Optimal Monetary Policy in a Model of Overlapping Price Contracts

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Abstract

This study estimates a model of overlapping nominal price contracts over three distinct monetary policy regimes, testing the stability of the parameters in the model across regimes. Upon finding a model that is stable over the three subsamples, the model that holds for the most recent monetary regime is used to compute the optimal policy frontier–the efficient combinations of output and inflation variances–and compared to actual policy performance. The study then evaluates the robustness of policy conclusions to particulars of the specification, and discusses the general properties that are required of a model in order to produce a plausible estimate of the optimal policy frontier.

The annual average rate of inflation in the GDP deflator for 1980 was 10.1 percent. By 1984, the same measure had dropped to 4.4 percent, and from 1990 through 1993, the rate of inflation has averaged 3.4 percent, deviating only modestly from that average over the period. From 1981 to 1984, the civilian unemployment rate averaged 8.6 percent, peaking at 10.7 percent in the fourth quarter of 1982. Many argue that much of the rise in unemployment was caused by intentionally contractionary monetary policy, and that the fall in inflation was a consequence of the high unemployment rate. Under this interpretation, the period from 1982 to the present may be characterized as a successful disinflation engineered by the Federal Reserve, successful at least in as far as it clearly accomplished the goal of lowering the inflation rate.

But was it an *optimal* disinflation? While the level of inflation at the end of the episode was evidently more satisfactory to the monetary authority than the level at the beginning of the episode, was the path that the real economy took in getting there just as satisfactory? Did the Federal Reserve move its instruments so as to obtain its desired target rate of inflation while minimizing the disruption to the real economy? If not, what course would have been better?

One commonly used measure of monetary policy performance is the weighted average of the *variances* of inflation and the output gap. This measure is a steady-state, rather than a path-specific, concept of policy optimality. An optimal policy according to this metric will systematically set the policy instrument in response to deviations of inflation and the output gap from their targets so as to minimize a weighted average of the unconditional variances of inflation and output. Other things equal, it is assumed that the monetary authority prefers less variance of inflation about its target to more, and less variance of real output about potential to more.[1] Given a description of the economy and a set of policy preferences over inflation and output variance, there exists a policy that achieves the minimum weighted average variance. This paper gives primary weight to this measure of optimality. A second, more path-specific measure of policy performance is the sacrifice ratio–the percentage shortfall of output below potential for each percentage point reduction in inflation. Fuhrer (1994) explores the implications of various monetary policy settings for the sacrifice ratio.

Because the notion of optimal monetary policy—the best policy that policymakers could have pursued—is inherently counterfactual, it must be addressed in the context of a model. It will be argued below that any model used to analyze monetary policy must accurately reproduce at least two important features of the data. If it does not, then its prescriptions about optimal monetary policy will be of little relevance. In particular, Taylor's [1980] model of staggered contracts implies insufficient persistence in inflation, and as a result yields an implausibly optimistic policy frontier.

The next sections describe the data and the model that will be used to assess the optimality of monetary policy. In the following sections, the model is estimated across three distinct monetary regimes, testing for the stability of the non-policy structural parameters that summarize the behavior of prices and aggregate demand. Finally, the optimal policy frontier is computed for the most recent monetary regime and its robustness examined with respect to modifications to the specification of the model.

[1]In the model used here for policy exercises, monetary policy cannot permanently move real output away from potential.

1 The Data

To adequately characterize monetary policy requires a description of the instrument of policy, taken here to be a short-term interest rate, as well as its primary targets, which include the rate of inflation, the rate of money growth, the growth rate of output, and the output gap. The transmission channel from policy instrument to ultimate goals also involves these variables. The data series that are used in this paper are itemized in Table 1.

As in earlier work (Fuhrer and Moore 1993a,b), a stationary representation of the fundamental data series is preferred. In the models that considered here, the stationarity of the rate of inflation, and hence of the nominal federal funds rate, depends upon the actions of the monetary authority. Unless the Federal Reserve responds to deviations of the level of inflation from a target (or to deviations of another nominal aggregate that is linked to the rate of inflation, such as the rate of money growth or nominal income), the rate of inflation and the short nominal rate will be nonstationary.[2] Thus, the estimates of the response of the funds rate to inflation (and/or money growth) in the reaction functions below will be critical in determining whether the Fed induces stationarity in the rate of inflation.

Table 2 presents the results of univariate augmented Dickey-Fuller tests for the data series of interest. The inflation rate and nominal rate appear to be at best borderline stationary. Because monetary policy can shift the mean of the inflation process (as well as its order of integration for a fixed mean), these full-sample tests may reflect a shifting mean across monetary policy regimes, and thus may not be terribly informative. However, tests based on the subsamples, reported in the last two panels of the table, include very few

[2]Similarly, the Fed could induce stationarity in the price *level* by responding to deviations of the price level from some target value. This has evidently not been the goal of the monetary authority over the last 30 years.

Table 1
Quarterly Data, 1966Q1–1993Q1

Mnemonic	Definition
y_t	log of per capita \$82 GDP
p_t	log of the implicit GDP deflator
f_t	Quarterly federal funds rate
π_t	Inflation rate, 4 Δp_t
$\tilde{y}_t$	Deviation of y_t from trend, 1959Q1–1993Q1
$m2_t$	log of M2

Table 2
Augmented Dickey-Fuller tests

Test regression: $\Delta x_t = \beta_0 x_{t-1} + \sum_{i=1}^{n} \beta_i \Delta x_{t-i} + \mu + \gamma t + \epsilon_t$

Series	n	$Q(12)$	β_0	τ_μ	τ_τ
1966:1-93:1					
π_t	2	13.8	−.14	−2.03	
f_t	3	13.4	−.09	−2.34	
y_t	2	17.5	−.12		−3.23
$\tilde{y}_t$	2	15.1	−.08	−3.10	
$m2_t$	1	15.5	.00	.82	
$m2_t$	1	12.4	−.03		−2.01
$\Delta m2_t$	0	13.7	−.33	−4.12	
1966:1-79:3					
π_t	2	11.5	−.25	−1.97	
f_t	3	6.6	−.17	−2.83	
$\Delta m2_t$	1	18.9	−.30	−3.25	
1982:4-93:1					
π_t	1	14.1	−.42	−2.70	
f_t	1	11.2	−.15	−2.86	
$\Delta m2_t$	0	8.2	−.33	−2.59	

observations, and thus are also suspect. In general, the magnitude of the coefficient β_0 for the subsample test regressions is larger, although the value of the ADF test statistic is not uniformly larger. A more complete discussion of the time-varying mean of inflation and the issue of the stationarity of the nominal variables in the model will be left to the discussion of the structural estimates.

The log of per capita output appears trend stationary. Not surprisingly, M2 is neither stationary in levels nor stationary about a trend, although its first difference is strongly stationary.[3]

Earlier work compared the behavior of an unrestricted vector autoregression for these variables to the behavior of a restricted structural model. Here, no attempt is made to estimate separate vector autoregressions for each monetary policy regime. Essentially, the structural model is taken as a reasonable description of the economy based on the evidence in Fuhrer and Moore (1993a,b), and the study moves on from there.

2 The Baseline Model

2.1 The I–S Curve

The real economy is represented with a simple I–S curve that relates the output gap to its own lagged values and one lag of the long-term real interest rate, ρ_{t-1}.

$$\tilde{y}_t = a_0 + a_1\tilde{y}_{t-1} + a_2\tilde{y}_{t-2} + a_\rho\rho_{t-1} + \epsilon_{y,t} \tag{1}$$

Monetary policy cannot affect the output gap in the long run; its equilibrium value is 0 for all feasible monetary policies. The long-term real rate is the

[3]The tests for M2 use data through 1982:Q3, the end of the nonborrowed reserves operating procedures and the end of the second subsample considered below.

yield to maturity on a hypothetical long-term real bond. In the maximum likelihood estimation discussed below, the realization of ρ_t is set equal to the weighted average of the expected real returns on federal funds forecast by the restricted structural model. Thus, the series of ρ_t realizations changes as the estimated structural parameters change.

The intertemporal arbitrage condition that equalizes the expected holding-period yields on federal funds and real long-term bonds is

$$\rho_t - D[E_t(\rho_{t+1}) - \rho_t] = f_t - E_t(\pi_{t+1}) \tag{2}$$

where D is a constant approximation to Macaulay's duration. Solving equation 2 for ρ_t in terms of ρ_{t+1} and $f_t - E_t(\pi_{t+1})$, then recursively substituting the result into itself, the long-term real rate is an exponentially weighted moving average of the forecast path of the real rate of return on federal funds.

$$\rho_t = \frac{1}{1+D}\sum_{i=0}^{\infty}\left(\frac{D}{1+D}\right)^i E_t(f_{t+i} - \pi_{t+i+1}) \tag{3}$$

Preliminary analysis suggests that a single lag of the long-term real rate is sufficient to explain the evolution of output; the coefficient on the contemporaneous value of ρ_t is not significantly different from zero in an instrumental variables estimation of the $\tilde{y}_t$ equation.

2.2 The reaction function

The systematic behavior of monetary policy is summarized with a reaction function in which the monetary authority moves the short-term nominal rate in response to deviations of target variables from target. The precise form of the reaction function varies over subsamples. In particular, on both empirical and *a priori* grounds, the short rate responds to money growth in the first two subsamples, but not in the third. Limited information estimates of the reaction functions find no evidence of a response to M2 growth once the response to inflation and output is included during the third subsample, and little or no evidence of a direct response to inflation in the first two subsamples, with a strong response to M2 growth. The general form of the reaction function is

$$f_t = \sum_{i=1}^{m} \alpha_{fi} f_{t-i} + \sum_{j=0}^{n} \alpha_{\pi_j} \pi_{t-j} + \sum_{k=0}^{p} \alpha_{yk} \tilde{y}_{t-k} + \sum_{l=0}^{q} \alpha_{ml} \Delta m_{t-l} + \epsilon_{f,t} \quad (4)$$

The monetary policy reaction function relates the quarterly federal funds rate to lags of the funds rate, contemporaneous and lagged levels of the inflation rate, contemporaneous and lagged levels of the output gap, and contemporaneous and lagged money growth.

The beginning of the first sample period, 1966, is dictated by the use here of the federal funds rate as the fundamental instrument of monetary policy. The federal funds rate, the overnight rate on interbank loans, traded below the Federal Reserve discount rate prior to the mid 1960s. Since that time, the funds rate has generally remained above the discount rate, and there has been a direct link between Federal Reserve open market transactions and movements in the funds rate.

2.3 Money demand

Because M2 appears in the reaction function for the first two subsamples, a money demand equation is required. If the only interaction between money and the rest of the model were through the reaction function, the specification of money demand would not be terribly important. However, since the long-term real rate that drives the I–S curve depends in part on expectations of the federal funds rate, money demand behavior will feed into expectations of the long-term real rate and have at least some effect on the output gap. Thus some care must be taken in specifying money demand.

A relatively simple error-correction specification is used for money demand. In the long run, the level of real money balances depends on real output and the opportunity cost of holding money

$$m2_t - p_t = b_0 + b_1 y_t + b_2 f_t + \epsilon_{m,t} \tag{5}$$

Two simplifications were made in the specification of the opportunity cost of holding M2. First, the own rate on M2 deposits is ignored, and second, the federal funds rate is used as the competing rate on assets with which money competes in the portfolios of money holders. The first simplification appears to have little or no effect on the fit or behavior of money demand, and it removes the difficult task of modeling deposit rates. The second simplification marginally sacrifices the fit of the money demand equation for simplicity of modeling. The alternative is to use the Treasury bill rate as the opportunity cost and then to model the bill rate, linking it to movements in the funds rate. The net loss for the purposes of this study from these simplifications seems small relative to the additional complication that they would add to the model.

Finally, the change in real money balances responds to the lagged discrepancy from long-run equation 5, as well as to lagged changes in real money and the funds rate.[4]

$$\Delta(m2_t - p_t) = c_0 + c_1 \epsilon_{m,t-1} + c_2 \Delta(m2_{t-1} - p_{t-1}) + c_3 \Delta f_{t-1} \tag{6}$$

[4]Other dynamic terms entered insignificantly in preliminary limited information estimates.

