) \\
&= \underbrace{-\frac{M_X^i}{M^i}\bar{B}\sigma_X^2}_{LRR_X} \underbrace{-\frac{M_{Y^i}^i}{M^i}\bar{B}\sigma_X\sigma_Y^i\rho_{XY^i}^i}_{LRR_Y} & (10) \\
\sigma_V^i &= \begin{bmatrix} \sigma_D^i & \frac{M_X^i}{M^i}\sigma_X & \frac{M_{Y^i}^i}{M^i}\sigma_Y^i \end{bmatrix} & (11) \\
dW &= [dW_D^i \quad dW_X \quad dW_{Y^i}]'
\end{aligned}$$

Proof: See Appendix.

In this economy, the only shocks that are priced are long-run risk shocks. Our model falls into the category of Inter-temporal Consumption-CAPM a la Merton (1973), where the representative agent hedges against changes in expected consumption growth rate, X_t. Risk-premia on portfolio i is given by (10) and accounts for exposure of cash-flows to shocks in both X_t and Y_t^i. The first term in (10) is due to exposure to the macro risk-factor, and we call it LRR_X. The second term is due to the correlation in the innovations of the fundamental factor with the macro risk-factor, and we call it LRR_Y. We can rewrite the risk-premium on asset i as

$$\mu_t^i = \overbrace{\left(\underbrace{\frac{M_X^i}{M^i}\sigma_X}_{\text{macro beta}} + \underbrace{\frac{M_{Y^i}^i}{M^i}\sigma_Y^i\rho_{XY^i}^i}_{\text{fundamental beta}}\right)}^{\text{consumption beta}} \times \underbrace{-\bar{B}\sigma_X}_{\text{risk price}}$$

Long-run risk impacts the risk-premium of an asset through consumption beta which can be broken down into two different parts. The first is the macro beta which arises due to asset i's cash-flow exposure to the macro risk-factor. The second is a novel risk exposure due to the covariance between macro risk-factor and fundamental factor shocks of asset i and we call it fundamental beta. This exposure vanishes if $\rho^i_{XY^i} = 0$ (i.e. the macro and fundamental shocks are uncorrelated), or if $\phi_2 = 0$ (i.e. the fundamentals do not predict cash-flow growth).

The macro beta and fundamental beta take the form

$$\text{macro beta}_t = \frac{(\phi^i_1 - 1)\sigma_X}{\kappa_X} \int_t^\infty \frac{m(X_t, Y^i_t, \tau)}{\int_t^\infty m(X_t, Y^i_t, \tau) ds} \left(1 - e^{-\kappa_X \tau}\right) ds \quad (12)$$

$$\text{fundamental beta}_t = \frac{\phi^i_2 \sigma^i_Y \rho^i_{XY^i}}{\kappa^i_Y} \int_t^\infty \frac{m(X_t, Y^i_t, \tau)}{\int_t^\infty m(X_t, Y^i_t, \tau) ds} \left(1 - e^{-\kappa^i_Y \tau}\right) ds \quad (13)$$

where $\tau = s - t$. Both betas are proportional to the "leverage" parameters. Since $\rho^i_{XY^i}$ is assumed to be positive, the "leverage" parameters control the sign of the risk exposure. Our model introduces a novel way to extend Abel's (1999) "leverage" effect. Zurek (2007) and BDL (2005) model dividend's exposure to consumption through the macro risk-factor X_t, and the resulting risk-premia is proportional to the "leverage" parameter ϕ^i_1. Our model adds another channel - firm-level fundamentals correlated with aggregate consumption, which distributes dividend's total exposure on consumption. The main role of the "leverage" parameters is to determine the sign of the betas, and hence, sign of the risk-premia. Both betas are time-varying and the time-variation in risk-premium is purely due to the time-variation in the betas. Cross-sectional variation in consumption beta is primarily due to variation in fundamental beta. Differences in volatility and autocorrelation of the fundamental factors across assets are the primary reasons for this cross-sectional variation.

For high values of ϕ^i_1 and/or ϕ^i_2, firm cash-flows do not provide a hedge against aggregate consumption growth and the agent requires a premium for holding it. Likewise, for low values of ϕ^i_1 and ϕ^i_2, cash-flows are less correlated with aggregate consumption growth and the agent is willing to hold the asset with a lower premium. In the extreme case, if $\phi^i_1 < 1$ and $\phi^i_2 < 0$, then the cash-flows of the asset provide a hedge against aggregate consumption growth and the agent is willing to hold it at a negative risk-premium (discount).

Our analysis of "momentum crashes" relies on an important observation on fundamental betas.

The following proposition shows that the changes in fundamental beta are positively related to changes in the fundamental factor.

Proposition 4.2. *The fundamental beta of each asset i increases with the fundamental factor Y_t^i, as long as $\rho^i_{XY^i} > 0$.*

Proof: *Taking derivative of (13) with respect to Y_t^i*

$$\frac{\partial fundamental\ beta_t}{\partial Y_t^i} = \frac{(\phi_2^i)^2 \sigma_Y^i \rho^i_{XY^i}}{{\kappa_Y^i}^2} \frac{\int_t^\infty m(\cdot)ds \int_t^\infty m(\cdot)h(\tau)^2 ds - \left(\int_t^\infty m(\cdot)h(\tau)ds\right)^2}{\left(\int_t^\infty m(\cdot)ds\right)^2}$$

where $h(\tau) = \left(1 - e^{-\kappa_Y^i \tau}\right)$ and $\tau = s - t$. The above expression is positive for $\rho^i_{XY^i} > 0$ under a straight-forward application of Cauchy-Schwartz inequality to $\sqrt{m(\cdot)}$ and $\sqrt{m(\cdot)}h(\tau)$ - both of which are integrable in the domain, as long as the transversality condition holds.

Although the Proposition shows that fundamental betas for all assets increase or decrease with fundamental factor shocks, the behavior of fundamental betas can be very different over longer horizons. For example, consider an economic recession where negative macro shocks last for multiple periods. If two assets have different half-lives, then the fundamental factor of the two assets will evolve differently. Let's assume, for example, that at the beginning of the recession the fundamental factor for the two assets are at the steady-state, mean-zero level i.e. $Y^i_{t_0} = Y^j_{t_0} = 0$. After an adverse X_t shock, the fundamental factor of an asset with higher half-life responds slower than the asset with lower half-life. It may take several periods for the fundamental factor of the higher half-life asset to start realizing this adverse economic shock. The lower half-life asset's fundamental factor reacts more quickly, drops with prolonged negative macro shocks and starts mean-reverting when the economy starts receiving positive shocks. Thus, by the time a recession ends, a lower half-life asset's fundamental factor would start to revert back towards its steady-state mean long before a higher half-life asset's fundamental factor has "bottomed out." Thus, a lower half-life asset would experience an increase in fundamental beta exiting a recession, and a higher life-asset, which probably hasn't even "bottomed out" yet, will decrease in fundamental betas exiting the same recession. Thus, a strategy of long high half-life asset and short low half-life asset experiences a net decrease in fundamental beta exiting a recession which decreases the risk-premia of this strategy.

5 Estimation of the model

In this section we describe how we estimate the aggregate parameters and macro risk-factor X_t from consumption growth data, and cross-sectional parameters and fundamental factor Y_t^i from portfolio-level dividend growth data for each portfolio i. First, we discretize our model and write-it in a state-space framework.

$$\Delta c_{t+1} = [\mu_C + X_t] + \sigma_C Z_{t+1}^1 \tag{14}$$

$$\Delta d_{t+1}^i = \left[\mu_D^i + \phi_1^i X_t + \phi_2^i Y_t^i\right] + \sigma_D^i Z_{t+1}^2 \tag{15}$$

$$X_{t+1} = e^{-\kappa_X} X_t + \sigma_X Z_{t+1}^3 \tag{16}$$

$$Y_{t+1}^i = \left(\frac{\rho^i \sigma_Y^i}{\sigma_X}\left(X_{t+1} - e^{-\kappa_X} X_t\right) + e^{-\kappa_Y^i} Y_t^i\right) + \sigma_Y^i \sqrt{1 - {\rho^i}^2} Z_{t+1}^4 \tag{17}$$

where Δc_{t+1} is aggregate consumption growth, Δd_{t+1}^i is dividend growth of portfolio i and $\{Z_{t+1}^j, j = 1, 2, 3, 4\}$ are iid normal. The dynamics of Y_t^i are expressed slightly differently by taking into consideration the underlying correlation between the state-variables. This is a linear state-space model, where the first two equations are the "observation" equations and the second two are "state-transition" equations. We use this state-space framework to jointly estimate these state-variables along with the parameters using a Bayesian approach called Markov Chain Monte Carlo (MCMC). The use of MCMC techniques in empirical finance literature is very prevalent. Some examples include Eraker (2001, 2004), Jacquier, Johannes and Polson (2007), Johannes, Polson and Stroud (2009), among others. A good overview is in Johannes and Polson (2009).

Here we provide a brief overview of the estimation procedure and leave the details for the

Appendix. Let the full parameter space and state variables that guide the system be given by

$$\Theta = \{\mu_C, \sigma_C, \kappa_X, \sigma_X, \mu_D^i, \sigma_D^i, \kappa_Y^i, \sigma_Y^i, \phi_1^i, \phi_2^i, \rho^i\}$$
$$(X, Y^i)$$

Here X and Y^i denote the full time-series $\{X_1, \cdots, X_T\}$ and $\{Y_1^i, \cdots, Y_T^i\}$. Our goal is to get joint estimates of $p(\Theta, X, Y^i|\text{data})$. The MCMC algorithm allows us to draw them conditional on each other, i.e.

$$p(X|\Theta, Y^i, \text{consumption data}) \tag{18}$$
$$p(Y^i|\Theta, X, \text{cash-flow data}) \tag{19}$$
$$p(\Theta|X, Y^i, \text{consumption or cash-flow data}) \tag{20}$$

The conditional distributions (18)-(19) are filtering problems. We observe consumption and dividend growth, but we do not observe their respective growth rates. We filter these growth rates using a Bayesian version of Kalman filter called Forward Filtering Backward Sampling (Carter and Kohn (1994), Frühwirth-Schnatter (1994)). Note, the second filtering problem in (19) is conditional on X as can be seen from the observation equation (15) and state-equation (17). Thus, the posterior distribution of Y^i is drawn by first factoring out the contribution of aggregate consumption on cross-sectional cash-flows. Having obtained the state-variables, we turn to estimating the parameters of the state-space in (20). This is also done one-at-a-time conditional on the other parameters, i.e. $\Theta_j|\Theta_{-j}, X, Y, \text{data}$, where Θ_{-j} is the rest of the parameters modulo the $j-$th one. In this simple, linear state-space setting all the posterior distributions of the parameters are available in elementary, conjugate form. The exact parameters of these posterior distributions is discussed in detail in Allenby, McCulloch and Rossi (2005). Details of the procedure are left in the Appendix. In addition, we set risk aversion $\gamma = 10$ and time-discount parameter $\beta = .05$.

5.1 Discussion on Priors

For our MCMC estimation, we need prior information for our parameters and state-variables. For the cross-sectional parameters, for every asset i we first obtain a proxy for Y_t^i via an iterative estimation method that minimizes prediction error.[2] This method also produces the parameter estimates of the cash-flow process. The prior means of the cross-sectional parameters are set to the point estimates obtained by this procedure. We use the same method to produce the prior means for the parameters governing the X_t process. While the prior means of the parameters are fairly informative, the distributions are all diffuse and the posterior distribution of the parameters reveal that the MCMC has explored a wide space in multiple dimensions.

6 Empirical Analysis

6.1 Parameters and State-Variables

In this section we explore our model contribution in understanding momentum returns. We first discuss the parameter estimates and latent state-variables obtained from the MCMC. Of particular interest are the dynamics of the fundamental factors in cash-flows which facilitates a new channel in which to explore cross-sectional and time-series variation in expected returns.

The introduction of the asset-specific factor enhances our understanding of macro linkages in the cross-section of average returns. Econometrically, this is accomplished by filtering the signals from cross-sectional cash-flows while conditioning on the macro risk-factor. First, we show that the fundamental factors provide greater statistical fit with their respective dividend growth series. For one, we visually inspect the fits provided by the two-factor model relative to the one-factor model containing the macro risk-factor only. Figures 1 and 2 show expected dividend growth rates from the bivariate and univariate models against actual dividend growth rates for two sets of momentum assets - in All Stocks (Figure 1) and in High/Small stocks (Figure 2). The

[2] This is accomplished by using the **ssest** routine in the System Identification Toolbox of Matlab.

fundamental factor is clearly picking up signals from dividend growth, over and above the macro risk-factor, which allows it to explain more of the dividend dynamics than the univariate model. Next, we also compute Bayes Factors (BF) for all portfolios in our sample and report them in Table 6. Based on the Kass and Raftery (1995) criterion of evaluating Bayes Factors, the case for the fundamental factor is strong for each of the portfolios considered.