2.4 The Contracting Specification

The contracting specification is identical to that used in Fuhrer and Moore (1993a), and the reader is referred to that paper for greater detail. Agents negotiate nominal wage contracts that remain in effect for four quarters. The aggregate log price index in quarter t, p_t, is a weighted average of the log wage contracts, x_{t-i}, that were negotiated in the current and the previous three quarters and are still in effect. The weights, ω_i, are the proportions of the outstanding contracts that were negotiated in quarters $t-i$,

$$p_t = \sum_{i=0}^{3} \omega_i x_{t-i} \tag{7}$$

where $\omega_i \geq 0$ and $\sum \omega_i = 1$. A downward-sloping linear function of contract length is used,

$$\omega_i = .25 + (1.5 - i)\, s, \qquad 0 < s \leq 1/6, \quad i = 0, \ldots, 3 \tag{8}$$

Let v_t be the index of real wage contracts that were negotiated on the contracts currently in effect,

$$v_t = \sum_{i=0}^{3} \omega_i (x_{t-i} - p_{t-i}) \tag{9}$$

Now suppose that agents set nominal wage contracts so that the current real contract wage equals the average real contract wage index expected to prevail over the life of the contract, adjusted for excess demand conditions.

$$x_t - p_t = \sum_{i=0}^{3} \omega_i E_t(v_{t+i} + \gamma \tilde{y}_{t+i}) + \epsilon_{p,t} \tag{10}$$

Substituting equation 9 into equation 10 yields the real version of Taylor's contracting equation,

$$x_t - p_t = \sum_{i=1}^{3} \beta_i (x_{t-i} - p_{t-i}) + \sum_{i=1}^{3} \beta_i E_t(x_{t+i} - p_{t+i}) + \gamma^* \sum_{i=0}^{3} \omega_i E_t(\tilde{y}_{t+i}) + \epsilon_t \quad (11)$$

where $\beta_i = \sum_j f_j f_{i+j} / (1 - \sum_j f_j^2)$, and $\gamma^* = \gamma / (1 - \sum_j f_j^2)$.

In their contracting decisions, agents compare the current real contract wage with an average of the real contract wages that were negotiated in the recent past and those that are expected to be negotiated in the near future; the weights in the average measure the extent to which the past and future contracts overlap the current one. When output is expected to be high, the current real contract wage is high relative to the real contract wages on overlapping contracts.

3 Motivation for the Subsample Estimation

An extensive literature is emerging to address the problem of determining breakpoints in econometric models. (See Andrews 1993, Zivot and Andrews 1992, and Banerjee, Lumsdaine and Stock 1992). Because this study is primarily concerned with the stability of structural parameters across monetary regimes, the subsamples are defined *a priori* as corresponding to the period before the nonborrowed reserves operating procedure (1966:I to 1979:III), the nonborrowed reserves operating procedure period (1979:IV to 1982:III), and the post-nonborrowed reserves period (1982:IV to 1993:I). While a change in operating procedures need not imply a change in the policy stance of the monetary authority, the documentation is sufficient to warrant a fairly close link between the changes in operating procedures in the 1980s and changes in monetary policy. For example, the October 1979 change in operating procedures coincided with the onset of a serious commitment to disinflation.

In addition to choosing breakpoints, this study must consider the way in which expectations change around the time of regime shifts. In principle, agents would be expected to take some time to learn about shifts in monetary regimes (Fuhrer and Hooker 1993). Here, the problems of expectation formation under changing regimes are ignored, and it is assumed that agents know about the regime shift as soon as it is implemented. While this is conceptually a large simplification, empirically it appears to be of small consequence in this specification: In the estimates presented below, no evidence was found of serious mis-fitting at the beginning of new regimes.

3.1 The Effects of Changing Policy Regimes on Model Dynamics

The introduction of a changing monetary policy regime in this model is of more than second-order interest. Monetary policy can dramatically alter the properties of the model.

Monetary policy can alter the mean of the inflation process. Nothing in the contracting specification pins down the mean of inflation. This is a feature that this model shares with many models, including the conventional expectations-augmented Phillips curve. In fact, it is only when the funds rate responds to the level of inflation (or equivalently, to money growth or the growth of nominal income), that inflation can be stationary. Thus significant changes in the reaction function can change the order of integration of all nominal variables, as well as the mean of the inflation process given stationarity.

Monetary policy can alter the dynamic path of expected long real rates. Because the ex ante long real rate equals the weighted average of expected short-term real rates in this model, and because changes in monetary policy will alter the expected path of both the funds rate and inflation, shifts in monetary policy will alter the time-series behavior of the long real rate (See Fuhrer and Moore 1993b for a fuller discussion of this point.)

4 Sketch of Estimation Method/Computations

Limited information estimates (ordinary least squares or instrumental variables estimates, where appropriate) are used as consistent starting values for the parameters of the model. The final estimation method is full information maximum likelihood (FIML); all parameters are estimated jointly. The log-likelihood is

$$\mathcal{L} = T(\log|\mathcal{J}| - .5\log|\hat{\Sigma}|)\ ,$$

where J is the Jacobian of transformation, and $\hat{\Sigma}$ is the estimate of the variance-covariance matrix of the errors in the structural model (the covariance matrix for ϵ_y, ϵ_f, ϵ_m, and ϵ_p). For the estimation results reported below, the residual *correlations* are fixed across all subsamples, while the *variances* are allowed to vary across subsamples.[5] Thus only the four innovation variances for the middle subsample must be estimated; the correlations (and implicitly, the covariances) are estimated with pooled information from all three samples. Under the assumption of jointly normal errors, the likelihood for each subsample is computed, the independent likelihoods for the three samples are summed to form the overall likelihood. The log-likelihood is maximized using a sequential quadratic programming algorithm, imposing the stability conditions required to generate a unique, stable solution for the structural model. For more details regarding the solution method and

[5] This relaxation is particularly important for the funds rate, which exhibited noticeable heteroskedasticity across these subsamples in earlier work.

retrieval of structural residuals, see the Appendix and Fuhrer and Moore (1993a).

5 Estimation Results

The full sample, which runs from 1966:I through 1993:I, is broken into three subsamples, corresponding to assumed shifts in the monetary policy regime. The first subsample runs from 1966:I to 1979:III, just before the well-known October 1979 change in Federal Reserve operating procedures. The second subsample runs from 1979:IV-1982:III, the end of the nonborrowed reserves operating procedure, and the third subsample runs from 1982:IV to 1993:I.

5.1 Limited Information Estimation of the Subsample Reaction Functions

Rather than estimating all possible reaction function specifications for all three subsamples jointly with all the other parameters in the model, limited-information estimates of the subsample reaction functions were obtained. The estimation strategy pursued here is to obtain a basic specification for the reaction function from limited information estimation on each subsample, and then to obtain more efficient estimates of the reaction function and other parameters via FIML. A brief description of the limited information specifications follows.

First subsample: 1966:I to 1979:III The funds rate responds to three lags of itself, two lags of the quarterly log growth rate of M2, and the contemporaneous output gap. Including contemporaneous and/or lagged inflation did not improve the fit of the equation, nor did including additional lags of the output gap or money growth. This specification is taken as the starting point for the FIML estimations below.

Second subsample: 1979:IV to 1982:III This subsample showed no discernible direct response to inflation or lagged inflation. However, the response to money growth increases dramatically compared to the first subsample (from about 0.1 to 0.8). Interestingly, the response to the output gap remains about the same as the estimated response for the first subsample.[6]

Third subsample: 1982:IV to 1993:I Only one lag of the funds rate is significant in this sample; its estimated coefficient is 0.8. While money growth is no longer significant, the coefficients on inflation (contemporaneous and once-lagged) sum to 0.53. The funds rate responds to both the output gap and the growth rate of real output.

Of all the equations, the reaction function equations show the least signs of misspecification, both in limited- and in full-information estimation. For example, the Ljung-Box $Q(12)$-statistics and associated p-values for the residuals from the three subsamples are 13.8 (0.31), 2.6 (Q(4) due to the limited number of observations, p-value = 0.63), and 2.9 (0.99) respectively.

5.2 FIML Estimates, Two Baseline Models

No attempt is made to estimate all the parameters of equations 1,4, 5, 6, 10,9 and 7 independently for the short middle sample. Instead, the parameters of the I–S curve and the contracting specification are constrained to be the same as those for the third sample, the money demand parameters to be the

[6] Preliminary maximum likelihood estimates yielded a coefficient on the lagged funds rate of 1. Thus the first-difference of the funds rate is used as the dependent variable in this equation from here on.

same as those for the first sample (there is no money demand function for the third sample), and the reaction function parameters are allowed to be estimated freely.

The exercise begins with an estimate of the model that holds all of the parameters fixed across the three subsamples *except* the reaction function parameters. All parameters are estimated jointly; the estimation method is as described above and in the Appendix. Tables 3 and 4 report the results of this estimation, along with asymptotic standard errors and equation summary statistics.

As a second benchmark, the model is estimated with all parameters free to vary across subsamples. The results of this exercise are reported in Tables 5 and 6.

Note that the overall deterioration in the likelihood function from constraining all of the non-reaction function parameters across the subsamples is 5.2. The likelihood ratio test for the six constraints imposed on s, γ, a_0, a_1, a_2, and a_ρ does not quite reject at the 10 percent significance level ($p-$value $= 0.11$). Thus the net effect of these constraints on the overall fit is relatively small. Still, this is essentially a portmanteau test of the six parameters which may hide significant instability in a key parameter or subset of the parameters. In the following subsections, this possibility will be tested.

Table 3
FIML Parameter Estimates: Independent Reaction Functions, Constrained

I–S, Contracting Equations

Equation	Parameter	Estimate	Standard error	t-statistic
I–S curve	a_0	.011	.003	3.1
	a_1	1.348	.041	32.6
	a_2	−.390	.042	−9.2
	a_ρ	−.379	.029	−13.1
Reaction function, 1966:I to 1979:III	α_0	−.029	.005	−5.5
	α_{r_1}	1.139	.059	19.2
	α_{f_2}	−.587	.046	−12.8
	α_{f_3}	.579	.060	9.6
	$\alpha_{\Delta m_1}$	−.013	.038	−.3
	$\alpha_{\Delta m_2}$	.115	.036	3.2
	α_y	.437	.046	9.5
Reaction function, 1982:IV to 1993:I	α_0	−.006	.003	−2.3
	α_{f_1}	.804	.034	23.8
	α_{π_0}	.301	.057	5.2
	α_{π_1}	.233	.061	3.8
	$\alpha_{\Delta y}$	.614	.079	7.7
Money demand 1966:I to 1982:III	b_0	−1.562	.154	−10.1
	b_1	1.713	.058	29.4
	b_2	−1.172	.115	−10.2
	c_0	.001	.001	.9
	c_1	−.016	.023	−.7
	c_2	.648	.050	12.9
	c_3	−.204	.042	−4.8
Contracting specification	s	.113	.010	10.7
	γ	.002	.001	1.9

Table 4

FIML Parameter Estimates: Summary Statistics

Overal Likelihood value: 2046.8	
Subsample Statistics:	
Sample 1: 1966:I–1979:III	
Ljung-Box Q(12) Statistics:	
I–S curve:	14.0
Reaction function:	15.0
Contracting equation:	38.5
Money Demand:	13.2
Decay Rate of Dominant Roots (complex): 6.1*percent* per quarter	
Sample 3: 1982:IV–1993:I	
Ljung-Box Q(12) Statistics:	
I–S curve:	17.1
Reaction function:	3.5
Contracting equation:	26.8
Dominant Roots Decay Rate (complex): 7.0 percent per quarter	

Table 5
FIML Parameter Estimates: Independent Parameters

Equation	Parameter	Estimate	Standard error	t-statistic
I–S curve 1966:I to 1979:III	a_0	.020	.013	1.6
	a_1	1.083	.082	13.2
	a_2	−.134	.085	−1.5
	a_ρ	−.895	.822	−1.1
I–S curve 1979:IV to 1993:I	a_0	.005	.010	.5
	a_1	1.533	.125	12.3
	a_2	−.583	.121	−4.8
	a_ρ	−.179	.267	−.7
Reaction function, 1966:I to 1979:III	α_0	1.141	.004	−6.9
	α_{f_1}	−.586	.050	23.0
	α_{f_2}	.574	.052	−11.2
	α_{f_3}	−.015	.044	13.0
	$\alpha_{\Delta m_1}$	.112	.023	−.6
	$\alpha_{\Delta m_2}$	.403	.024	4.6
	α_y	−.027	.041	9.8
Reaction function, 1982:IV to 1993:I	α_0	−.006	.003	−2.3
	α_{f_1}	.809	.037	21.6
	α_{π_0}	.316	.078	4.1
	α_{π_1}	.220	.083	2.7
	$\alpha_{\Delta y}$	.619	.090	6.9
Money demand 1966:I to 1982:III	b_0	−1.664	.331	−5.0
	b_1	1.635	.044	37.0
	b_2	−1.451	.418	−3.5
	c_0	.008	.007	1.2
	c_1	−.020	.005	−3.8
	c_2	.643	.039	16.6
	c_3	−.197	.044	−4.5
Contracting spec., 66-79:3	s	.122	.011	11.0
	γ	.001	.001	.8
Contracting spec., 1979:IV to 1993:I	s	.103	.013	8.1
	γ	.002	.003	.9

Table 6
FIML parameter estimates, Summary Statistics

Overal Likelihood value: 2052.0	
Subsample Statistics:	
Sample 1: 1966:I–1979:III	
Ljung-Box Q(12) Statistics:	
I–S curve:	13.9
Reaction function:	13.3
Contracting equation:	38.2
Money Demand:	12.9
Dominant Roots Decay Rate (complex): 5.5 percent per quarter	
Sample 1: 1982:IV–1993:I	
I–S curve:	13.3
Reaction function:	3.7
Contracting equation:	26.9
Dominant Roots Decay Rate (complex): 8.5 percent per quarter	

5.3 Comparison of Vector Autocovariance Functions for the Two Specifications

A compact but comprehensive comparison of the dynamics implied by the constrained and unconstrained models may be obtained through the vector autocovariance function.[7] Figure 1 displays the vector autocovariance function for the constrained and unconstrained specifications. Note that there is little discernible difference between the two functions, suggesting that whatever restrictions are imposed, they will not alter the fundamental dynamic interactions among the funds rate, inflation, and the output gap, and thus will not dramatically affect the analysis of the optimal policy frontier.

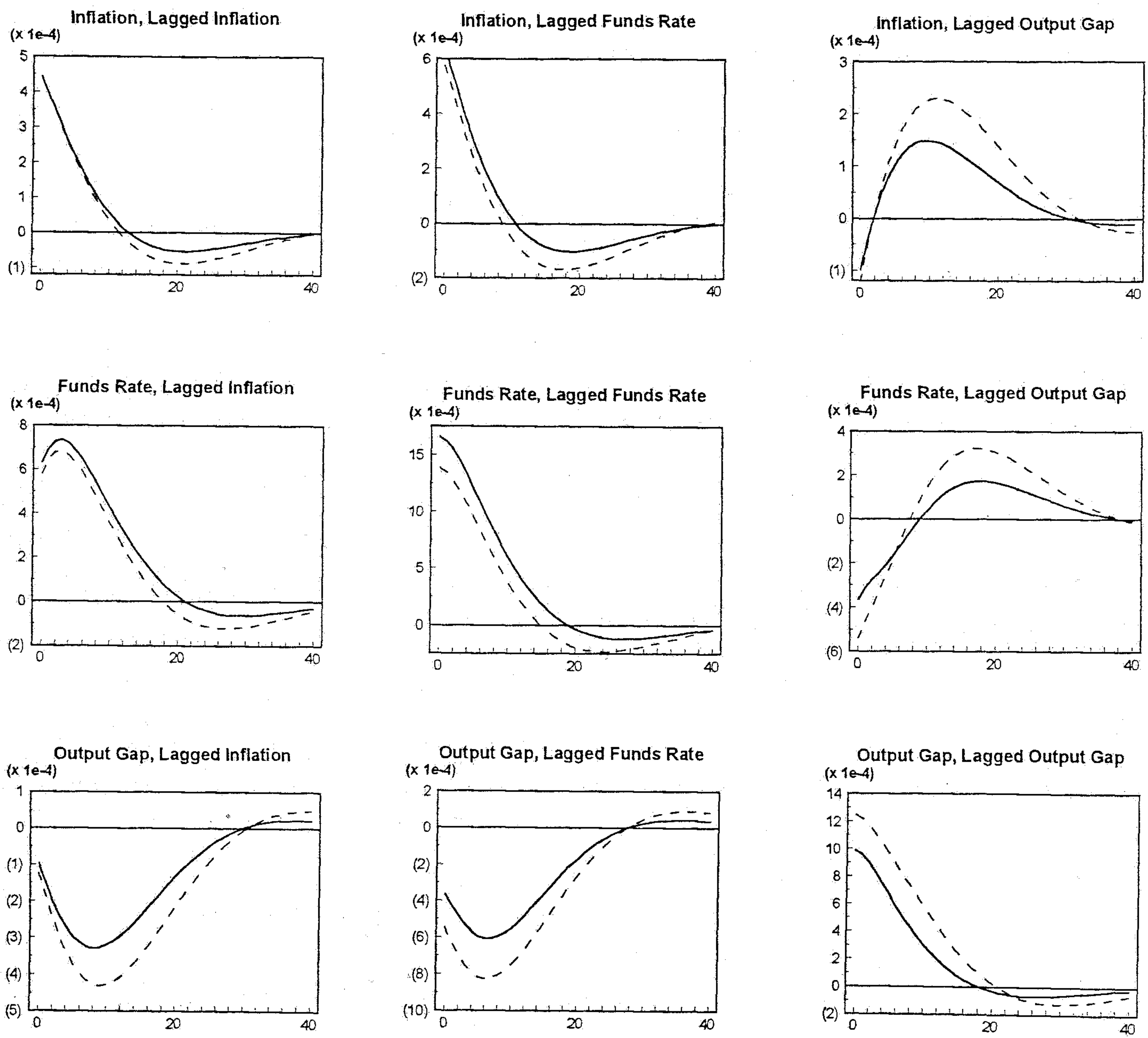
[7]For this linear model, the vector autocovariance function contains *all* of the information in the likelihood for the model. The details of computation for the autocovariance function may be found in the Appendix.

Figure 1

Comparison of Autocovariance Functions

Unconstrained vs. Fully Constrained Models

Free ———— Constrained - - - - - -



5.4 FIML Estimates, Freeing Up a Variety of Constraints

Table 7 displays the results from an array of subsample parameter stability tests. The table includes an almost exhaustive set of restrictions that lie between the restricted model (line 0 in the table) and the completely unconstrained model (the "Baseline" line in the table). The last three columns of the table list the log-likelihood value obtained under that row's set of restrictions, the number of restrictions imposed (the degrees of freedom for the likelihood ratio test), and the p-value of the likelihood ratio test for those restrictions.

The bottom line from these tests is that the only parameters that reject subsample stability are the lags of the output gap in the I–S curve. Sets of restrictions that include the restriction on these lags yield the lowest p-values. The test that restricts only these lags produces a likelihood value of 2049.3, thus accounting for more than half the reduction in the likelihood from the baseline model to the fully restricted model. The restriction that all of the parameters *except* the lags are stable across all three subsamples (specification 5) yields a p-value of 0.493. Only one set of restrictions that does not include the I–S lags produces a p-value lower than 0.4 (restricting the slope of the contract distribution, s, yields a p-value of 0.13).

Final specification. A final specification is chosen that restricts the contracting specification parameters as well as the I–S interest elasticity across the policy regimes. The lags in the I–S curve are estimated separately for the periods 1966:I to 1979:III and 1979:IV to 1993:I. With these restrictions imposed, it is possible to estimate the second subsample reaction function jointly with the other parameters. A summary of the results is reported in Tables 8 and 9. Figure 2 displays the vector autocovariance functions for

Table 7
Likelihood Ratio Tests for Subsample Parameter Stability

Model	Coefficient Constrained?					Log Likelihood	Degrees of Freedom	p-value, LR Test
	s	γ	a_ρ	$a_{1,2}$	a_0			
0	√	√	√	√	√	2046.8	6	.109
1		√	√	√	√	2047.8	5	.129
2	√		√	√	√	2047.0	5	.075
3			√	√	√	2048.1	4	.099
4	√	√		√	√	2047.3	5	.094
5	√	√	√		√	2050.3	4	.493
6	√	√			√	2050.3	3	.493
7	√	√	√			2050.3	3	.493
8		√		√	√	2048.1	4	.099
9		√	√		√	2051.2	3	.659
10					√	2051.7	1	.439
11	√					2050.9	1	.131
12		√				2051.8	1	.538
13			√			2051.7	1	.446
14				√		2049.3	1	.063
15			√	√		2048.0	2	.045
Baseline						2052.0	-	(*Baseline*)

Table 8
FIML Parameter Estimates: Final Specification

Equation	Parameter	Estimate	Standard error	t-statistic
I–S curve	a_0	.012	.004	2.8
	a_ρ	−.350	.094	−3.7
1966:I to 1979:III	a_1	1.054	.078	13.6
	a_2	−.152	.087	−1.7
1979:IV to 1993:I	a_1	1.527	.115	13.3
	a_2	−.551	.115	−4.8
Reaction function, 1966:I to 1979:III	α_0	−.0261	.004	−6.8
	α_{f_1}	1.143	.054	21.1
	α_{f_2}	−.570	.069	−8.3
	α_{f_3}	0.552	.054	10.2
	$\alpha_{\Delta m_1}$	−.014	.032	−.4
	$\alpha_{\Delta m_2}$	.108	.031	3.5
	α_y	.387	.055	7.1
Reaction function, 79:4-82:3	α_0	−.011	.015	−.7
	$\alpha_{\Delta m_1}$	.211	.171	1.2
	α_y	.364	.154	2.4
1982:IV to 1993:I	α_0	−.003	.004	−.8
	α_{f_1}	.838	.048	17.4
	α_{π_0}	.271	.091	3.0
	α_{π_1}	.142	.097	1.5
	α_y	.113	.035	3.3
	$\alpha_{\Delta y}$	.424	.117	3.6
Money demand 1966:I to 1982:III	b_0	−1.538	.809	−1.9
	b_1	1.701	.056	30.3
	b_2	−1.611	.342	−4.7
	c_0	.002	.021	.1
	c_1	−.027	.019	−1.4
	c_2	.643	.063	10.2
	c_3	−.202	.050	−4.0
Contracting specification	s	.112	.010	11.1
	γ	.002	.001	1.6

Table 9
FIML Parameter Estimates, Summary Statistics

Overal Likelihood value: 2052.0	
Subsample Statistics:	
Sample 1: 1966:I–1979:III	
Ljung-Box Q(12) Statistics:	
I–S curve:	14.1
Reaction function:	12.9
Contracting equation:	38.6
Money Demand:	12.6
Dominant Roots Decay Rate (complex): 7.4 percent per quarter	
Sample 2: 1979:IV–1982:III	
Ljung-Box Q(4) Statistics:	
I–S curve:	2.1
Reaction function:	4.0
Contracting equation:	6.5
Money Demand:	4.2
Dominant Roots Decay Rate (complex): 6.2 percent per quarter	
Sample 3: 1982:IV–1993:I	
Ljung-Box Q(12) Statistics:	
I–S curve:	12.8
Reaction function:	3.6
Contracting equation:	26.8
Dominant Roots Decay Rate (complex): 6.5 percent per quarter	