The fundamental factor also provides a new channel for risk-premia since it extracts information from the cash-flows of each portfolio which are correlated with the macro risk-factor. Movements in cash-flows could be attributable to both idiosyncratic and systematic reasons. However, the Bayesian Kalman Filter is able to filter the signal from dividend growth which is systematic in nature. Figures 3 and 4 graphically examine the relationship between innovations in the two factors (standardized to have unit variance) in the extreme momentum portfolios of All Stocks and High/Small stocks. In these figures, the dashed-line showing changes in X_t are the same across all assets, whereas the solid line showing changes in Y_t^i are asset specific. Notice from both figures that the Bayesian methodology has extracted signals which share correlated dynamics with the macro risk-factor. Interestingly, the correlations are not strikingly high. In the extreme portfolios, posterior medians of correlations range between 0.22 (Low/Small/Loser) and 0.50 (High/Small/Winner).

The fundamental factors across the momentum portfolios differ through their parameter heterogeneities. We summarize these heterogeneities through two statistics - half-lives and stationary volatilities reported in Table 7. The fundamental factors of Loser portfolios tend to have lower half-lives, while for Winners the half-lives are roughly twice as high as the Losers. The fundamental factors of Losers also have lower stationary volatilities than Winners, however, the difference is not as stark. The momentum cash-flows also differ on "leverage" parameters reported in Table 5. The "leverage" parameter of the fundamental factor tends to be lower (and often even negative) for Losers, and higher for Winners, but the variation is nowhere near the variation of the "leverage" parameter in BDL (2005) and Zurek (2007). On the other hand, the "leverage" parameter on the macro risk-factor has almost no variation across assets.

6.2 Asset Pricing Quantities

6.2.1 Unconditional Results

Based on the draws from the posterior distributions of the parameters and state-variables we compute risk-premium (10) for each portfolio. The time-series average of the median, 5-th and 95-th percentiles of risk-premium are reported in Table 8. Table 8 also exhibits the model implied average risk-premium of the "Winner minus Loser" portfolios in different sub-sections. Our model is able to produce the unconditional average momentum returns observed in the data. For example, the average return of long Winner and short Loser strategy in All Stocks is 2.40% in the data, and the average risk-premium of this strategy is 2.34% in the model. Similarly, the long/short strategy yields 4.29% return on average for Low/Small stocks, and 3.96% in the model. Sharpe Ratios of portfolios based on data are also in line with the model estimates, except for the Winner portfolios. There we are able to generate only about 50-80% of the observed Sharpe Ratio.

Notice that there is considerable variation in average return across the portfolios. The Value premium generates significant difference in the momentum portfolios between High/Small and Low/Small. The average returns of the Winner and Loser portfolios in High/Small is much higher than Low/Small. Similarly, the size premium implies high average returns in Winner and Loser in Small stocks. Table 8 shows that our model is able to match this heterogeneity across the momentum assets. Two interesting portfolios stand out - the Loser portfolio constructed out of all stocks and the Loser portfolio in Low/Small. They both have negative average returns in the data, while in our model their negative average risk-premium is due to negative macro and fundamental betas. These betas are negative because the median "leverage" parameters of these portfolios are either less than one (for ϕ_1^i) or negative (for ϕ_2^i). Negative beta assets benefit investors through hedging against adverse economic risk. In the case of momentum strategy, they provide positive risk-premium because these assets are being held short.

We now turn to explain which of the factors determines the lion's share of average risk-premium generated by the model. Let us first consider the long-run risk due to exposure to the macro factor, X_t. The loading on the macro factor, ϕ_1^i, is small. This makes the macro beta small while the corresponding long-run risk-premium, LRR_X, is insignificant, as shown in Table 9. In absolute value terms, the highest median risk-premium from this channel is 44 Basis Points for Portfolio 8 (under All Stocks) and 80 Basis Points for the Winner Portfolio (under Small). Perhaps more discouraging is the fact that most risk-premia from this channel is negative. Moreover, there is no discernable cross-sectional differences in ϕ_1^i. In fact, the "macro beta" is slightly negatively related to average returns as can be seen from the top plot of Figure 5. Clearly, this channel is not suitable for asset pricing purposes.

In our model, the fundamental factor is the key variable for all asset pricing results. Table 9 shows that almost all risk-premia is due to long-run risk generated from the exposure of the fundamental factor to the macro risk-factor, LRR_Y . Table 9 also shows that the corresponding "fundamental beta" is large, and monotonically increasing with risk-premia. The positive relation between "fundamental beta" and average return in the data is shown in the bottom plot of Figure 5. Clearly, the fundamental factor is responsible for not only generating high risk-premia, but also in generating the cross-sectional variation in risk-premia which lines up quite well with the data.

Given the large variation of fundamental betas across the momentum portfolios, we examine which cash-flow parameters create this heterogeneity. In Figure 6, we plot fundamental betas against different cash-flow parameters. Three cash-flow variables - "leverage (ϕ_2^i)", half-life, and stationary volatility, are positively correlated with fundamental betas. The result is consistent with recursive preferences of an agent who prefers early resolution of uncertainty. Exposure to factors which are volatile and take longer to resolve their uncertainty generate high risk-premia. The result is also consistent with Bansal and Shaliastovich (2010) who show that when investors are not confident about firm fundamentals, the risk-premia rises. Taking together, recursive preferences and volatile cross-sectional dynamics seem to explain key unconditional asset pricing quantities.

6.2.2 Time-Series Results

Having been able to match key unconditional properties of the momentum payoffs, we turn to assess the time-series properties implied by our economic setup. First, our theory generates time-varying momentum risk premia as shown in Figures 7 and 8. The time-variation is fairly modest among All Stocks (top graph of Figure 7), but it is sizeable among Small, High/Small and Low/Small stocks. Exiting a recession, the risk-premia of momentum typically diminishes. To understand this cyclical property, we first show the drivers of time-variation in risk-premia focusing on the extreme momentum portfolios.

Recall, that total risk-premium, μ_t^i, of an asset is the sum of long-run risk premium attributable to cash-flow exposure to the macro risk-factor, LRR_X, and exposure of the fundamental factor to the macro risk-factor LRR_Y.

$$\mu_t^i = LRR_{X}{}_t^i + LRR_{Y^i}{}_t^i$$

Figures 9-12 plot the two ingredients establishing the long-run risk-premia, LRR_X and LRR_Y, for the extreme momentum portfolios. Figure 9 shows the time-series of each premia for Winner and Loser portfolios in All Stocks. Clearly, the unconditional difference in risk-premia between Winner and Loser stocks is due to the difference in LRR_Y in the two portfolios, yet the time-variation in LRR_Y is modest. In contrast, Figures 10-12 exhibit much higher time-variation of LRR_Y in the momentum portfolios in Small, Low/Small and High/Small deciles. However, LRR_X shows almost no time-variation in these portfolios. Thus, long-run risk from exposure of the fundamental factor to the macro risk-factor drives time-series variation in risk-premia in the extreme Winner and Loser portfolios. Each of the long-run risk components can be expressed as

$$\begin{aligned} LRR_{X}{}_t^i &= \text{macro beta}_t^i \times \text{Market Price of risk of X} \\ LRR_{Y}{}_t^i &= \text{fundamental beta}_t^i \times \text{Market Price of risk of X} \end{aligned}$$

In our economy, the market price of risk is time invariant. Thus, all time-variation in risk-premia is due to fundamental beta. The much higher time-variation in fundamental beta can be attributed to higher volatility in the fundamental factor Y_t^i for these portfolios, whereas macro beta is roughly constant for every portfolio because X_t is relatively flat in comparison.

The time-variation in fundamental beta allows one to provide a counterpart to the beta-reversal phenomenon pointed out by Daniel and Moskowitz (2014). In particular, we show a rapid increase (decrease) of fundamental beta in Losers (Winners) following a prolonged stressful economic regime. Although the directional changes are not high enough to have a net negative fundamental beta, we generate a sizeable decrease in fundamental beta of the model implied momentum strategy. Our explanation is based purely on the cross-sectional variation in half-life. For this analysis, it is important to recall the implication of Proposition 4.2 for the momentum portfolios. The proposition implies that fundamental betas for both Winner and Loser portfolios should decrease (increase) with negative (positive) shock to the fundamental factor. However, the fundamental factors for Winners and Losers display different mean reversion due to difference in half-lives. Consequently, their fundamental factors evolve differently around an economic crisis which have differential effects in fundamental betas. The end result is that when the economy exits a recession, Loser portfolios tend to increase in fundamental betas while Winner portfolios diminish. This implies that the momentum strategy of long Winner and short Loser experiences a net decrease in fundamental beta which lowers momentum risk-premia upon exiting a recession. We explain this phenomenon in greater detail below.

We start with some anecdotal evidence on the dynamics of the fundamental factor. Figure 13 plots the fundamental factors (standardized to have unit standard deviation) for Winner and

Loser portfolios in Small stocks during two past recessionary periods. The top graph of Figure 13 shows the fundamental factors for the recession between June 1981 and November 1982, while the bottom graph shows the same for the recession between February 2001 and November 2001. Both graphs show that the fundamental factor for the Loser portfolio mean-reverts much quicker than the Winner portfolio. Entering a recession, both Winner and Loser portfolios receive negative macro-shocks. However, the fundamental factor for Winners does not respond immediately, as it is highly persistent and it continues on its previous growth path. The successive negative macro factor shocks eventually force the Winner fundamental factor to mean-revert, but the effect of past negative shocks last much longer and it stays on the negative growth path for many more quarters after the economy exits the recession. The fundamental factor for the Loser portfolio, on the other hand, shows much quicker mean-reversion. It responds to the negative macro shocks almost immediately. That is, it starts dipping when the economy enters a recession and starts mean-reverting almost as soon as the recessions end. After exiting the recession, the fundamental factor rises sharply for Losers, and for several quarters, stays higher than the Winners'.

This holds not just anecdotally but for the full time-series as well. Recall that cross-sectional variation in half-lives has been a major determinant of the cross-sectional variation in risk-premia. Revisiting Table 7, it is evident that half-lives for Loser portfolios are much smaller than for Winner portfolios, explaining the difference in dynamics in the fundamental factor. Its effect on fundamental betas now directly follow from Proposition 4.2. The fundamental beta of Loser portfolios rise quickly right after a recession, but fundamental beta of Winners continue to fall. Thus, Loser portfolios have higher fundamental beta after a recession, while Winner portfolios have lower. A full time-series of fundamental betas of Winners and Losers are displayed in Figures 14-15.

We tabulate the differences in fundamental betas in Winners and Losers among different equity portfolios in Table 10. We use all the NBER recessionary time-periods, except for Jan 1980-June 1980, because this recession lasted only for six months. For each momentum portfolio, we report the percentage change in average fundamental beta from during recession to four quarters after the recession. For example, consider the Winner and Loser portfolios in Small Stocks. Following the recession from February 2001 to November 2001, fundamental beta on Losers (Winners) increased (decreased) by 63.32 (10.81) percent on average for the next four quarters. Loser portfolios in Small stocks experience positive changes in fundamental beta following five out of six recessions, and Winner portfolios experience negative changes for four out of six. Sizeable changes in fundamental beta post-recession are also recorded in Low/Small and High/Small, as can be seen in Figures 14-15. However, the changes are modest in the momentum portfolios in All Stocks, where the time-variation in risk-premium is also small. In all, out of 22 different portfolio/recessionary time-period pairings, Loser portfolios increase in fundamental beta 18 times and Winner portfolios decrease in fundamental beta 16 times. Coming out of the recession a momentum strategy of long Winners and short Losers experiences a net decrease in fundamental beta 15 out of 22 times, which implies risk-premia from this strategy also ought to be lower upon exiting a recession. We provide evidence in favor of that hypothesis in Figures 7-8.