Table 10
Target Inflation Rates and Asymptotic Standard Errors

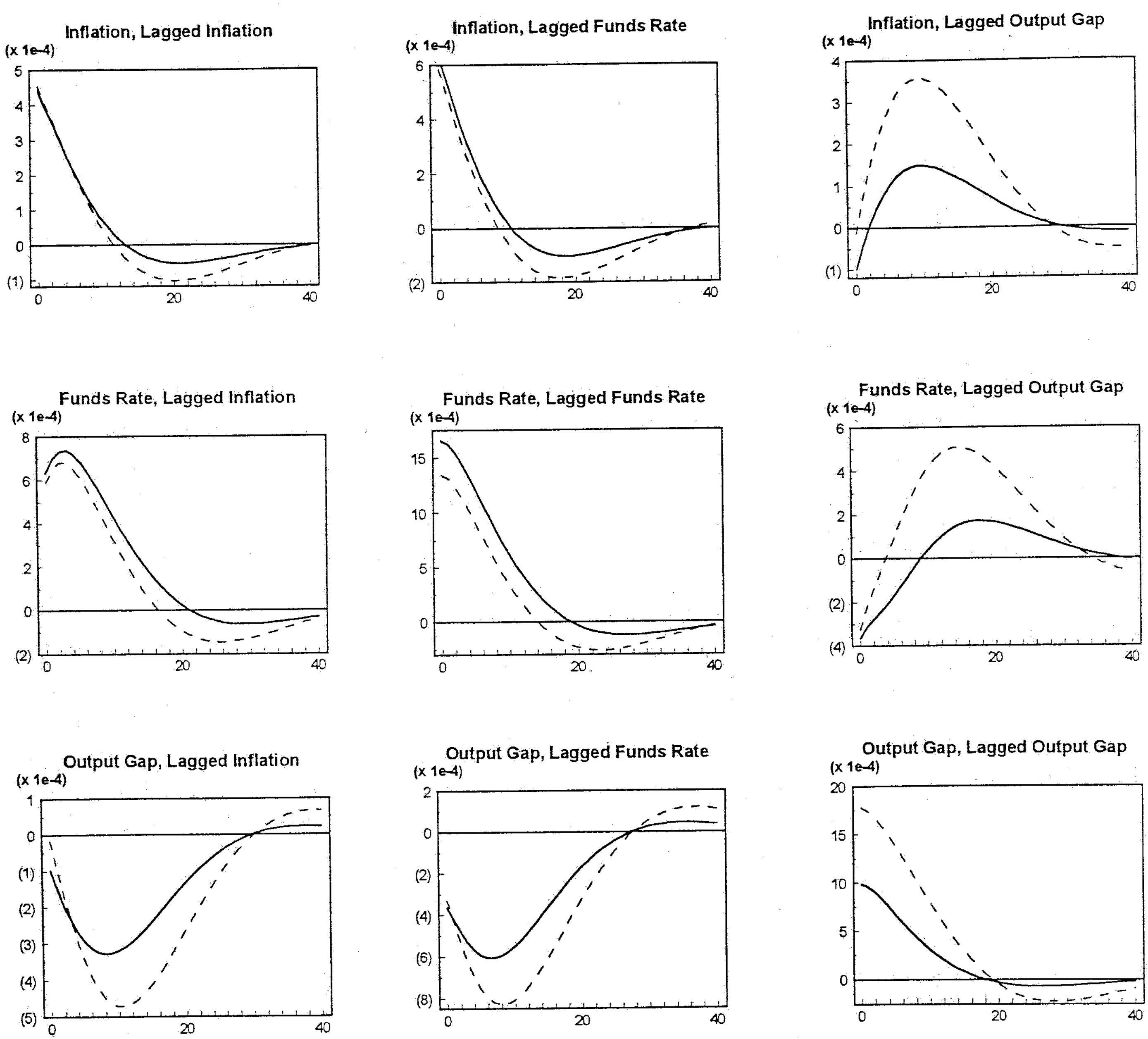
Subsample	Target Inflation Rate	Standard Error	t-Statistic
1966:I to 1979:III	9.95	1.4	6.9
1979:4-82:3	5.32	3.0	1.8
1982:IV to 1993:I	3.43	.9	3.9

Figure 2

Comparison of Autocovariance Functions

Unconstrained vs. Final Specification

Free ——————— Final - - - - - -



the unconstrained and final specifications. As the figure indicates, the restrictions imposed in the final specification do not greatly alter the dynamic interactions of the model variables.

5.5 The Target Rate of Inflation

For each subsample, the estimated reaction function implies a target rate of inflation. The target rate of inflation is of interest because its value in different subsamples could indicate a change in the Fed's target rate of inflation, and its precision can indicate whether the target is significantly different across subsamples. In addition, an estimated inflation target that is insignificantly different from zero may indicate that the Fed did not act so as to return the inflation rate to a specific target. In this case, as discussed above, the inflation rate will not be stationary.

The target inflation rates implied by each subsample reaction function, π_i^*, are displayed in equation 12.

$$
\begin{aligned}
\pi_1^* &= \bar{\rho}\frac{(1-\sum_i \alpha_{f_i}) - \alpha_0}{\sum_j \alpha_{\Delta m_j} - (1-\sum_i \alpha_{f_i})} \\
\pi_2^* &= \frac{-\alpha_0}{\alpha_{\Delta m_1}} \\
\pi_3^* &= \frac{\bar{\rho}(1-\alpha_{f_1}) - \alpha_0}{\sum_i \alpha_{\pi_i} - (1-\alpha_{f_1})} \qquad (12)
\end{aligned}
$$

where $\bar{\rho}$ is the equilibrium real interest rate, obtained from the I–S curve as $-a_0/a_1$.[8] The approximate asymptotic standard errors for the π_i^*s are

[8] The estimate of $\bar{\rho}$ from the final specification is .034, with an asymptotic standard error of 0.01, not significantly different from the estimate of $\bar{\rho}$ reported in Fuhrer and Moore (1993b).

computed from

$$V(\pi_i^*) = \frac{\partial g(\theta)}{\partial \theta} \Sigma \frac{\partial g(\theta)'}{\partial \theta}$$

where $g(\theta)$ is the function summarized in equation 12 mapping the estimated parameters into the π^*s and Σ is the variance-covariance matrix of the estimated parameters. Table 10 displays the estimated target inflation rates and their asymptotic standard errors and t-ratios. The model implies a declining target rate of inflation from the earliest through the latest sample periods. The target inflation rate of almost 10 percent in the earliest sample that includes the 1970s is more than two standard errors away from the two standard error region around the target inflation rate for the most recent sample. Thus the evidence is consistent with a significantly lower target inflation rate in the most recent sample. Note that only in the middle sample is the evidence of a specific target rate weakly significant. Overall, this model implies that the Fed has always behaved as if it had a target rate of inflation, thus inducing stationarity in the inflation rate. The target rate has dropped by 1/3 since the 1970s.

5.6 Modifications to the Model

Forward-looking monetary policy rule. In the reaction functions considered in the previous section, the funds rate responds only to current and lagged policy targets. A reasonable alternative is that monetary policy looks forward in determining where to set the federal funds rate, so that expected policy targets should enter the reaction function. A simple way to accomplish this is to allow as many leads of policy targets to enter as there are lags in the estimated reaction function, restricting the coefficients on the lead variables to be proportional to the coefficients on the lagged variables. Denoting the weight on the past and future targets as λ and $1-\lambda$, this implies a simplified reaction function

$$f_t = \lambda(\alpha_\pi \pi_{t-1} + \alpha_y \tilde{y}_{t-1}) + (1-\lambda)E_t(\alpha_\pi \pi_{t+1} + \alpha_y \tilde{y}_{t+1})$$

The estimate of λ for the final specification is 0.97 with standard error 0.53, indicating little evidence in the data for a forward-looking reaction function.

A less restricted version of the forward-looking reaction function allows the funds rate to react to the four-quarter moving averages of the expected inflation rate, the expected growth rate of real output, and the expected output gap

$$f_t = \alpha_\pi \pi_{t-1} + \alpha_y \tilde{y}_{t-1} + E_t(1/4)\sum_{i=1}^{4}(\alpha_\pi^f \pi_{t+i} + \alpha_y^f \tilde{y}_{t+i} + \alpha_{\Delta y}^f \Delta y_{t+i})$$

The full information estimates of α_π^f, α_y^f, and $\alpha_{\Delta y}^f$ for the third subsample are all insignificantly different from zero; the forward-looking coefficients' asymptotic t-statistics are .02, .69, and 1.34, respectively.[9]

At least two explanations can be offered for the lack of forward-looking behavior in these estimates. First, the forecasts in this specification are model-consistent expectations of future output and inflation; they may not closely resemble forecasts assembled by the staff of the Federal Reserve System. Staff forecasts may resemble fairly unrestricted projections of actuals on lagged values; the estimated reaction function already captures this. Second, voting members of the Federal Open Market Committee (FOMC) are not *required* to base their decisions on the staff forecasts. Thus while the staff may provide considerable forward-looking information, it may not have

[9]The standard errors on the backward-looking reaction function coefficients are increased as well, with the exception of the coefficient on the lagged output gap. This suggests that it is difficult to separately identify the influence of expected policy targets and the influence of lagged and current policy targets on the funds rate.

been reflected in movements of the policy instrument.

Alternative error covariance computations. Allowing the full error covariance matrix to differ across subsamples changes the results reported in the previous section very little. Again, the I–S curve shows the only compelling evidence of subsample instability. Constraining all the subsamples to have the same error covariance matrix (for those variables that overlap the subsamples) yields essentially the same results.

6 The Optimal Policy Frontier

A widely used measure of optimality for monetary policy suggests that policy attempts to minimize the weighted average of the unconditional variances of inflation and output (or unemployment) around target values.[10] For many reasonable characterizations of the economy, there will exist an "optimal policy frontier" that depicts the efficient combinations of inflation variance and output variance attainable by policymakers. The policy frontier is generally expected to be convex to the origin; that is, one must trade higher inflation variance for lower output variance, and vice versa. The frontier describes the variance combinations that are possible; it says nothing about what combinations are desirable. However, any reasonable set of preferences over inflation and output variance will lead to an interior solution in which the policymakers accept some of both inflation and output variance.[11]

[10]It may be that the monetary authority cares about the unconditional variance of its instrument as well. This concern does not enter the implicit objective function in this paper, in part because it is not clear why, given policies that yield stable economies, the variance of the instrument matters once the variance of the ultimate targets are minimized.

[11]Taylor (1994) provides analytical motivation for the long-run inflation/output variability trade-off in the context of a textbook model with sticky prices. Note that the frontier and preferences have convexities opposite to those in the standard prefer-

Note that this description of optimal policy allows the possibility that the optimal policy is no policy. While the estimated parameters indicate that the Fed has leaned against the wind, to varying degrees in different periods, the optimal policy may be laissez-faire. Interestingly, for all combinations of preferences and model specification, this study finds that activist monetary policies are optimal.

Because the policy frontier is inherently a counterfactual concept (what is the best that monetary policy could have done?), the frontier can only be constructed using an explicit model. It should come as no surprise, then, that the location and shape of the optimal policy frontier can depend critically upon the properties of the model used to construct the frontier.

What characteristics must a model have to produce an accurate estimate of the policy frontier (both its location relative to the origin and its slope)? It appears that a model must replicate at least two important features of the data in order to be considered adequate to provide an accurate estimate of the policy frontier. These features are required in order to produce realistic estimates of the (unconditional) variances of the variables determined by the model. To motivate a focus on the two properties, consider the simplest autoregressive model for variable x.

$$x_t = \rho x_{t-1} + \epsilon_t; \quad V(\epsilon_t) = \sigma_\epsilon^2 \tag{13}$$

The unconditional variance for x depends upon both the conditional variance (the variance of the error term, σ_ϵ^2), and the dynamics in the process for x, completely captured by the parameter ρ for this simple model. The unconditional variance, V(x), is $\sigma_\epsilon^2/(1-\rho^2)$. The larger is the conditional

ences/technology Edgeworth box. Their counterintuitive curvature arises because the "goods" under consideration—the variance of inflation and output—are really "bads".

variance, and the larger is ρ, the larger is the unconditional variance.

The analogues for a multivariate model are straightforward. The larger are the conditional variances, and the larger are the dominant roots in the system (the more slowly the system damps toward its equilibrium), the larger are the unconditional variances of the variables in the system. Thus, a model that is to accurately reflect the trade-offs in unconditional variances of inflation and output must at least[12]

1. Yield a reasonable estimate of the conditional variance. A model that fits the data poorly will overstate the conditional variances, thus overstating the unconditional variances and shifting the locus of the policy frontier outward from the origin.

2. Capture the dynamics in the data well,at least matching the dominant roots in the data. A model that overestimates the rate of decay of the policy targets toward their equilibria will underestimate the unconditional variances (for given conditional variance), shifting the locus of the policy frontier inward toward the origin. The properties of an unconstrained vector autoregression are used here as a benchmark against which to judge these properties of a candidate structural model. [13]

The final specification detailed in Table 8 satisfies both of these criteria. Fuhrer and Moore (1993a) documents the fit of the real contracting model,

[12]These are necessary, not sufficient, conditions for a model to yield a plausible optimal policy frontier.

[13]The unconditional variance is computed as the limit of the sequence $\sum_{i=0}^{k-1} A^i \Psi (A^i)'$, where A is the state transition matrix for the solved model, and Ψ is the residual variance-covariance matrix. The powers of A may be written $A^i = VD^iV^{-1}$ where V is the matrix of right eigenvectors of A, and D is the diagonal matrix of corresponding eigenvalues. Thus the rate at which terms in the sequence of $A^i\Psi(A^i)'$ s converge depends on the magnitude of the dominant roots of A.

demonstrating that it dominates the traditional contracting model, particularly for matching the sample properties of inflation.[14] An unconstrained vector autoregression for the variables π, f, and $\tilde{y}$ estimated over the last two subsamples has dominant roots that decay toward equilibrium at 6.4 percent per quarter. The estimated model yields dominant roots with decay rates of 6.2 percent and 6.5 percent for the last two subsamples. Thus our model should yield a plausible estimate of the optimal policy frontier.

6.1 Computing the optimal policy frontier

Here the optimal policy frontier is computed by tracing out the minimum weighted unconditional variances at different slopes along the frontier (implicitly, at different relative preferences for inflation versus output gap variance). Denote the relative weight attached to inflation variance as μ. Given the model specification for the last subsample, the following optimization is performed:

$$\min_{\theta} \{\mu V(\pi - \pi^*, \theta) + (1-\mu) V(\tilde{y}, \theta)\} \tag{14}$$

over a grid for μ from 0.05 to 0.95 in increments of 0.05. θ includes the parameters in the monetary policy reaction function $[\alpha_{f_1}, \alpha_{\pi_0}, \alpha_{\pi_1}, \alpha_{\Delta y}, \alpha_{\tilde{y}}]$. Departures from the estimated form of the reaction function and the cross-sample restrictions imposed on the other parameters in the model are explored below. Details of the computation of the unconditional variances may be found in the Appendix.

[14]Direct comparison of the structural model with a VAR in inflation, funds rate, and output gap is not appropriate. In the the structural model, the price *index* is the observable variable, not the inflation rate. In addition, the presence of the contract price in the structural model implicitly includes an infinite history of price indices in the model.

6.2 Results

Figure 3 displays the optimal policy frontier computed from the last subsample reaction function, the full sample contracting specification, and the partially constrained I–S curve. The asterisk indicates the combination of unconditional variances that arises for this model at the estimated parameter values. The estimated frontier has several interesting implications:

- The actual policy outcome, summarized by the combination of unconditional variances at the estimated parameter values, lies somewhat outside the optimal frontier. Policy in the 1980s has not been far from optimal according to this metric.

- The actual policy outcome lies just outside the frontier at the point $\mu = 0.8$. The actual policy outcome implies a 4 to 1 relative distaste for inflation variability relative to output variability.

- While the distance in inflation variance/output variance space does not appear great, the policy responses required to move to the frontier are considerably larger than those estimated historically, particularly for the parameter on the level of the output gap. The optimal reaction function parameters along the frontier are displayed in Table 11.

- Decreasing inflation variance (a move to the left and upward along the frontier) would entail a substantial increase in the variation of the output gap.

Figure 4 displays the results of a similar exercise for the first subsample. Here the actual outcome is also quite close to the frontier. The unconditional variance of inflation is larger in this subperiod, due to the large supply shocks that disrupted the economy during this period (the larger conditional variance-covariance matrix estimated for the 1960s and 1970s). In addition,

Figure 3

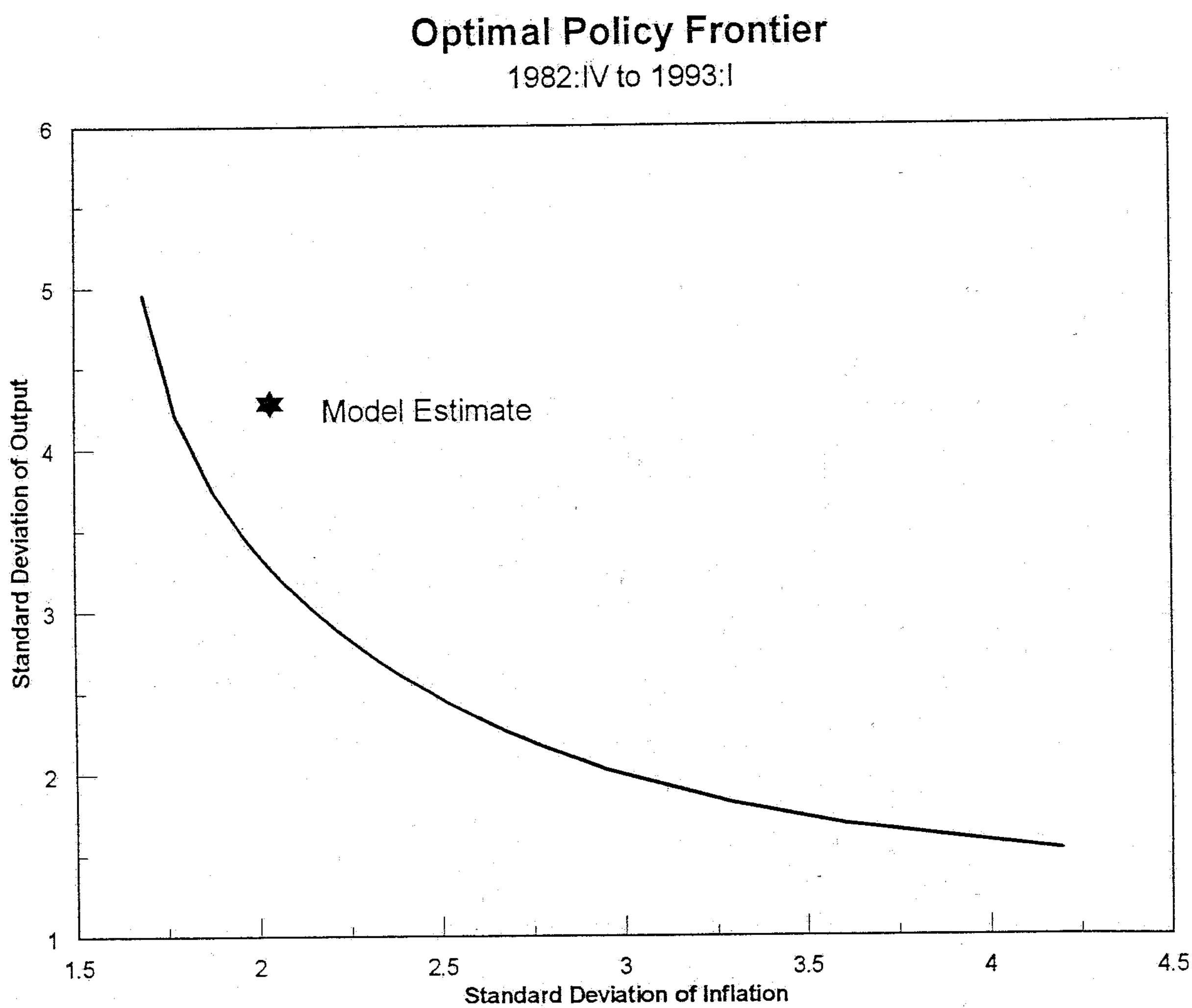
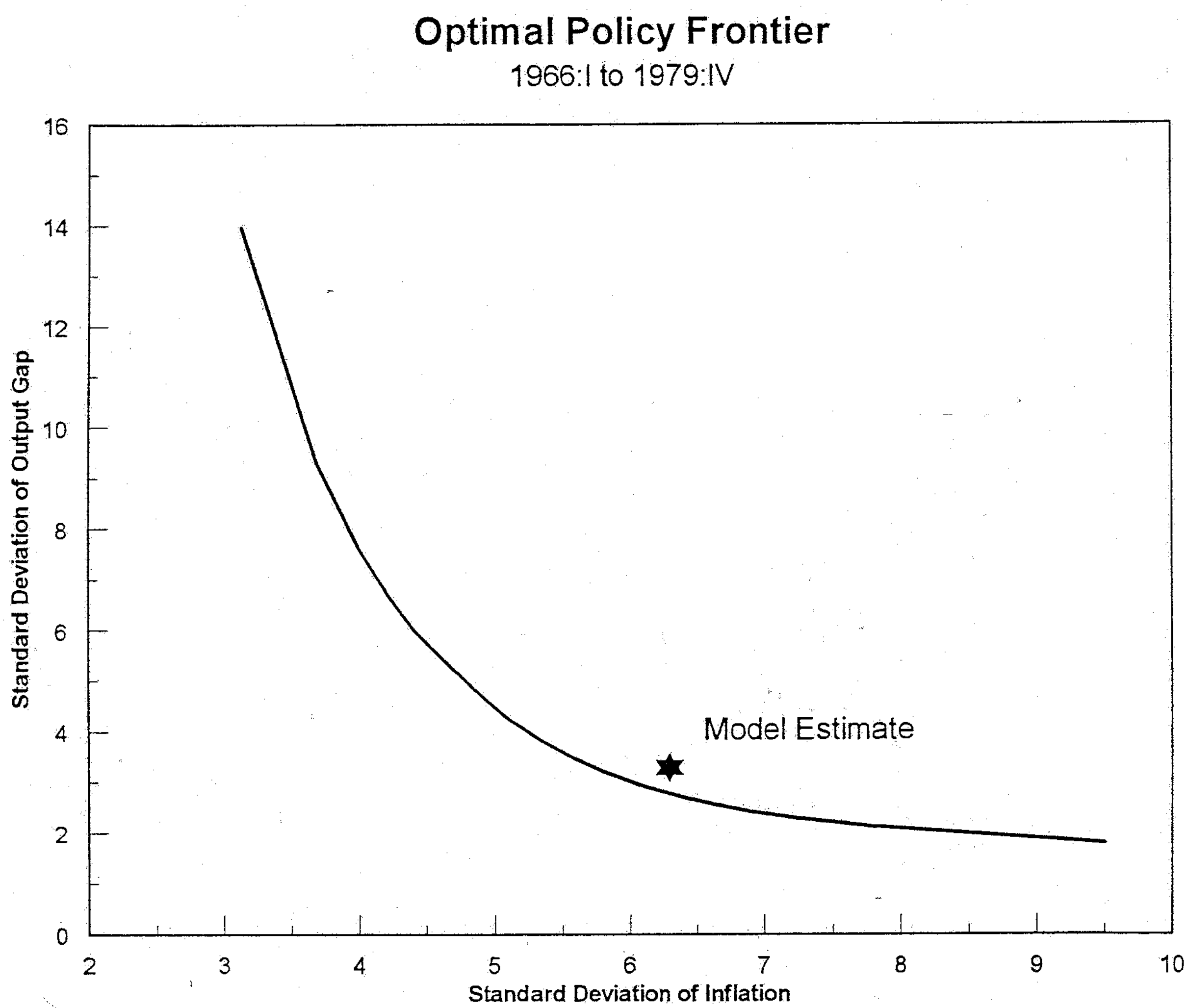

Optimal Policy Frontier
1982:IV to 1993:I
Model Estimate
Standard Deviation of Output
Standard Deviation of Inflation
6
5
4
3
2
1
1.5
2
2.5
3
3.5
4
4.5

Figure 4


Optimal Policy Frontier
1966:I to 1979:IV
16
14
12
10
8
6
4
2
0
Standard Deviation of Output Gap
Model Estimate
2
3
4
5
6
7
8
9
10
Standard Deviation of Inflation

the dominant roots of the system imply a somewhat slower decay rate in the first subsample, so that shocks keep the system away from equilibrium longer, increasing the unconditional variances.

Table 11
Policy Parameters along the Optimal Policy Frontier

	Reaction Function Parameters							
	Estimated	Optimal parameter values for $\mu =$						
Mnemonic	Value	.06	.22	.42	.54	.70	.82	.98
α_{π_0}	.27	.40	.40	.41	.41	.40	.41	.48
α_{π_1}	.14	.18	.24	.26	.27	.28	.29	.37
α_y	.11	2.91	1.56	.91	.67	.42	.28	.19
$\alpha_{\Delta y}$	.42	.57	.54	.52	.51	.50	.50	.48
α_r	.84	.76	.81	.83	.84	.85	.87	1.00

6.3 The Robustness of the Frontier to Variations in Structural Parameters

A simple way to gauge the robustness of the computed optimal policy frontier with respect to the exact specification detailed in Table 8 is to compute the derivatives of the unconditional variances at the frontier with respect to the (non-policy) structural parameters at their estimated values.[15] In addition to the parameters in the final specification, a parameter λ is included that indexes the degree of "forward-lookingness" in monetary policy, using a simple modification of the reaction function.

$$f_t = \lambda(\alpha_\pi \pi_{t-1} + \alpha_y \tilde{y}_{t-1}) + (1-\lambda)E_t(\alpha_\pi \pi_{t+1} + \alpha_y \tilde{y}_{t+1})$$

[15]The derivatives of the weighted sum of the unconditional variances with respect to the policy parameters are 0 by construction.

The derivative with respect to λ is taken about 0, the value imposed in the estimation of the final specification. Because the optimal policy frontier is a highly nonlinear function of the underlying structural parameters in the model, the derivatives are only local, first-order approximations of the sensitivity of the frontier to changes in the parameters.