The time-variation in risk-premia also generates predictability in our model. The model implied risk-premia, μ_t^i, is an estimate of conditional expected return, and, as such, gives us a forecast of future returns of asset i. We can directly assess the predictive power of risk premia using Mean Squared Errors (MSE) with respect to realized future returns. For any return horizon, k, let

$$\mathrm{MSE}_1 = \frac{1}{T-k-1}\sum_{t=1}^{T-k}(r_{t+k}^i(\mathrm{data}) - \mu_t^i(\mathrm{model}))^2$$

where r_{t+k}^i is the k-period ahead real cumulative return of asset i. In order to compute μ_t^i one has to know Y_t^i. However, Y_t^i is not estimated in real-time. It is filtered using the full-sample of dividend growth rate of each asset implying a certain look-ahead-bias to our risk-premia estimate. With that in mind, we test the in-sample performance of our model relative to another in-sample performance of return predictability. We compare MSE_1 from our model with the in-sample MSE_2

based on a standard predictive regression (see, e.g., Campbell and Shiller (1988))

$$r^i_{t+k} = a^i + b^i \frac{D^i}{P}_t + \epsilon^i_{t+k}$$

Our performance metric is the ratio $\mathrm{MSE}_1/\mathrm{MSE}_2$. The results of this analysis is reported in Table 11 for horizons $k = 1, 2, 3$ quarters. Notice that performance varies across the portfolios. For momentum portfolios in All Stocks, there is better performance in the lower return deciles. In the higher return deciles our formulation performs just as good, often worse, than the traditional predictive regressions. Our formulation performs particularly poorly in the middle Portfolios 3 and 5. However, much better performance is recorded for the other momentum assets. It coincides with the fact that we have been able to generate higher time-variation in risk-premia for these assets. Moreover, the ratio of mean squared errors remains stable across horizons, which seems to indicate that the results are not driven by mere chance. Ultimately, the time-series analysis shows that the fundamental factor, extracted out of cash-flow growth, generates variation in expected return that does a credible job in predicting future returns.

7 Conclusion

This paper presents a flexible dynamic model of cash-flows which explains significant amount of momentum in equity prices. In addition, we provide a theoretical basis of momentum crashes discussed in Daniel and Moskowitz (2014). The work horse in our model is a state-variable filtered from cash-flows of each asset and aggregate consumption growth. Indeed, cross-sectional variation in average return and Sharpe ratios among various equity portfolios are explained by the exposure of that state variable to aggregate consumption growth rate. Our model has been successful in explaining both conditional and unconditional features of momentum payoffs. Looking forward, we conjecture that the rich cross-sectional dynamics of fundamentals introduced here can be used to study other anomalous patterns in the cross section of average returns. We leave that for future work.

References

[1] Abel, A., 1999. Risk Premia and Term Premia in General Equilibrium. Journal of Monetary Economics 43, 3-33.

[2] Allenby, G.M., Rossi, P.E., McCulloch R., 2005. Bayesian Statistics and Marketing, Wiley.

[3] Asness, C.S., Moskowitz, T.J., Pedersen, L.J., 2013. Value and Momentum Everywhere. Journal of Finance 68, 929-985.

[4] Avramov, D., Chordia, T., 2006. Asset Pricing Models and Financial Market Anomalies. Review of Financial Studies 19, 1001-1040.

[5] Avramov, D., Chordia, T., Jostova, G., Philipov, A., 2007. Momentum and Credit Rating. Journal of Finance 62, 2503-2520.

[6] Bansal, R., Dittmar, R.F., Lundblad, C.T., 2005. Consumption, Dividends and the Cross Section of Equity Returns. Journal of Finance 60, 1639-1672.

[7] Bansal, R., Shaliastovich, I., 2010. Confidence Risks and Asset Prices. American Economic Review P&P 100, 537-541.

[8] Bansal, R., Yaron, A., 2004. Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles. Journal of Finance 59, 1481-1509.

[9] Barberis, N., Schleifer, A., Vishny, R., 1998. A Model of Investor Sentiment. Journal of Financial Economics 49, 307-343.

[10] Berk, J.B., Green, R.C., Naik, V., 2002. Optimal Investment, Growth Options and Security Returns. Journal of Finance 54, 1553-1607.

[11] Breeden, D.T., 1979. An Intertemporal Asset Pricing Model with Stochastic Consumption and Investment Opportunities. Journal of Financial Economics 7, 265-296.

[12] Campbell, J.Y., Shiller, R.J., 1988. The Dividend-Price Ratio and Expectations of Future Dividends and Discount Factors. Review of Financial Studies, 195-228.

[13] Carter, C.K., Kohn R., 1994. On Gibbs Sampling for State Space Models. Biometrika 81, 541-553.

[14] Cooper, M.J., Gutierrez, R.C., Hameed, A., 2004. Market States and Momentum. Journal of Finance 59, 1345-1365.

[15] Daniel, K., Hirshleifer, D., Subrahmanyam, A., 2002. Investor Psychology and Security Market Under- and Overreactions. Journal of Finance 53, 1839-1885.

[16] Daniel, K., Moskowitz, T.J., 2014. Momentum Crashes. Working Paper, University of Chicago.

[17] Duffie, D., Epstein, L.G., 1992. Asset Pricing with Stochastic Differential Utility. Review of Financial Studies 5, 411-436.

[18] Eraker, B., 2001. MCMC Analysis of Diffusion Models with Application to Finance. Journal of Business and Economic Statistics 19, 177-191.

[19] Eraker, B., 2004. Do Stock Prices and Volatility Jump? Reconciling Evidence from Spot and Option Prices. Journal of Finance 59, 1367-1403.

[20] Fama, E.F., French, K.R., 1993. Common Risk Factors in the Returns on Stocks and Bonds. Journal of Financial Economics 33, 3-56.

[21] Fama, E.F., French, K.R., 1996. Multifactor Explanation of Asset Pricing Anomalies. Journal of Finance 51, 55-84.

[22] Fama, E.F., French, K.R., 2008. Dissecting Anomalies. Journal of Finance 63, 1653-1678.

[23] Ferson, W.E., Nallareddy, S.K., Xie, B., 2013, The "Out-of-Sample" Performance of Long-Run Risk Models. Journal of Financial Economics 107, 537-556.

[24] Frühwirth-Schnatter, S., 1994. Data Augmentation and Dynamic Linear Models. Journal of Time Series Analysis 15, 183-202.

[25] Geczy, C.C., Samonov, M., 2013. 212 Years of Price Momentum (The World's Longest Backtest: 1801-2012). Working Paper, University of Pennsylvania.

[26] Gomes, J.F., Kogan, L., Zhang, L., 2003. Equilibrium Cross-Section of Returns, Journal of Political Economy 111, 693-732.

[27] Hong, H., Lim, T., Stein, J.C., 2002. Bad News Travels Slowly: Size Analyst Coverage, and the Profitability of Momentum Strategies. Journal of Finance 55, 265-295.

[28] Jacquier, E., Johannes, M., Polson, N., 2007. MCMC Maximum Likelihood for Latent State Models. Journal of Econometrics 127, 615-640.

[29] Jegadeesh, N., Titman, S., 1993. Returns to Buying Winners and Selling Losers: Implications for Stock Market Efficiency. Journal of Finance 48, 35-91.

[30] Jegadeesh, N., Titman, S., 2001. Profitability of Momentum Strategies: An Evaluation of Alternative Explanations. Journal of Finance 56, 699-720.

[31] Jegadeesh, N., Titman, S., 2002. Cross-Sectional and Time-Series Determinants of Momentum Returns. Review of Financial Studies 15, 143-157.

[32] Johannes, M., Polson, N., Stroud, J., 2009, Optimal Filtering of Jump-Diffusions: Extracting Latent States from Asset Prices. Review of Financial Studies 22, 2259-2299.

[33] Johannes, M., Polson, N., 2009, MCMC Methods for Financial Econometrics. Handbook of Financial Econometrics (editors Yacine Ait-Sahalia and Lars Peter Hansen), 1-72.

[34] Johnson, T.C., 2002. Rational Momentum Effects. Journal of Finance 57, 585-608.

[35] Kass, R.E., Raftery, A.E., 1995. Bayes Factors. Journal of the American Statistical Association 90, 773-795.

[36] Lucas, R., 1978. Asset Prices in an Exchange Economy. Econometrica 46, 1429-1446.

[37] Merton, R.C., 1973. An Intertemporal Capital Asset Pricing Model. Econometrica 41, 867-887.

[38] Pastor, L., Veronesi, P., 2003, Stock Valuation and Learning about Profitability. Journal of Finance 58, 1749-1789.

[39] Schwert, G.W., 2003. Anomalies and Market Efficiency, in: Constantinides, G.M., Harris, M., Stulz, R.M. (Eds.), Handbook of the Economics of Finance. Elsevier, Amsterdam, pp. 939-974.

[40] Weil, P., 1990. Non-Expected Utility in Macroeconomics. Quarterly Journal of Economics 105, 29-42.

[41] Yang, W., 2007. Time-Varying Exposure to Long-Run Consumption Risk. Working Paper, University of Rochester.

[42] Zurek, P., 2007. Momentum and Long-Run Risks. Working Paper, University of Pennsylvania.

Table 1: The table presents descriptive statistics of returns and cash-flow (dividend+repurchases) growth of momentum portfolios. The underlying assets are ordinary common shares of stocks traded in NYSE, AMEX or NASDAQ. Loser represents the lowest decile and Winner the highest decile portfolio. The breakpoints for the deciles come from Ken French's Prior Return Breakpoints. Both returns and cash-flow series are value-weighted and converted to real using the PCE deflator. The data are quarterly and cover the time period 1963-2013.

Portfolio	Avg. Real Return	Std. Dev.	Sharpe Ratio	Average no. of Firms	Avg. Real Div. Growth	Winner - Loser	t-stat	p-value
Loser	-1.07	6.57	-0.16	588	-0.84	2.40	3.94	0.000
2	-0.50	5.28	-0.10	356	0.45			
3	0.07	4.35	0.02	295	1.13			
4	0.11	3.96	0.03	269	1.31			
5	0.16	3.79	0.04	258	1.42			
6	0.37	3.75	0.10	247	1.47			
7	0.68	3.84	0.18	251	1.98			
8	0.72	3.98	0.18	264	2.28			
9	0.98	4.33	0.23	292	2.26			
Winner	1.33	5.73	0.23	445	2.68			

Table 2: The table presents descriptive statistics of returns and cash-flow (dividend+repurchases) growth of momentum portfolios in different size deciles. The underlying assets are ordinary common shares of stocks traded in NYSE, AMEX or NASDAQ. Big contains stocks which are above the 70th percentile in market cap and Small contains stocks below the 30th percentile. The breakpoints for the size deciles come from Ken French's ME (Market Equity) breakpoints. Within each size portfolio, three momentum portfolios are formed based on prior returns. The breakpoints for the momentum portfolios are such that each portfolio has roughly the same number of stocks. Both returns and cash-flow series are value-weighted and converted to real using the PCE deflator. The data are quarterly and cover the time period 1963-2013.

Portfolio		Avg. Real		Sharpe	Average no.	Avg. Real	Winner -		
Size	Mom.	Return	Std. Dev.	Ratio	of Firms	Div. Growth	Loser	t-stat	p-value
Small:	Loser	2.03	17.20	0.12	622	-0.81	2.69	-1.79	0.074
	Middle	3.63	12.80	0.28	611	1.42			
	Winner	4.72	13.03	0.36	610	2.20			
Medium:	Loser	2.13	12.58	0.17	257	-0.01	1.65	-1.43	0.155
	Middle	2.86	9.64	0.30	262	0.96			
	Winner	3.78	10.89	0.35	262	1.64			
Big:	Loser	1.37	9.17	0.15	137	-0.10	1.34	-1.50	0.135
	Middle	1.80	7.71	0.23	138	0.35			
	Winner	2.71	8.98	0.30	137	0.92			

Table 3: The table presents descriptive statistics of returns and cash-flow (dividend+repurchases) growth of momentum portfolios in different Book-to-Market (BM) and Size deciles. The underlying assets are ordinary common shares of stocks traded in NYSE, AMEX or NASDAQ. High contains stocks which are above the 70th percentile in book value and Low contains stocks below the 30th percentile. The breakpoints for the BM deciles come from Ken French's BM breakpoints. Within each BM portfolio, three size portfolios are formed based on market cap. Within each size portfolio, three momentum portfolios are formed based on prior returns. The breakpoints for the sub-portfolios are such that each portfolio has roughly the same number of stocks. Both returns and cash-flow series are value-weighted and converted to real using the PCE deflator. The data are quarterly and cover the time period 1971-2013.

B/M	Portfolio Size	Mom.	Avg. Real Return	Std. Dev.	Sharpe Ratio	Average no. of Firms	Avg. Real Div. Growth	Winner - Loser	t-stat	p-value
Low:	Small:	Loser	-0.72	20.85	-0.04	117	-3.25	4.29	-2.14	0.033
		Middle	1.42	16.90	0.08	112	-1.09			
		Winner	3.57	16.78	0.21	111	1.70			
	Medium:	Loser	0.97	15.79	0.06	146	-0.79	2.43	-1.55	0.122
		Middle	2.48	12.81	0.19	148	1.09			
		Winner	3.41	13.77	0.25	153	1.94			
	Big:	Loser	0.93	10.46	0.09	169	-0.29	1.76	-1.58	0.115
		Middle	1.82	9.17	0.20	174	0.70			
		Winner	2.70	10.59	0.26	176	1.70			
Moderate:	Small:	Loser	1.64	18.75	0.09	98	-0.04	2.88	-1.65	0.100
		Middle	2.46	13.63	0.18	94	1.70			
		Winner	4.52	13.85	0.33	91	2.83			
	Medium:	Loser	2.33	14.16	0.16	130	0.72	1.63	-1.19	0.234
		Middle	2.77	10.84	0.26	130	1.29			
		Winner	3.96	11.61	0.34	128	2.42			
	Big:	Loser	1.27	10.38	0.12	147	0.08	1.13	-1.11	0.267
		Middle	1.78	7.85	0.23	148	0.54			
		Winner	2.40	8.71	0.28	144	1.35			
High:	Small:	Loser	2.27	22.08	0.10	68	-0.96	3.47	-1.66	0.097
		Middle	3.49	16.42	0.21	62	1.15			
		Winner	5.73	16.88	0.34	58	4.50			
	Medium:	Loser	2.58	17.26	0.15	95	0.10	2.07	-1.28	0.201
		Middle	3.86	12.86	0.30	90	1.71			
		Winner	4.65	12.90	0.36	87	2.70			
	Big:	Loser	2.21	12.29	0.18	111	0.12	0.84	-0.71	0.479
		Middle	2.39	9.37	0.26	110	0.91			
		Winner	3.05	10.00	0.31	106	1.57			

Table 4: The table presents parameter estimates of the consumption growth dynamics in (14) and (16). The state-space is estimated via Bayesian MCMC. The posterior distribution is formed by iterating the Gibbs sampler 25,000 times and discarding the first 10,000 for burn-in. For each parameter, we present the posterior median along with the 5th and 95th percentiles of the simulated posterior draws. The data used for consumption growth is Personal Consumption Expenditures (PCE) in Nondurables and Services and converted to real using the PCE price deflator. The sample period covers from 1963-2013.

$\mu_C X10$	0.05 (0.05, 0.05)
$\sigma_C X10$	0.06 (0.04, 0.06)
κ_X	0.17 (0.14, 0.22)
$\sigma_X X10$	0.04 (0.03, 0.05)

Table 5: The table presents parameter estimates of the cash-flow (dividends+repurchases) growth dynamics in (15) and (17). The state-space is estimated via Bayesian MCMC. The posterior distribution is formed by iterating the Gibbs sampler 25,000 times and discarding the first 10,000 for burn-in. For each parameter, we present the posterior median along with the 5-th and the 95-th percentiles of the simulated posterior draws. The first panel presents posterior distributions of parameters for five momentum portfolios constructed from All Stocks which are described in Table 1. The second panel presents posterior distributions of parameters for three momentum portfolios constructed from the stocks in the lowest size decile (Small) portfolio described in Table 2. The third panel presents posterior distributions of parameters for three momentum portfolios constructed from the stocks in the lowest BM and Size deciles (Low/Small) portfolio described in Table 3. The fourth panel presents posterior distributions of parameters for three momentum portfolios constructed from the stocks in the highest BM and lowest Size deciles (High/Small) portfolio described in Table 3. Cash-flow growth data is value weighted and converted to real using the PCE price deflator. The sample period covers from 1963-2013 (for All Stocks and Small Stocks) and 1971-2013 (for Low/Small Stocks and High/Small Stocks).

		Portfolios				
		Loser	3	5	8	Winner
All Stocks	μ_D	-0.01	0.01	0.01	0.02	0.03
		(-0.03, 0.01)	(-0.00, 0.03)	(0.00, 0.03)	(0.01, 0.04)	(0.01, 0.05)
	σ_D	0.11	0.09	0.08	0.08	0.08
		(0.11, 0.12)	(0.07, 0.10)	(0.06, 0.10)	(0.08, 0.09)	(0.05, 0.11)
	ϕ_1	0.75	0.88	0.93	0.63	1.13
		(-2.62, 1.88)	(-2.77,2.00)	(-1.32, 2.16)	(0.06, 2.25)	(0.038, 2.22)
	ϕ_2	-1.92	-0.77	1.62	1.82	1.59
		(-3.08,-0.73)	(-2.89, 1.58)	(-0.25, 2.73)	(-1.13, 3.25)	(0.49, 2.68)
	κ_Y	0.13	0.14	0.14	0.08	0.07
		(0.05,0.18)	(0.08, 0.21)	(0.08, 0.19)	(0.11, 0.20)	(0.05, 0.09)
	$\sigma_Y X10$	0.03	0.02	0.02	0.04	0.05
		(0.01,.04)	(0.01, 0.03)	(0.02, 0.05)	(0.03, 0.10)	(0.02, 0.07)
	ρ	0.49	0.15	0.10	0.25	0.43
		(0.22,0.68)	(-0.15, 0.40)	(-0.15, 0.37)	(-0.02, 0.46)	(0.12, 0.65)
		Loser	Middle	Winner		
Small Stocks	μ_D	-0.01	0.01	0.02		
		(-0.03,0.01)	(0.01, 0.06)	(0.01, 0.04)		
	σ_D	0.08	0.06	0.06		
		(0.75, 0.09)	(0.04, 0.81)	(0.02, 0.09)		
	ϕ_1	0.42	1.05	0.28		
		(-0.16, 2.25)	(-0.19, 1.19)	(-1.15, 1.72)		
	ϕ_2	0.89	2.02	1.41		
		(-0.56, 3.81)	(0.56, 3.38)	(0.48, 2.27)		
	κ_Y	0.12	0.08	0.06		
		(0.06, 0.15)	(0.06, 0.11)	(0.04, 0.09)		
	σ_Y	0.01	0.01	0.01		
		(0.00, 0.02)	(0.01, 0.02)	(0.00, 0.02)		
	ρ	0.35	0.49	0.46		
		(0.08, 0.57)	(0.17, 0.60)	(0.32, 0.76)		
Low Small Stocks	μ_D	-0.03	-0.01	0.02		
		(-0.08, 0.01)	(-0.04, 0.02)	(-0.04, 0.05)		
	σ_D	0.15	0.12	0.11		
		(0.10, 0.19)	(0.08, 0.19)	(0.07, 0.18)		
	ϕ_1	-0.18	0.45	0.65		
		(-1.68, 2.03)	(-1.26, 2.15)	(-1.05, 2.26)		
	ϕ_2	-1.05	1.51	1.63		
		(-2.26, 0.81)	(0.36, 2.93)	(0.42, 2.82)		
	κ_Y	0.10	0.07	0.06		
		(0.04, 0.12)	(0.01, 0.14)	(0.05, 0.15)		
	σ_Y	0.01	0.02	0.02		
		(0.00, 0.02)	(0.00, 0.02)	(0.00, 0.03)		
	ρ	0.22	0.26	0.49		
		(-0.24, 0.42)	(0.06, 0.60)	(0.18, 0.73)		
High Small Stocks	μ_D	-0.01	0.01	0.05		
		(-0.06, 0.043)	(-0.04, 0.06)	(0.00, 0.09)		
	σ_D	0.16	0.12	0.11		
		(0.12, 0.18)	(0.08, 0.16)	(0.06, 0.17)		
	ϕ_1	1.11	0.67	0.90		
		(-0.22, 2.70)	(-1.15, 2.13)	(0.06, 2.98)		
	ϕ_2	1.13	1.33	1.48		
		(0.53, 2.5)	(0.75, 2.89)	(0.20, 2.47)		
	κ_Y	0.07	0.07	0.06		
		(0.01, 0.14)	(0.07, 0.15)	(0.03, 0.11)		
	σ_Y	0.01	0.01	0.01		
		(0.00, 0.02)	(0.00, 0.01)	(0.00, 0.02)		
	ρ	0.34	0.41	0.50		
		(0.04, 0.65)	(0.08, 0.52)	(0.10, 0.64)		

Table 6: This Table reports the Bayes Factor of the Alternate Model relative to the Base Model. The parameters and state-variable of the Alternate Model are from (14)-(17), estimated using MCMC and reported in Table 5. The parameters and state-variables of the Base Model are also from (14)-(16), but by shutting off the dynamics of Y_t^i for every asset in the cross-section. Bayes Factor(BF) is computed as $BF = \frac{P(D^i|A^i)}{P(D^i|B^i)}$, where D^i is the dividend data of every asset i. We compute each of the conditional probabilities by integrating out the parameter space, i.e. $P(D|A) = \int_{\Theta_A} P(D|A, \Theta_A)P(\Theta_A|A)d\Theta_A$. Similarly, we compute $P(D|B)$. Finally, we compute BF using a Monte Carlo method. We take the entire set of draws from the posterior distribution of parameters and state-variables, evaluate the likelihood at each of those draws and take an average. The sample period covers from 1963-2013 (for All Stocks and Small Stocks) and 1971-2013 (for Low/Small Stocks and High/Small Stocks).

	Portfolios	Bayes Factor
All Stocks	Loser	12.6
	3	3.11
	5	9.78
	8	2.18
	Winner	15.14
Small Stocks	Loser	4.11
	Middle	12.18
	Winner	9.35
Low Small Stocks	Loser	8.17
	Middle	11.91
	Winner	9.79
High Small Stocks	Loser	12.25
	Middle	16.88
	Winner	14.11

Table 7: The table presents stationary volatility and half-life of the fundamental shocks of each portfolio. Stationary volatility is computed as $\frac{\sigma_Y^i}{\sqrt{2\kappa_Y^i}}$ and half-life as $\frac{\ln 2}{\kappa_Y^i}$. These quantities are computed at the posterior median of each parameter reported in Table 5. The sample period covers from 1963-2013 (for All Stocks and Small Stocks) and 1971-2013 (for Low/Small Stocks and High/Small Stocks).

	Portfolios	Stationary Volatility	Half-life
	Loser	$0.06 \text{ X } 10^{-1}$	5.33
All	3	$0.04 \text{ X } 10^{-1}$	4.99
Stocks	5	$0.04 \text{ X } 10^{-1}$	4.81
	8	$0.10 \text{ X } 10^{-1}$	8.66
	Winner	$0.14 \text{ X } 10^{-1}$	10.50
	Loser	0.02	5.77
Small	Middle	0.03	9.24
Stocks	Winner	0.03	10.83
Low	Loser	0.02	6.93
Small	Middle	0.04	10.35
Stocks	Winner	0.06	11.55
High	Loser	0.03	9.76
Small	Middle	0.03	10.66
Stocks	Winner	0.04	12.60

Table 8: The table presents comparative statistics from the data and the model. The columns under data represent summary statistics of different momentum portfolios from Tables 1-3. The columns under Model present the corresponding quantities from the model evaluated at the posterior distribution of parameters and state-variables reported in Table 5. Risk-premia is obtained from (10), Sharpe Ratio is the ratio of risk-premia and volatility obtained from (11), and Winner-Loser is obtained from subtracting the model-implied risk-premia of Loser from the Winner. We evaluate each of the quantities at the posterior distributions of parameters and state-variables obtained from the MCMC and report the time-series average of the median, the 5-th and the 95-th percentiles. The sample period covers from 1963-2013 (for All Stocks and Small Stocks) and 1971-2013 (for Low/Small Stocks and High/Small Stocks).