Table 12 displays the derivatives of the unconditional variances and the objective function from equation 14 with respect to each of the parameters at a grid of values for μ.[16] Increasing λ, which increases the forward-lookingness of monetary policy, can shift the optimal policy frontier either outward or inward, depending on the emphasis placed on inflation variability. In neither case does the magnitude of λ have any appreciable influence on the location or shape of the optimal frontier. As shown in Figure 5, an increase in λ from 0 to 0.3 yields an almost imperceptible shift in the frontier. This result suggests that the omission of explicitly forward-looking monetary policy responses is of little importance in ascertaining the shape and location of the optimal policy frontier.[17]

Increased sensitivity of the I–S curve (as summarized in a_ρ) affects the unconditional variances as expected. It allows greater control over the inflation rate through the standard monetary transmission channel, thus decreasing the variance of inflation. The trade-off is an increase in the variation of the output gap for heavy emphasis on inflation. As with λ, the effect on the policy frontier of changing the a_ρ is quite small, as shown in Figure 6.

[16] That is, Table 12 reports the value of D $V(\pi)/D\theta$, D $V(\tilde{y})$ / $D\theta$, and μ D $V(\pi)/D\theta$ + (1-μ) D $V(\tilde{y})/D\theta$ for μ = [.15, .25, .45, .55, .65, .85, .95]. The derivatives are computed numerically; the derivatives reported here are not sensitive to the finite-differencing interval used.

[17] This result holds up the response to expected inflation and output growth is defined as λ times the unweighted average of the next four quarters of expected inflation and output growth. In addition, the optimal policy frontier with unrestricted forward-looking policy responses, as in section 5.6 above, lies in nearly the same location and has nearly identical contours as the frontier in Figure 3.

Table 12
Derivatives of Unconditional Variances with respect to Non-policy Parameters

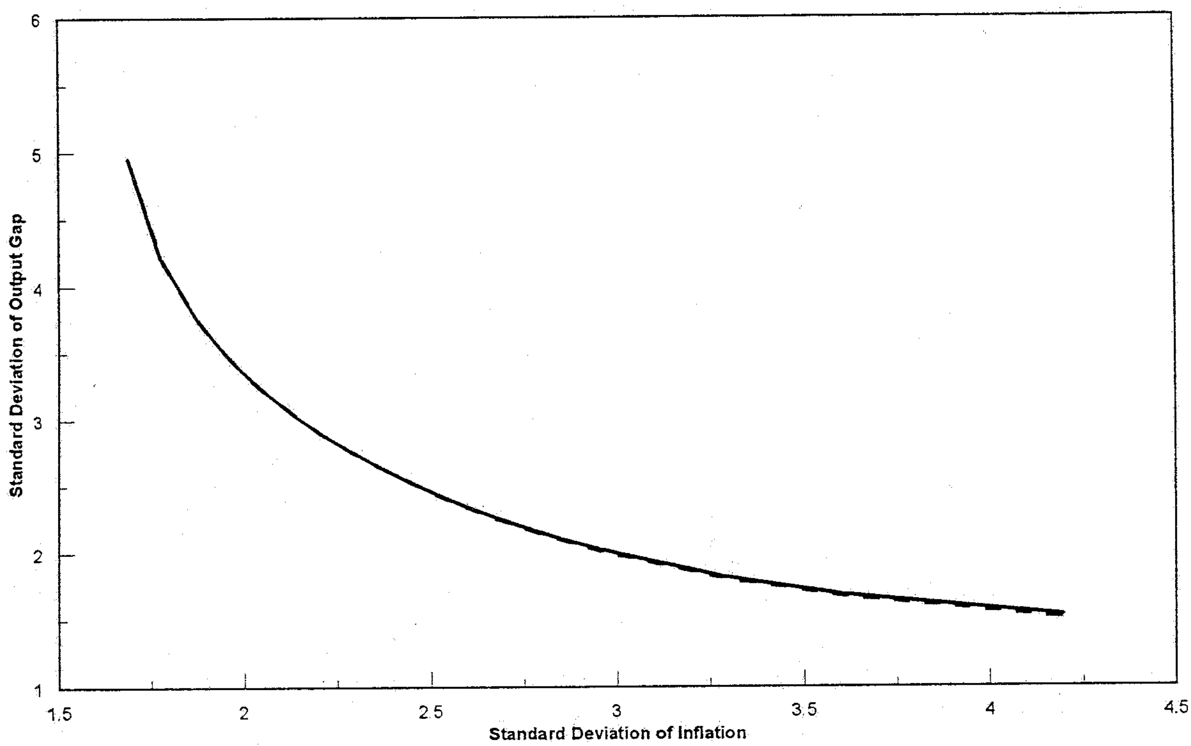
Parameter	.15	.25	.45	.55	.65	.85	.95
			Weight on Inflation Variance				
Derivative of unconditional variance of inflation with respect to:							
a_1	−.039	−.061	−.053	−.057	−.058	−.037	−.001
a_2	−.041	−.063	−.052	−.054	−.053	−.029	0.006
a_ρ	−.000	−.001	−.002	−.003	−.005	−.009	−.009
s	1.659	1.239	.938	.810	.701	.547	.459
γ	−35.205	−27.459	−20.955	−17.992	−15.049	−9.755	−6.060
λ	−.003	−.002	−.001	−.000	.000	.001	.001
Derivative of unconditional variance of output gap with respect to:							
a_1	.051	.117	.113	.148	.198	.338	.595
a_2	−.018	.021	.021	.044	.076	.160	.340
a_ρ	−.015	−.023	−.018	−.016	−.011	.022	.080
s	.069	.120	.254	.335	.436	.732	1.335
γ	−1.577	−2.977	−7.056	−10.343	−15.391	−34.457	−73.170
λ	−.004	−.001	−.002	−.001	−.001	−.003	−.018
Derivative of objective function with respect to:							
a_1	.037	.073	.038	.035	.031	.019	.028
a_2	−.021	−.000	−.012	−.010	−.008	−.001	.022
a_ρ	−.013	−.018	−.011	−.009	−.007	−.004	−.005
s	.307	.400	.562	.596	.608	.575	.503
γ	−6.621	−9.098	−13.311	−14.550	−15.169	−13.461	−9.416
λ	−.004	−.001	−.001	−.001	−.000	.000	.000

All figures are multiplied by 100.

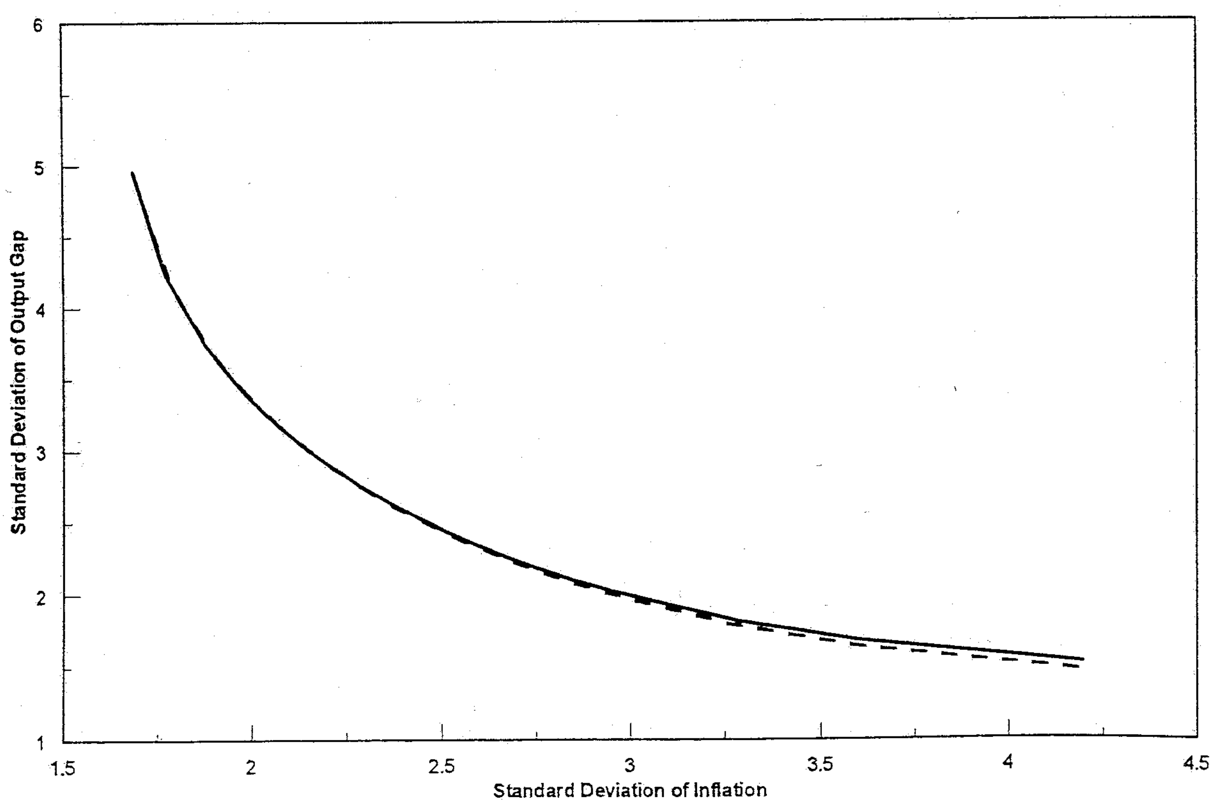
Figure 5

Shift in Optimal Policy Frontier with Decreases in Forward-Lookingness of Policy

Due to 10% increase in lambda



Due to 30% increase in lambda



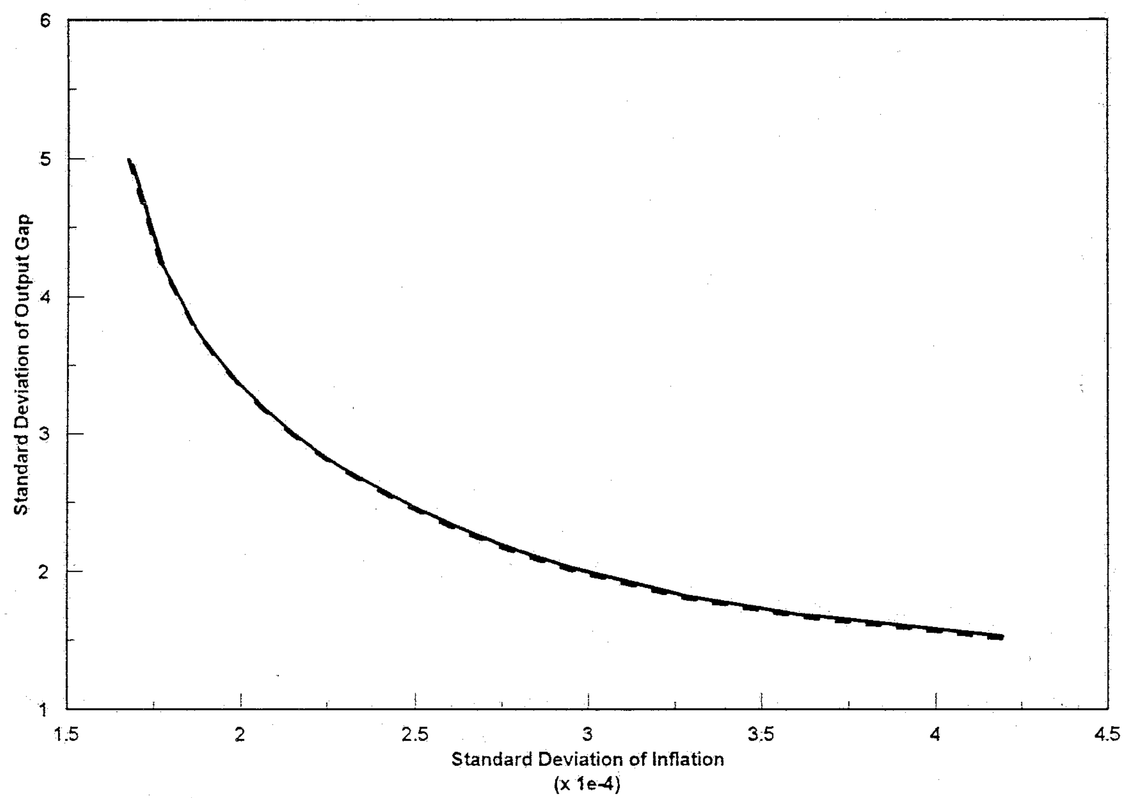
A decrease in the estimated slope of the contract distribution, s, unambiguously lowers both unconditional variances. Decreasing the slope of the contract distribution is equivalent to increasing the average age of price contracts. Holding other effects constant, this change should impart greater inertia to the inflation rate, reducing its response to shocks in the steady state, and reducing the unconditional variance of inflation. The less vigorous policy response required for a less volatile inflation rate implies less volatile output. Thus a significantly smaller estimate of the slope of the contract distribution shifts the policy frontier noticeably inward, as shown in Figure 7. However, note that the slope parameter is one of the most precisely estimated, with an asymptotic standard error less than one-tenth the magnitude of the parameter. Thus, it is unlikely that the estimate of the slope is so far off as to produce an inaccurate representation of the policy frontier.