		Data			Model		
	Portfolios	Av. Real Excess Return	Sharpe Ratio	Winner-Loser	Risk Premia	Sharpe Ratio	Winner-Loser
All Stocks	Loser	-1.07	-0.16	2.40	-0.92 (-1.48,-0.08)	-0.08 (-0.13,-0.00)	2.34 (1.75,2.55)
	3	0.07	0.02		-0.23 (-0.73,0.23)	-0.02 (-0.02,0.03)	
	5	0.16	0.04		0.14 (-0.13,0.45)	0.02 (-0.01,0.03)	
	8	0.72	0.18		0.68 (-1.65,1.33)	0.06 (-0.03,0.08)	
	Winner	1.33	0.23		1.42 (0.29,2.50)	0.14 (0.10,0.18)	
Small Stocks	Loser	2.03	0.12	2.69	1.86 (-0.27,3.43)	0.11 (-0.08,0.15)	2.28 (-0.05,3.56)
	Middle	3.63	0.28		3.16 (0.61,4.88)	0.26 (0.21,0.28)	
	Winner	4.72	0.36		4.14 (2.43,6.62)	0.21 (0.11,0.30)	
Low Small Stocks	Loser	-0.72	-0.04	4.29	-0.79 (-1.12,0.47)	-0.05 (-0.10,0.03)	3.96 (0.88,5.20)
	Middle	1.42	0.08		1.19 (0.14,2.35)	0.07 (0.00,0.12)	
	Winner	3.57	0.21		3.18 (1.52,4.98)	0.16 (0.05,0.23)	
High Small Stocks	Loser	2.27	0.10	3.47	2.11 (0.20,3.13)	0.11 (0.01,0.18)	3.22 (-0.75,6.63)
	Middle	3.49	0.21		3.45 (0.72,4.64)	0.17 (0.06,0.20)	
	Winner	5.73	0.34		5.39 (0.67,7.60)	0.20 (0.04,0.28)	

Table 9: The table presents break-down of model implied risk-premia from Table 8 into long-run risk components due to exposure to the macro factor (LRR_X) and due to exposure to the fundamental factor (LRR_Y). LRR_X is the first term and LRR_Y is the second term of (10). Also shown are corresponding macro beta from (12) and fundamental beta from (13). We evaluate each of the quantities at the median of the posterior distribution of the parameters and state-variables obtained from the MCMC and report the time-series average. The sample period covers from 1963-2013 (for All Stocks and Small Stocks) and 1971-2013 (for Low/Small Stocks and High/Small Stocks).

	Portfolios	LRR_X	LRR_Y	Macro Beta X 100	Fundamental Beta X 100
	Loser	-0.06	-0.87	-0.25	-3.94
	3	-0.13	-0.10	-0.57	-0.46
All	5	-0.08	0.22	-0.37	1.01
Stocks	8	-0.44	1.12	-1.91	4.85
	Winner	0.14	1.28	0.65	5.80
	Loser	-0.61	2.48	-2.65	10.70
Small	Middle	0.05	3.11	0.23	13.43
Stocks	Winner	-0.80	4.94	-3.45	21.36
Low	Loser	-0.23	-0.56	-0.98	-2.42
Small	Middle	-0.10	1.29	-0.45	5.58
Stocks	Winner	-0.06	3.12	-0.26	13.48
High	Loser	0.11	2.00	0.47	8.63
Small	Middle	-0.35	3.80	-1.53	16.42
Stocks	Winner	-0.09	5.48	-0.46	27.21

Table 10: This table reports the percentage change in average fundamental beta from the recessionary time-period to four quarters after the recession. The first column reports the recessionary time-period reported by the NBER. For of these recession dates, we compute average fundamental beta during these periods. Then we compute average fundamental beta for four quarters after each recession period. We report the percentage change between these quantities. We obtain fundamental beta from (13). Taking the median of the posterior distribution of parameters and state-variables, we compute a time-series of fundamental beta for each portfolio and perform our analysis using this time-series. The sample period covers from 1963-2013 (for All Stocks and Small Stocks) and 1971-2013 (for Low/Small Stocks and High/Small Stocks).

	All Stocks		Small Stocks		Low/Small Stocks		High/Small Stocks	
Recessions	Loser	Winner	Loser	Winner	Loser	Winner	Loser	Winner
Nov69-Oct70	1.76	-2.22	19.47	-8.83	N/A	N/A	N/A	N/A
Oct73-Feb75	0.75	-3.50	6.97	-1.05	10.65	-1.95	-6.83	-36.29
June81-Nov82	0.10	-0.23	10.11	-6.62	-0.61	-12.25	2.23	-17.20
June90-Feb91	1.84	2.76	-7.80	2.18	4.94	6.45	84.39	108.70
Feb01-Nov01	10.18	-8.15	63.32	-10.81	17.87	-7.87	-31.00	23.21
Nov07-May09	2.06	5.32	2.66	0.00	-3.44	0.69	26.55	-28.78

Table 11: The table presents a comparison of predictability results between two separate models. The numerator is the Mean Squared Error (MSE) from the model in this paper. Taking the median estimate of risk-premia for each portfolio i, we compute MSE_1 from our model as $\frac{1}{T-k-1}\sum_{t=1}^{T-k}(r^i_{t+k}(\text{data}) - \mu^i_t(\text{model}))^2$. The denominator is the MSE_2 from a standard predictability regression from the data - $r^i_{t+k} = a^i + b^i \frac{D}{P}{}_t^i + \epsilon^i_{t+k}$. We pick three different horizons 1, 2, and 3 quarters. The sample period covers from 1963-2013 (for All Stocks and Small Stocks) and 1971-2013 (for Low/Small Stocks and High/Small Stocks).

	Portfolios	$\text{MSE}_1/\text{MSE}_2$		
		k=1	k=2	k=3
	Loser	0.96	0.75	0.95
All	8	0.98	0.98	0.99
Stocks	5	1.02	1.00	1.04
	3	1.04	1.05	1.05
	Winner	1.02	1.02	1.02
	Loser	0.96	1.00	1.00
Small	Middle	0.99	0.96	0.99
Stocks	Winner	0.97	0.97	0.95
Low	Loser	0.90	0.97	1.00
Small	Middle	0.98	0.98	0.96
Stocks	Winner	1.01	0.91	0.93
High	Loser	0.98	0.98	0.99
Small	Middle	0.89	0.92	0.97
Stocks	Winner	0.92	0.93	0.83

Figure 1: Real dividend growth (demeaned) vs. expected dividend growth for momentum portfolios (Loser, Portfolio 5 and Winner) in All stocks. Dividend growth is constructed from dividends and repurchases, and adjusted for inflation. Two different measures of expected dividend growth from the state-space (1)-(2) are presented - Base and Alternate. Base is the one factor model and Alternate is the two-factor model. In each case, we present expected growth rates, AS_t, constructed from the median of the posterior distribution of parameters and state-variables. BF denotes the Bayes factor for the Alternate Model relative to the Base Model. The sample period covers from 1963-2013.

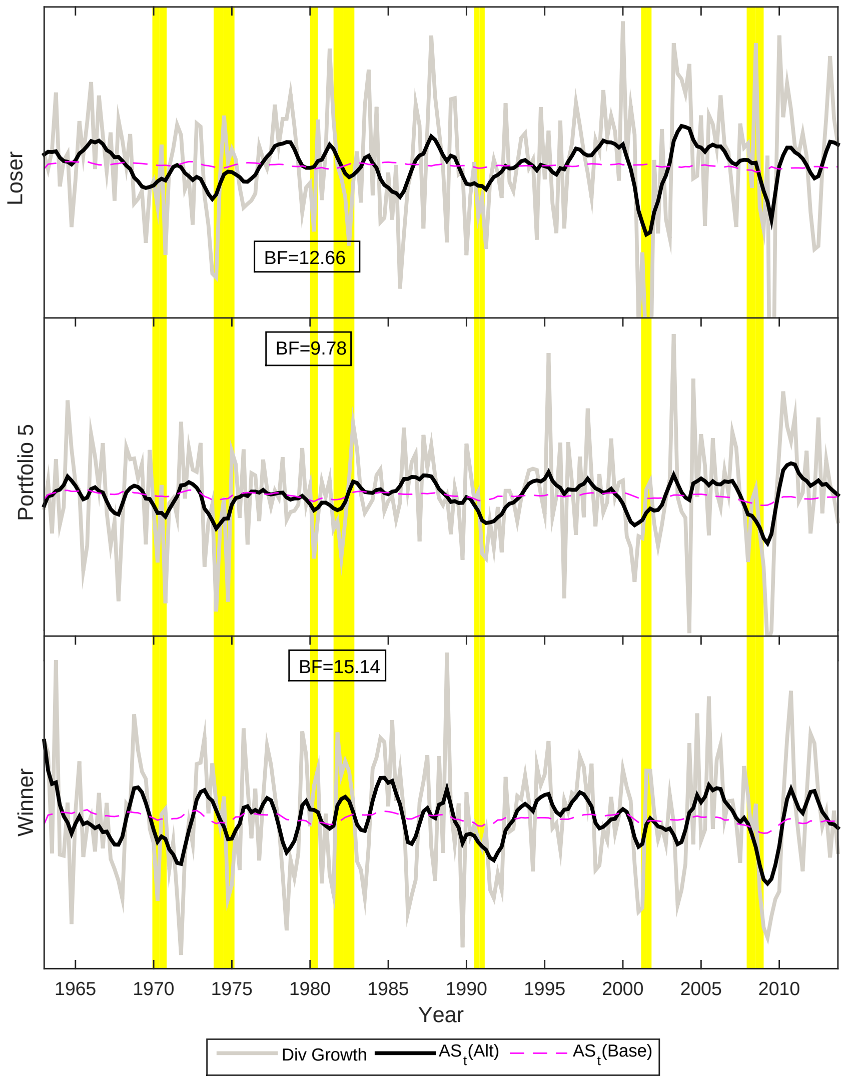

Figure 2: Real dividend growth (demeaned) vs. expected dividend growth for momentum portfolios (Loser, Middle and Winner) in High/Small stocks. Dividend growth is constructed from dividends and repurchases, and adjusted for inflation. Two different measures of expected dividend growth from the state-space (1)-(2) are presented - Base and Alternate. Base is the one factor model and Alternate is the two-factor model. In each case, we present expected growth rates, AS_t, constructed from the median of the posterior distribution of parameters and state-variables. BF denotes the Bayes factor for the Alternate Model relative to the Base Model. The sample period covers from 1971-2013.

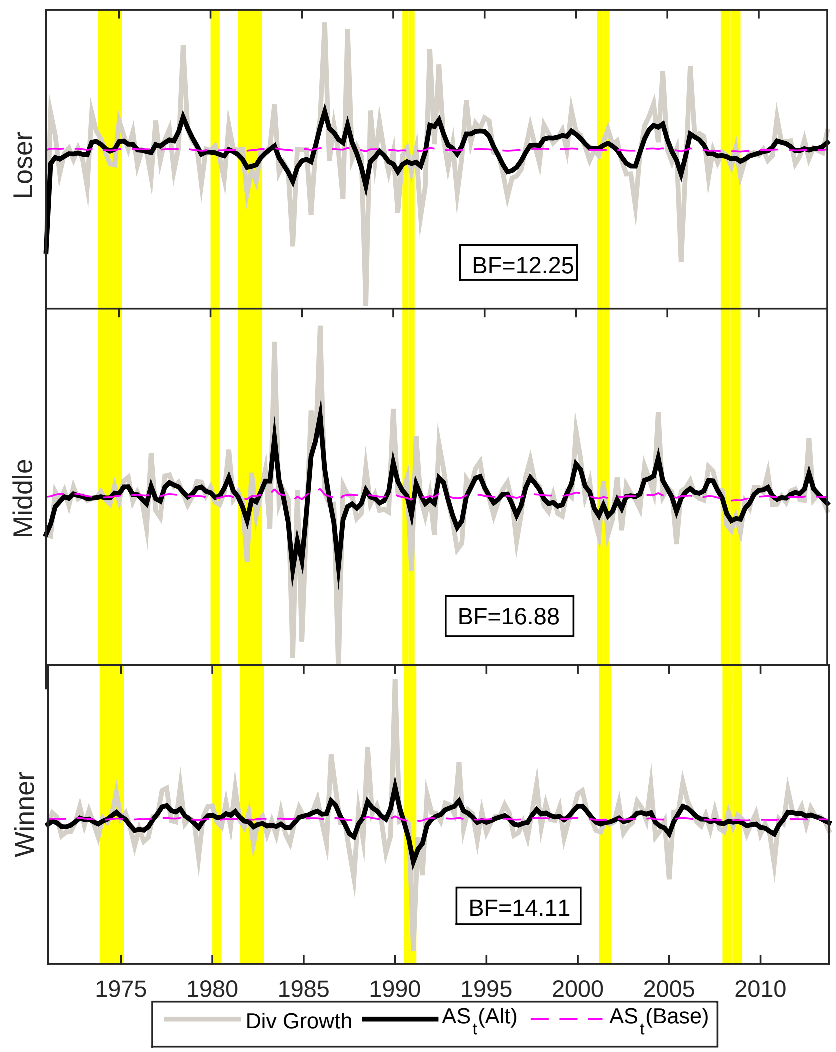

Figure 3: Changes in the macro factor ΔX_t relative to changes in the fundamental factor ΔY_t^i for momentum portfolios (Loser and Winner) in All Stocks. We construct the time-series from the median of the posterior distribution for each state-variable. The sample period covers from 1963-2013.

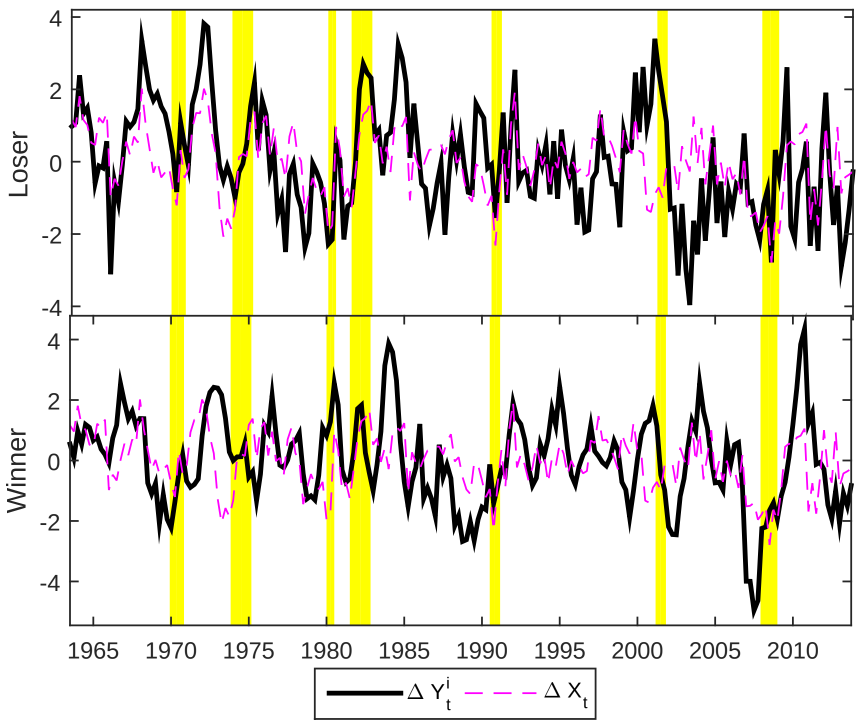

Figure 4: Changes in the macro factor ΔX_t relative to changes in the fundamental factor ΔY_t^i for momentum portfolios (Loser, Middle and Winner) constructed out of High/Small stocks. We construct the time-series from the median of the posterior distribution for each state-variable. The sample period covers from 1971-2013.

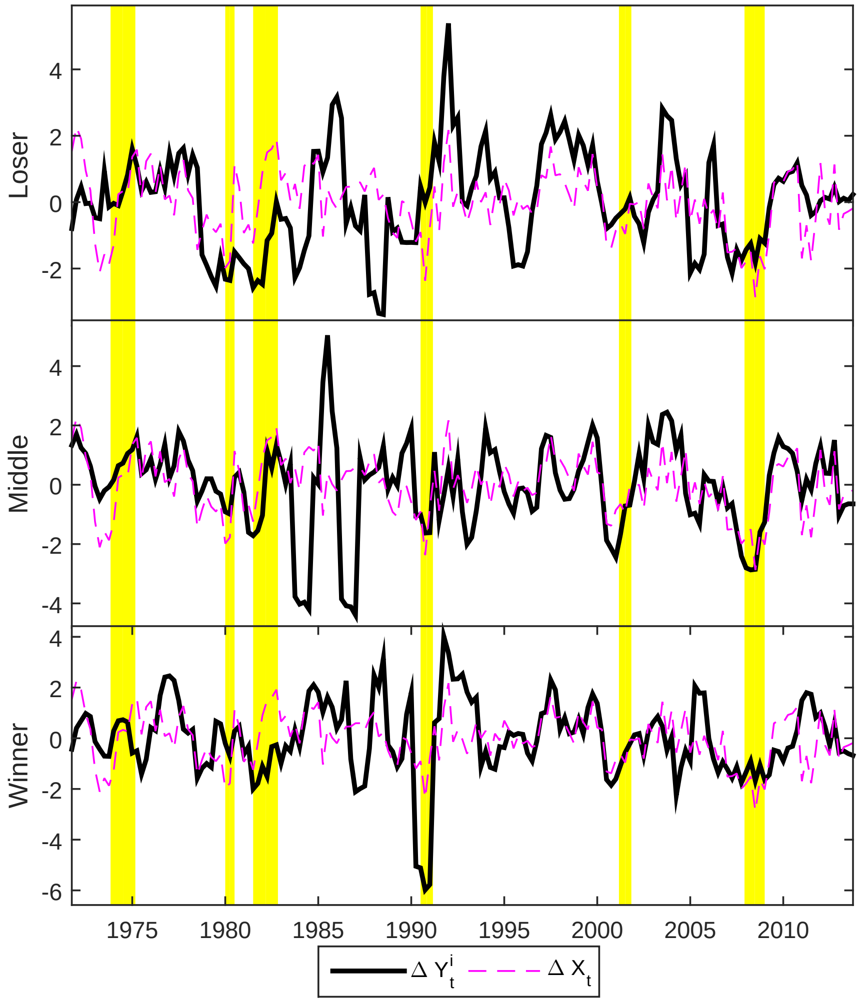

Figure 5: Beta vs Average Return. The top panel shows the cross-sectional relationship between macro beta in (12) and average return from the data. The bottom panel shows the cross-sectional relationship between fundamental beta in (13) and average return. Average return is reported in Tables 1-3. We compute a time-series of macro and fundamental beta by using the posterior median of parameters and state-variables. The figure uses time-series average of each quantity and for each asset. The sample period covers from 1963-2013 (for All Stocks and Small Stocks) and 1971-2013 (for Low/Small Stocks and High/Small Stocks).

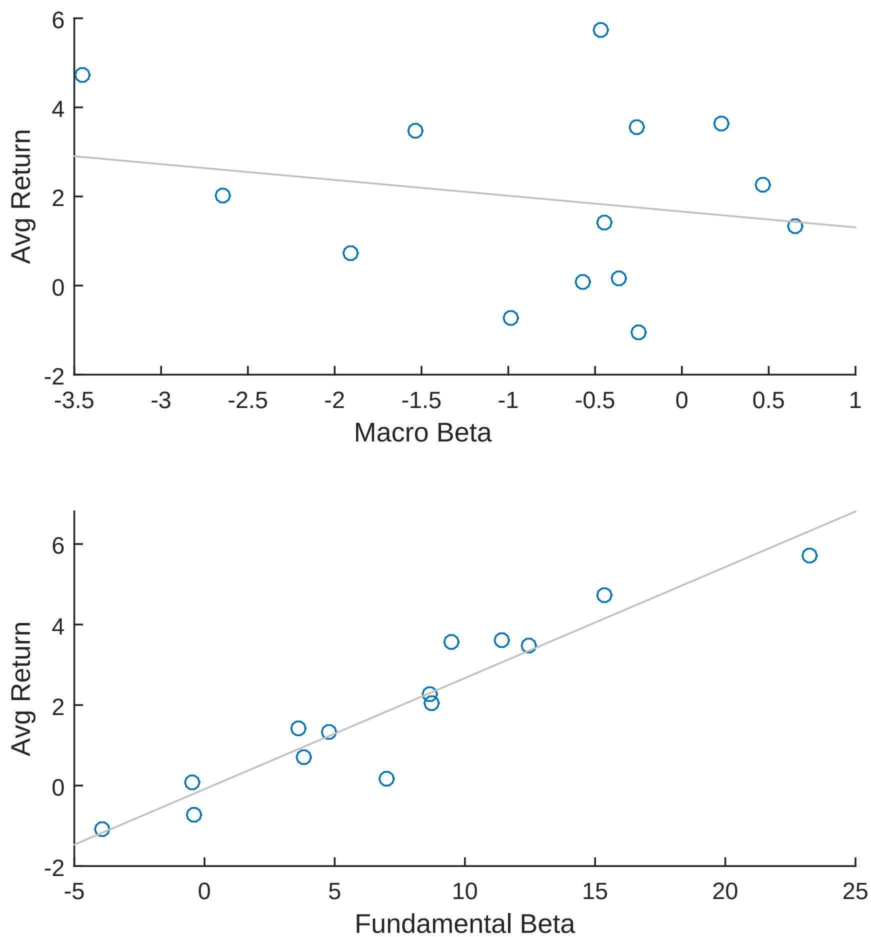

Figure 6: Fundamental beta vs cash-flow parameters. We show cross-sectional relationship between fundamental beta and different parameters of the cash-flow process. Here ϕ_2 represents the "leverage" parameter on the fundamental factor reported in Table 5. Half-life and stationary volatility of the fundamental factor are reported in Table 7. Fundamental beta is reported in Table 9. The sample period covers from 1963-2013 (for All Stocks and Small Stocks) and 1971-2013 (for Low/Small Stocks and High/Small Stocks).

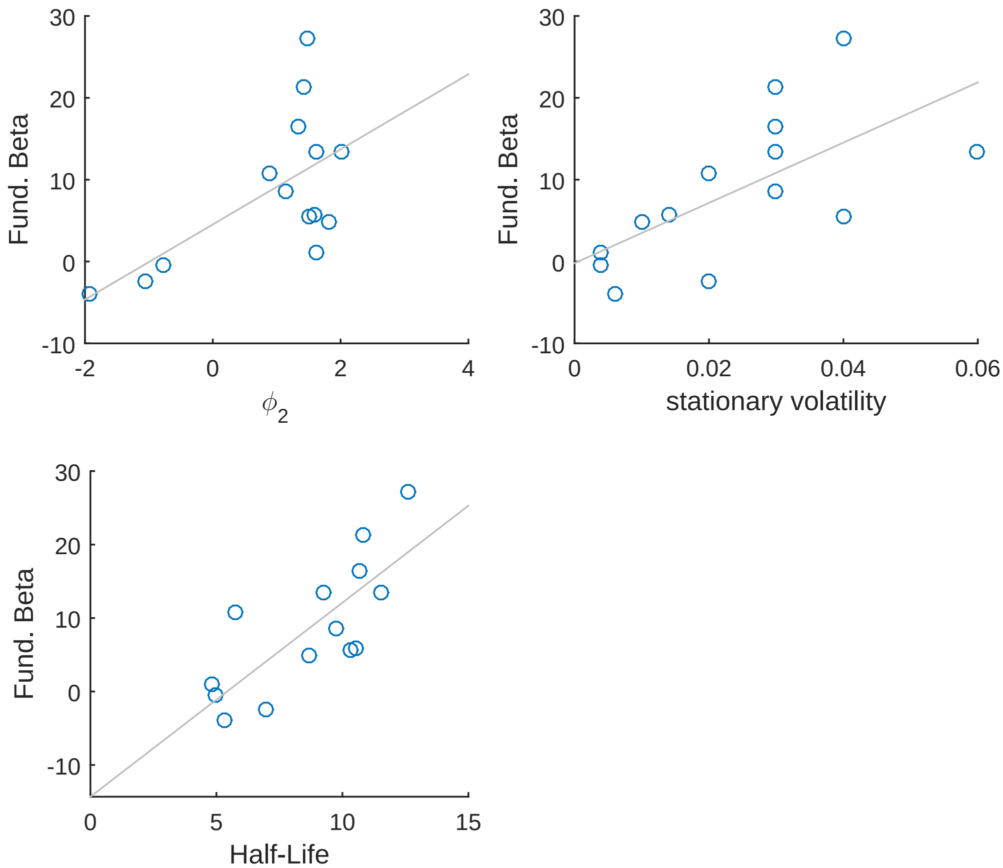

Figure 7: Time-series of risk-premia from a momentum strategy of going long on the Winner and shorting the Loser according to the model. The top panel shows momentum premia in All Stocks. The bottom panel shows momentum premia in Small stocks. We compute risk-premia, μ_t^i, using (10) for Winner and Loser portfolios in each cross-section, and produce the time-series of model-implied momentum risk-premia by $\mu_t^{\text{Winner}} - \mu_t^{\text{Loser}}$. We use the median values from the posterior distributions of all parameters and state-variables. The solid line shows the momentum premia in the data for each cross-section. The sample period covers from 1963-2013.