Both unconditional variances are unambiguously and significantly decreased by an increase in the effect of excess demand on real contract prices, γ. When a smaller change in the output gap yields a larger impact on inflation, monetary policy has more control over inflation, thus lowering its variance, and need not disrupt output as much to effect its inflation goal, thus lowering output gap variations. The effect of a 30 percent increase in γ on the policy frontier is shown in Figure 8. The frontier flattens out a bit, implying a slightly smaller output variance cost for a given decrease in inflation variance. The frontier also shifts inward, decreasing the inflation and output variances along the frontier by 10 to 15 percent. This parameter suggests the most uncertainty about the policy frontier, since it is not estimated with great precision (its one-tailed t-test is not quite in the right-hand 5 percent tail). Thus a 30 percent smaller value is well within the 95 percent confidence region for the parameter. If the frontier lies in the region indicated by the dashed line in Figure 8, the actual inflation/output gap variance realization

Figure 6

Shift in Optimal Policy Frontier with Decreased Sensitivity of the I-S Curve

Due to 10% increase in a_ρ



Due to 30% increase in a_ρ

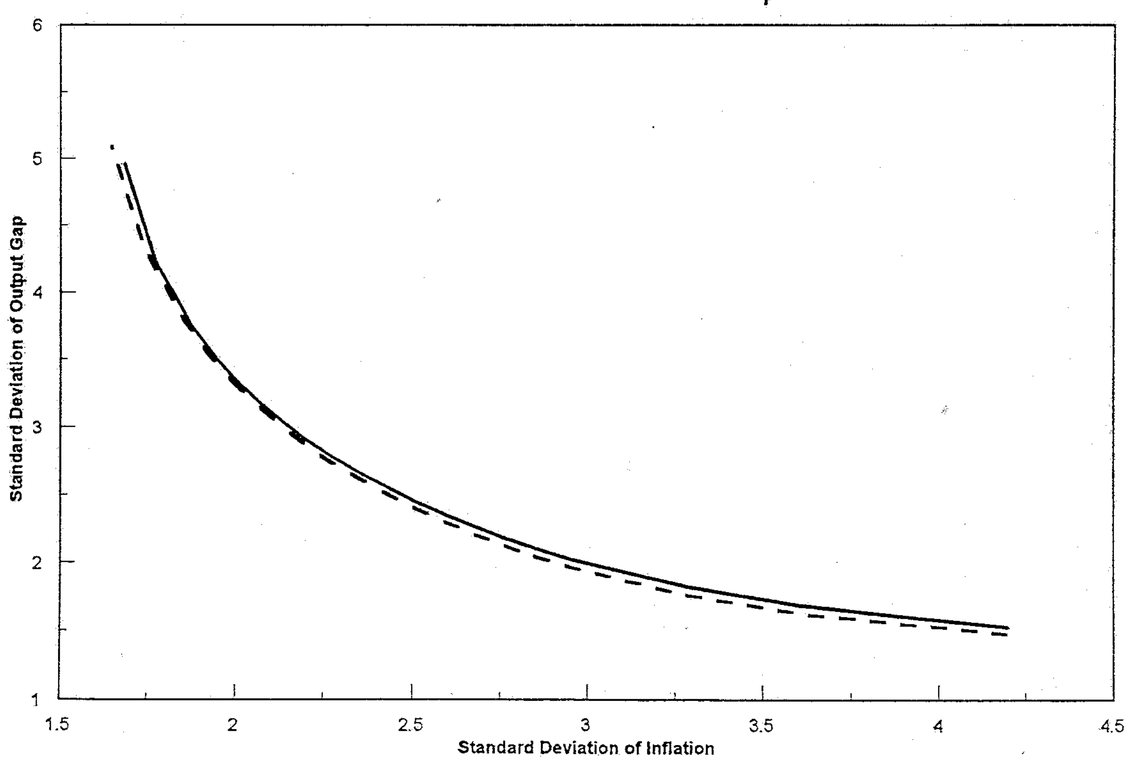
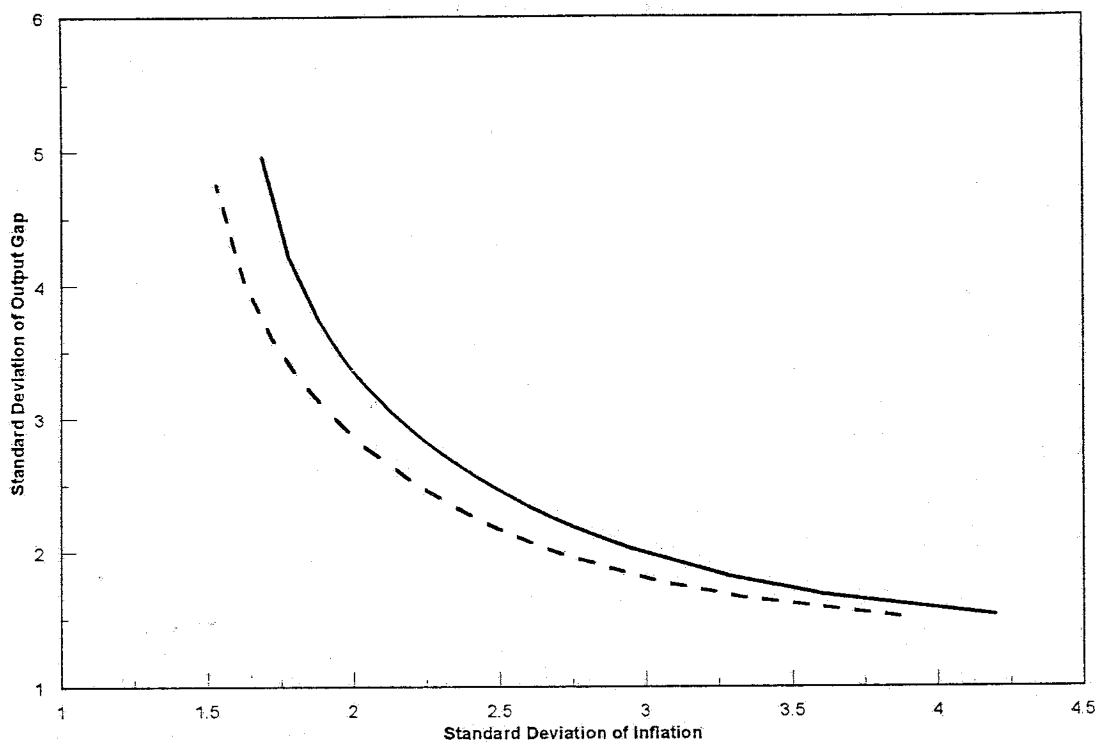


Figure 7

Shift in Optimal Policy Frontier with a Decrease in the Estimated Slope of the Contract Distribution

Due to 10% decrease in slope



Due to 30% decrease in slope

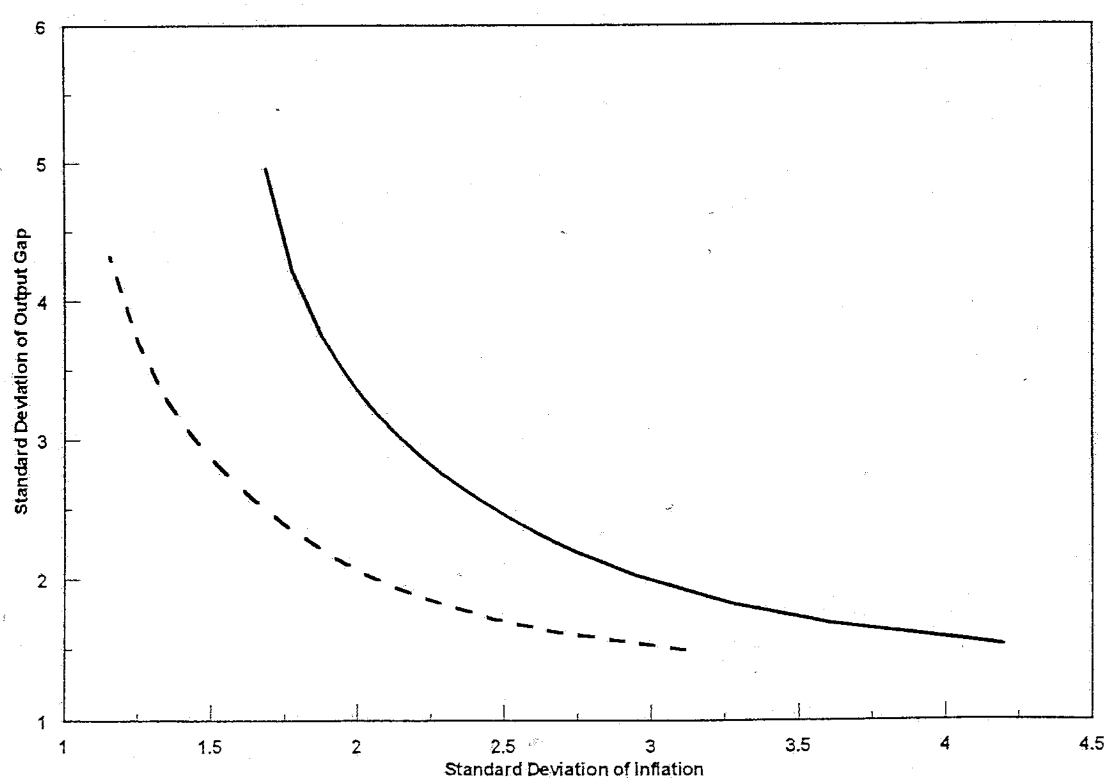
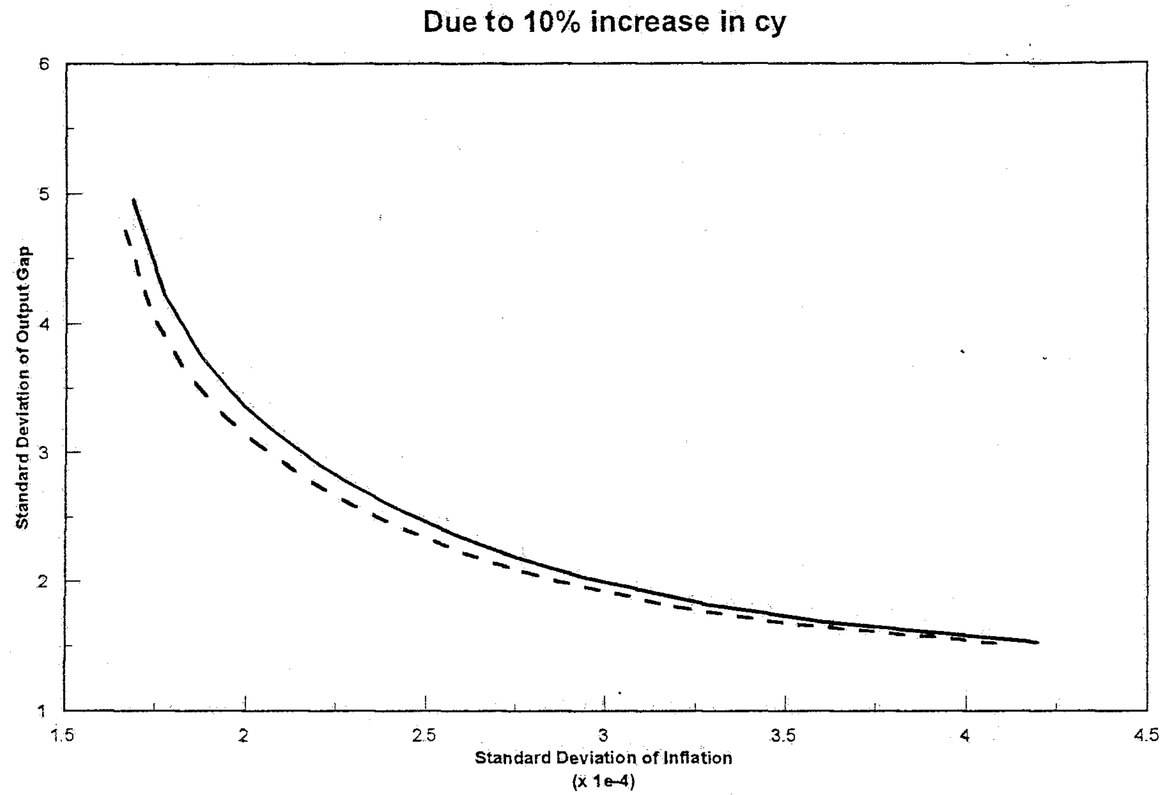
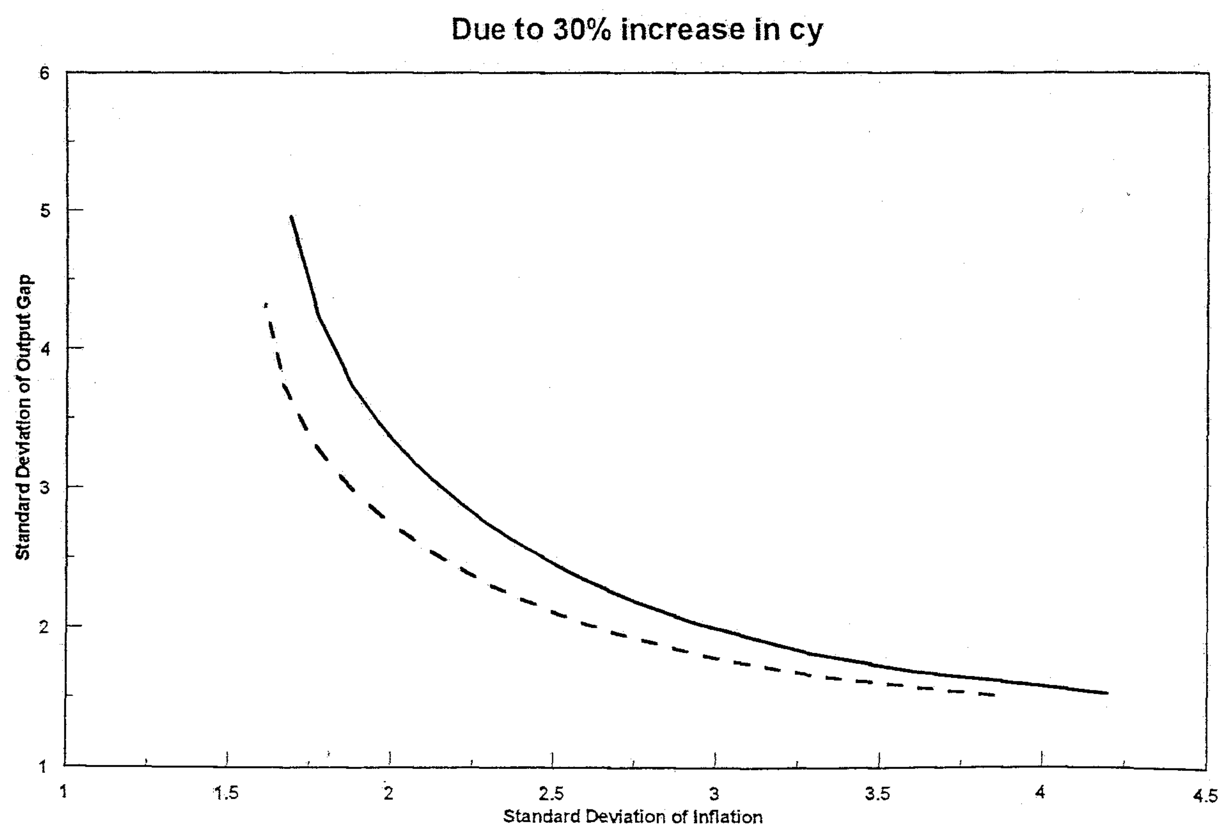