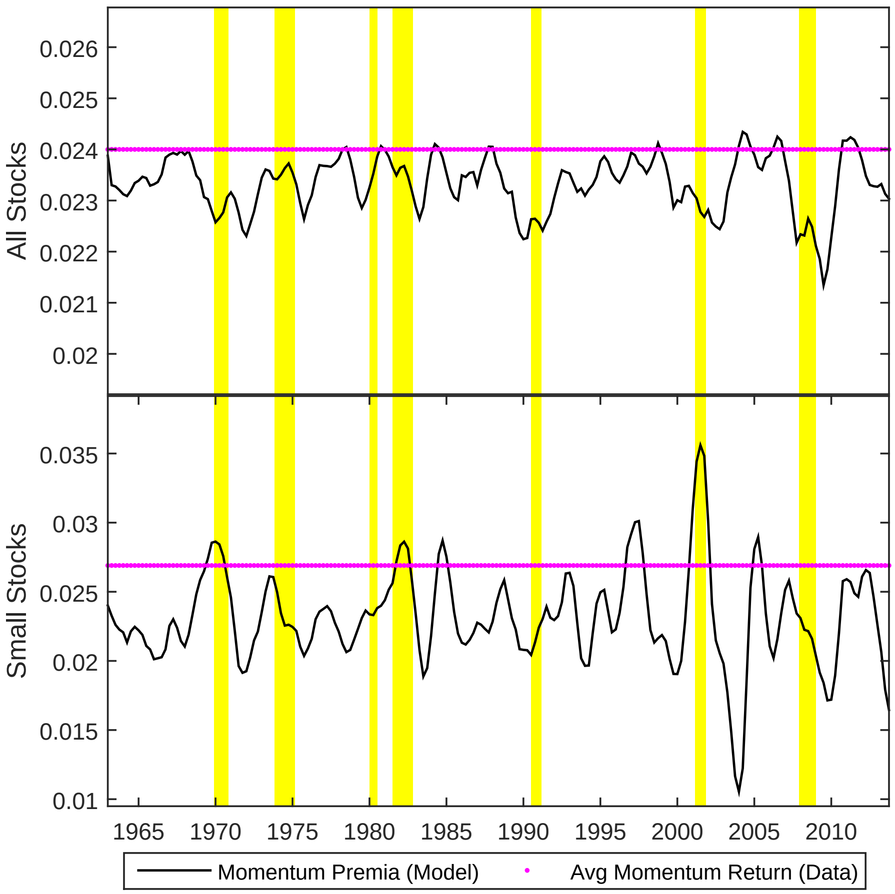

Figure 8: Time-series of risk-premia from a momentum strategy of going long on the Winner and shorting the Loser according to the model. The top panel shows momentum premia in Low/Small stocks. The bottom panel shows momentum premia in High/Small stocks. We compute risk-premia, μ_t^i, using (10) for Winner and Loser portfolios in each cross-section, and produce the time-series of model-implied momentum risk-premia by $\mu_t^{\text{Winner}} - \mu_t^{\text{Loser}}$. We use the median values from the posterior distributions of all parameters and state-variables. The solid line shows the momentum premia in the data for each cross-section. The sample period covers from 1971-2013.

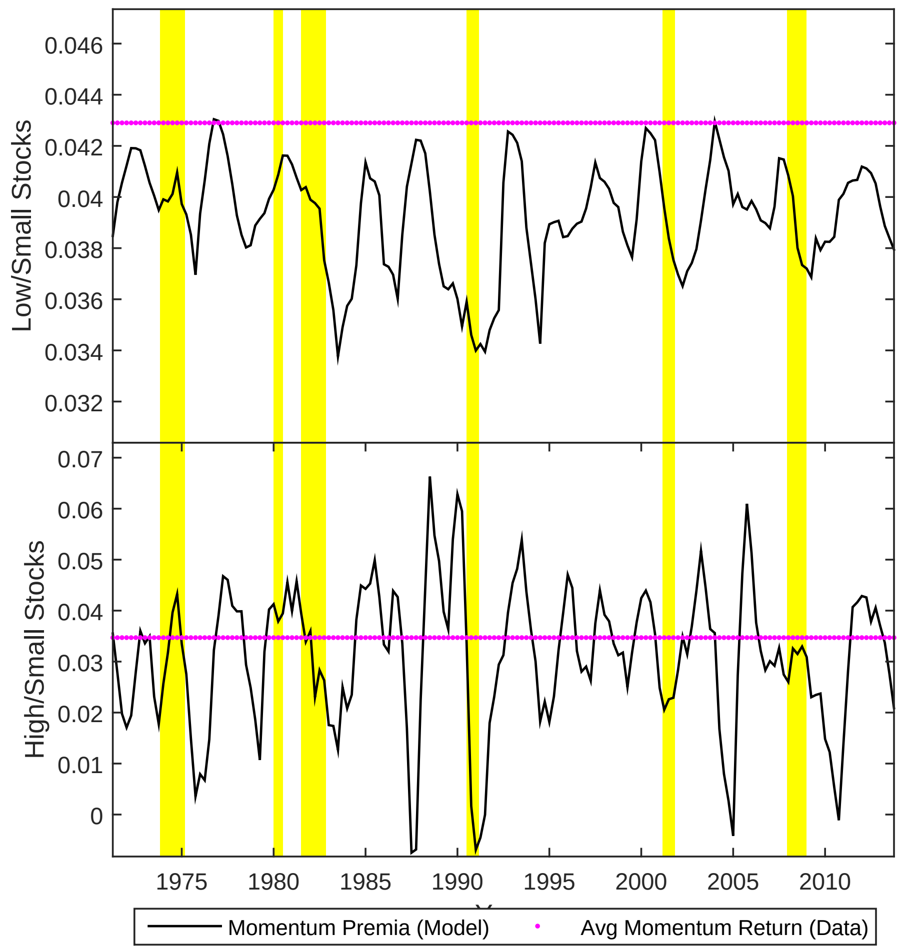

Figure 9: Time-series of long-run risk premia for momentum portfolios (Loser and Winner) in All Stocks. LRR_X corresponds to long-run risk due to exposure to the macro factor (the first term of risk-premia given by (10)) and LRR_Y corresponds to long-run risk due to exposure of the fundamental factor to the macro factor (the second term of risk-premia given by (10)). Each of these quantities are created from the median of the posterior distribution of the parameters and state-variables. The sample period covers from 1963-2013.

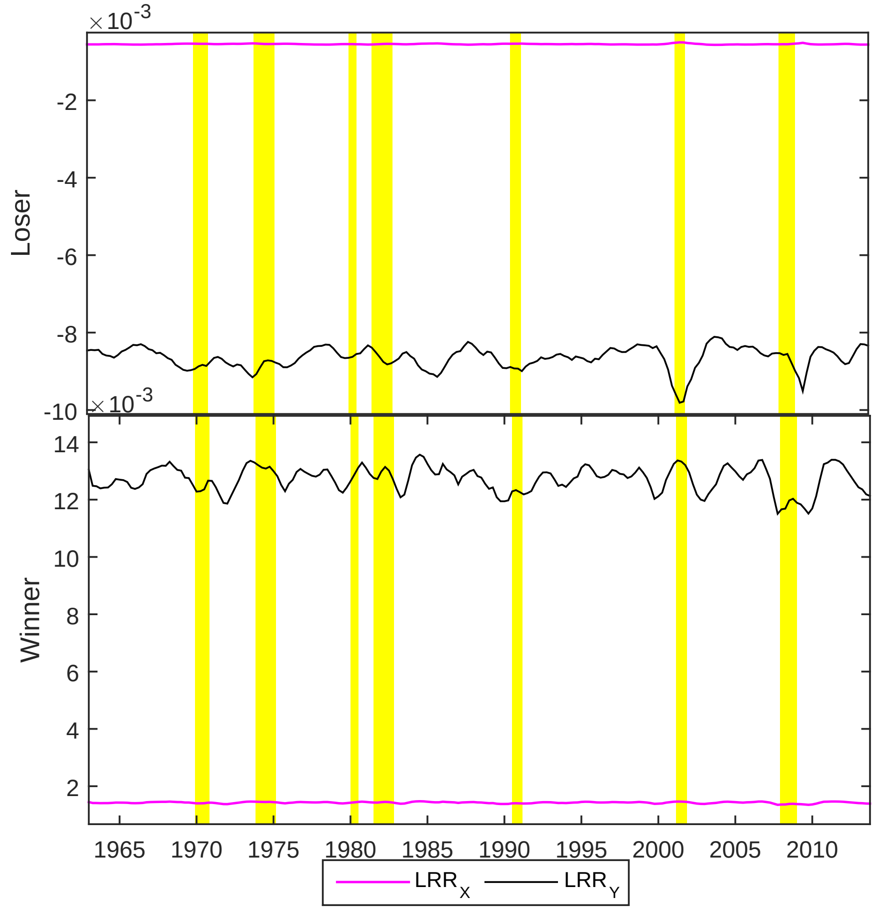

Figure 10: Time-series of long-run risk premia for momentum portfolios (Loser and Winner) in Small stocks. LRR_X corresponds to long-run risk due to exposure to the macro factor (the first term of risk-premia given by (10)) and LRR_Y corresponds to long-run risk due to exposure of the fundamental factor to the macro factor (the second term of risk-premia given by (10)). Each of these quantities are created from the median of the posterior distribution of the parameters and state-variables. The sample period covers from 1963-2013.

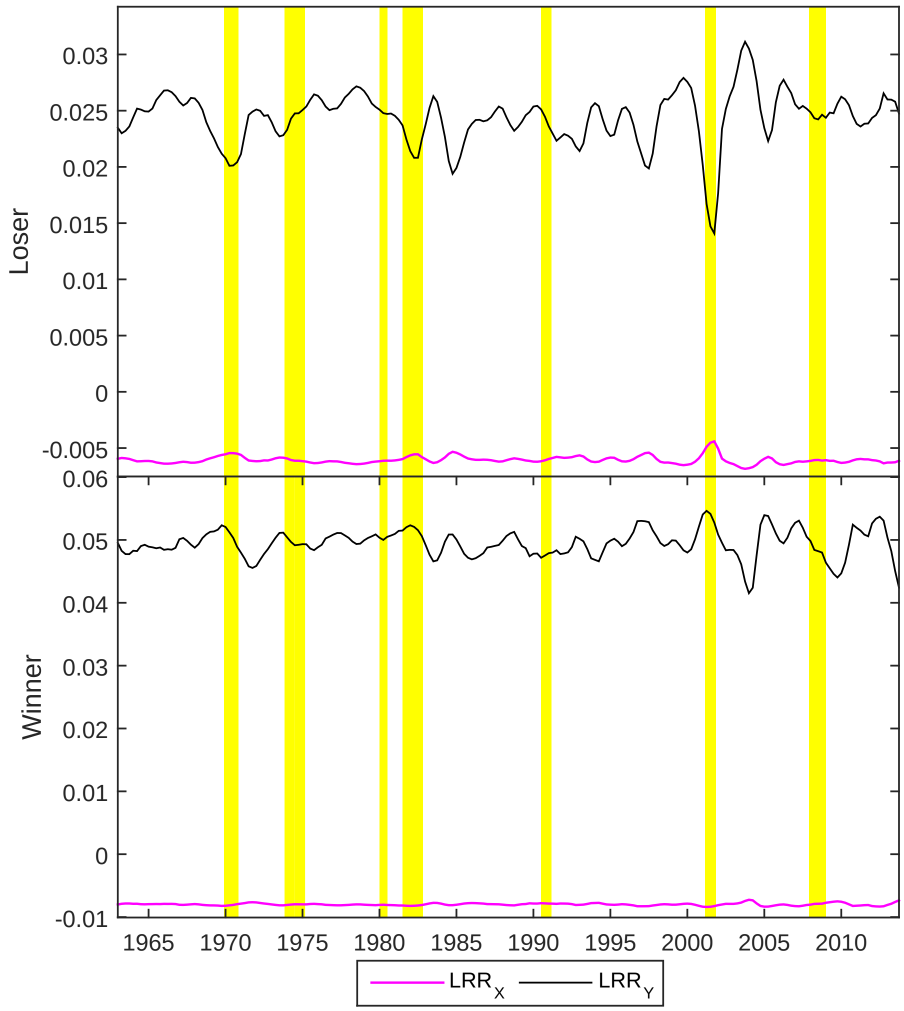

Figure 11: Time-series of long-run risk premia for momentum portfolios (Loser, Middle, Winner) constructed from stocks in the High/Small stocks. LRR_X corresponds to long-run risk due to exposure to the macro factor (the first term of risk-premia given by (10)) and LRR_Y corresponds to long-run risk due to exposure of the fundamental factor to the macro factor(the second term of risk-premia given by (10)). Each of these quantities are created from the median of the posterior distribution of the parameters and state-variables. The sample period covers from 1971-2013.

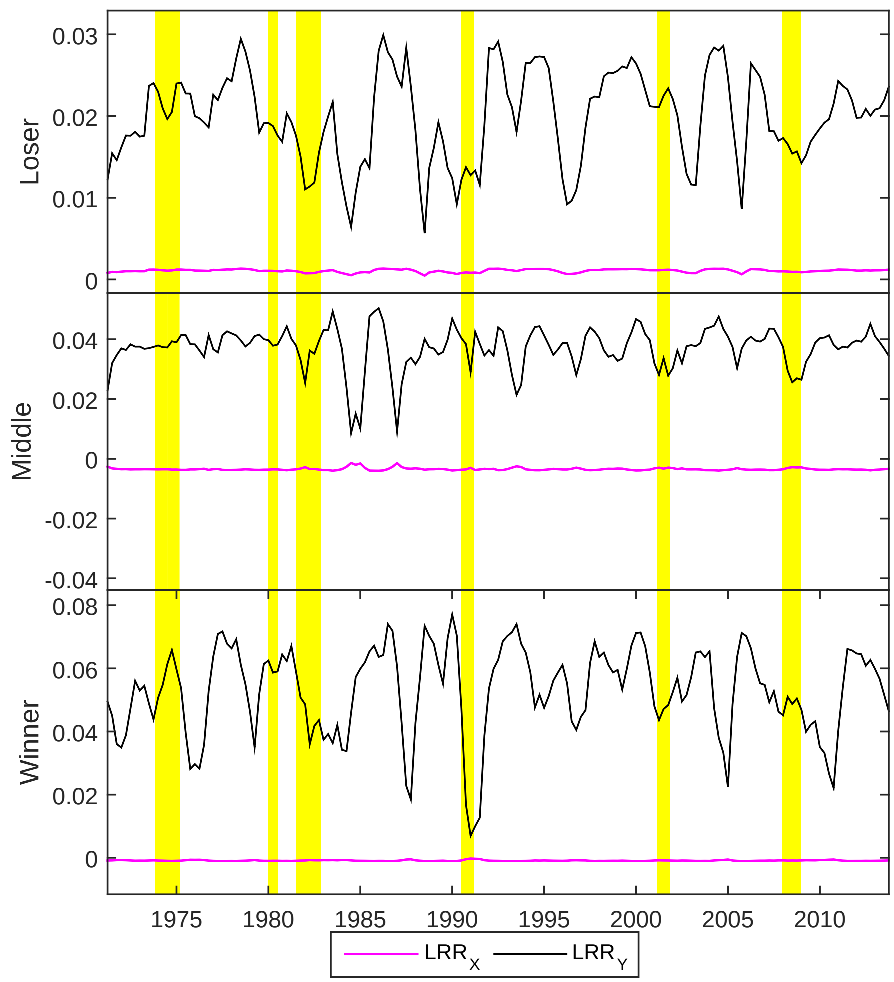

Figure 12: Time-series of long-run risk premia for momentum portfolios (Loser, Middle, Winner) constructed from stocks in the Low/Small stocks. LRR_X corresponds to long-run risk due to exposure to the macro factor (the first term of risk-premia given by (10)) and LRR_Y corresponds to long-run risk due to exposure of the fundamental factor to the macro factor(the second term of risk-premia given by (10)). Each of these quantities are created from the median of the posterior distribution of the parameters and state-variables. The sample period covers from 1971-2013.

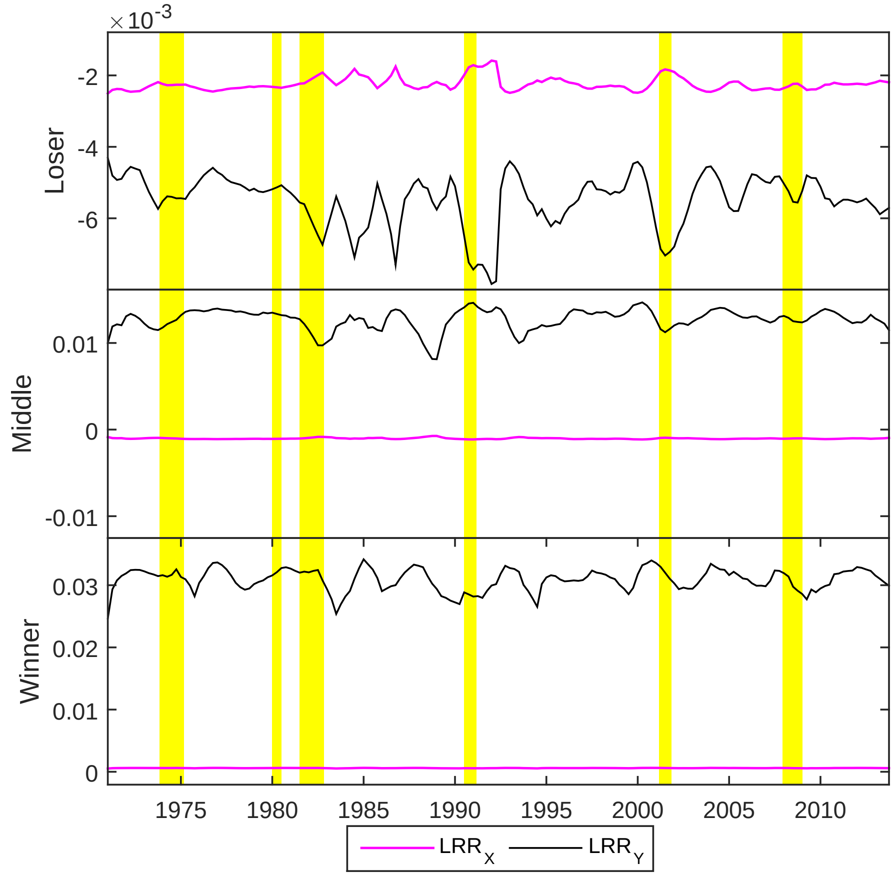

Figure 13: Time-series of the median from the posterior distribution of Y_t^i in Winner and Loser portfolios in Small Stocks during two recessions. The recession in the top graph is between June 1981 to November 1982, and the recession in the bottom graph is from February 2001 to November 2001. Both series are standardized to have unit standard deviation.

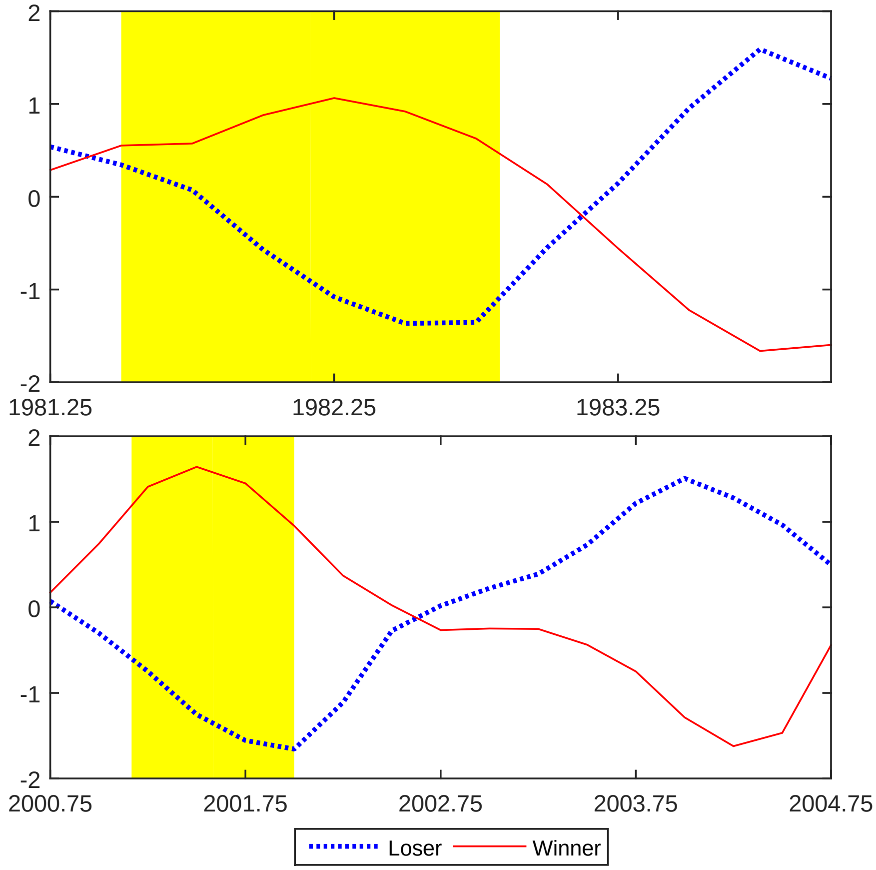

Figure 14: Time-series of fundamental beta in Winner and Loser portfolios in All Stocks and Small Stocks. Loser is in the left-axis and Winner in the right-axis. Fundamental beta is computed according to (13) using the median of the posterior distribution of parameters and state-variables. The sample period covers from 1963-2013.

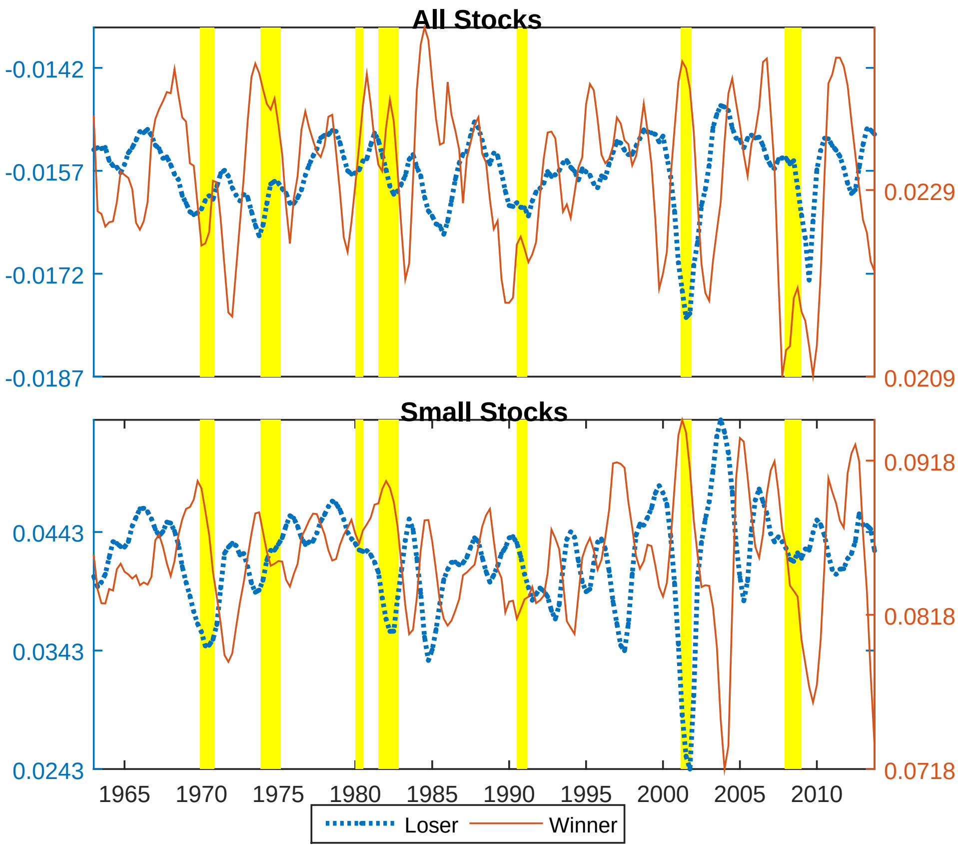

Figure 15: Time-series of fundamental beta in Winner and Loser portfolios in Low/Small Stocks and High/Small Stocks. Loser is in the left-axis and Winner in the right-axis. Fundamental beta is computed according to (13) using the median of the posterior distribution of parameters and state-variables. The sample period covers from 1971-2013.