Figure 8

Shift in Optimal Policy Frontier with an Increase in the Effect of Excess Demand on Real Contract Prices





depicted by the asterisk in figure 3 looks somewhat less like a success story. Still, the general position and contours of the frontier are not dramatically different from the frontier consistent with the estimated parameter values.

6.4 A Second Robustness Check: Alternative Structural Models

As a second check on this study's estimate of the locus and slope of the optimal policy frontier, the frontier is computed for structural models with different price specifications. The first model uses the simple Phillips curve

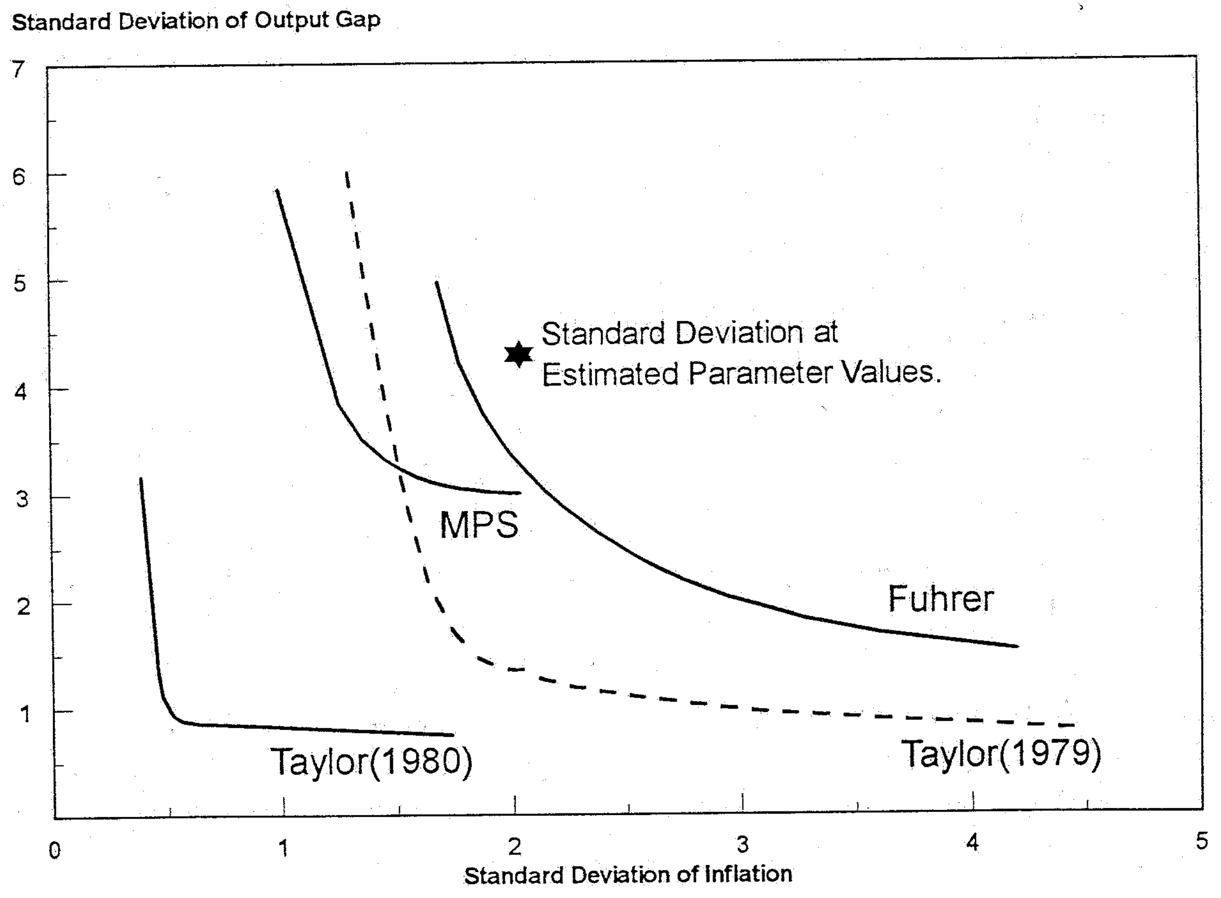
$$\pi_t = \sum_{i=1}^{3} \delta_i \pi_{t-i} + \Gamma \tilde{y}_t + \epsilon_\pi \tag{15}$$

where $\sum \alpha_i = 1$ is imposed. This is a simplified version of the type of expectations-augmented Phillips curve that appears in the MPS quarterly model (see Brayton and Mauskopf 1985). Equation 15 is estimated jointly with the I–S and third subsample reaction functions from above to obtain estimates of δ_i and Γ. The estimates are not reported here, but the overall fit of the model is good. The dominant roots for the companion form of the model are a complex pair with modulus 0.982, implying a decay rate of less than 2percent per quarter, considerably slower than the structural model and the unconstrained VAR.[18] As shown in Figure 9, the optimal frontier for this MPS-style model lies in about the same position as the frontier for the real contracting model. The contours of the MPS frontier are a bit different from the real contracting model; the frontier flattens out at a higher output gap standard deviation, suggesting a less severe penalty in output variation for

[18] As expected, the trace and determinant of the unconditional variance-covariance matrix for this model at the estimated parameter values are somewhat larger than their counterparts for the contracting model.

Figure 9

Optimal Policy Frontiers



a decrease in inflation variation at that point. However, the output penalty for decreasing the standard deviation of inflation below 1.5percent is severe, as it is for the real contracting model.

The second model is the overlapping nominal contracts model of Taylor (1980). The definition of the price index is the same as the real contracting model; the contracting equation is

$$x_t = \sum_{i=1}^{3} \beta_i [x_{t-i} + E_t x_{t+i}] + \gamma^* \sum_{i=0}^{3} \omega_i E_t(\tilde{y}_{t+i}) + \epsilon_t \qquad (16)$$

As documented in Fuhrer and Moore (1993a), the dynamic correlations among inflation, its own lags, and lags of the nominal rate and the output gap decay much more rapidly than those in a benchmark vector autoregression. This suggests that the nominal contracting model will estimate considerably smaller unconditional variances, implying an optimal policy frontier that lies well inside the frontier for the real contracting model.[19]

The policy reaction function for the third subsample and the I–S curve are held at their estimates from Table 8. The maximum likelihood estimates for s and γ for the third subsample for this model are 0.105 and 0.00233, with standard errors of 0.02 and 0.002.[20] Instead of using the ML estimates, the excess demand parameter, γ, is boosted to .24, as reported in Roberts (1993).

As shown in Figure 9, the policy frontier for the nominal contracting model lies well inside the frontiers for the Phillips curve and the real contracting models. The general contours are similar to the other models' frontiers.

[19]Taylor has not computed a frontier in variance of output, variance of inflation space for this model.

[20]Note that the estimate of γ is higher than the full-sample estimate presented in *Inflation Persistence*. However, as we point out in that paper, the dynamics of the nominal contracting model depend very little on the magnitude of γ.

Note that Taylor (1992) presents a similar juxtaposition of policy frontiers. In that paper, he also displays a policy frontier from a model developed in Taylor (1979) , which is reproduced as the dashed line in Figure 9. That frontier lies much closer to the MPS and real contracting frontiers. However, the properties of that model differ substantially from the overlapping contracts model of Taylor (1980). In Taylor (1979), the inflation equation is $\pi_t - \pi_{t-1} = \alpha E_{t-1}\tilde{y}_t, \alpha \geq 0$. This formulation is a simplified version of the MPS Phillips curve equation 15, with the maximum lag set to unity. The inflation equation in Taylor (1980) is $E_t\pi_{t+1} - \pi_t = -\alpha\tilde{y}_t, \alpha \geq 0$. A more extensive discussion of the difference in dynamic behaviors of these two specifications may be found in Fuhrer and Moore (1992). Finally, the Taylor (1979) frontier was computed using data from 1953 to 1975, which misses the second oil price shock. Thus the nature of the shocks assumed to disrupt the economy may be more like the shocks in the 1980s and 1990s, and the frontier may appropriately fit in with the rest of the frontiers in Figure 9.

6.5 What about the 1990s?

At considerable econometric hazard, the reaction function can be estimated for the period 1988 to the present and the unconditional variances implied by that policy response computed. The funds rate reaction function for this sample is well represented by

$$f_t = 1.24 * (1/4)\sum_{i=1}^{4}\pi_{t-i} + .52\tilde{y}_{t-1} + .028$$

Note that no evidence appears of interest rate smoothing (the lagged funds rate does not appear significantly in the equation), and the emphasis on inflation has more than doubled over the estimate for the period 1982 to 1993. The actual and fitted values for this equation appear in the top panel of Figure 10.

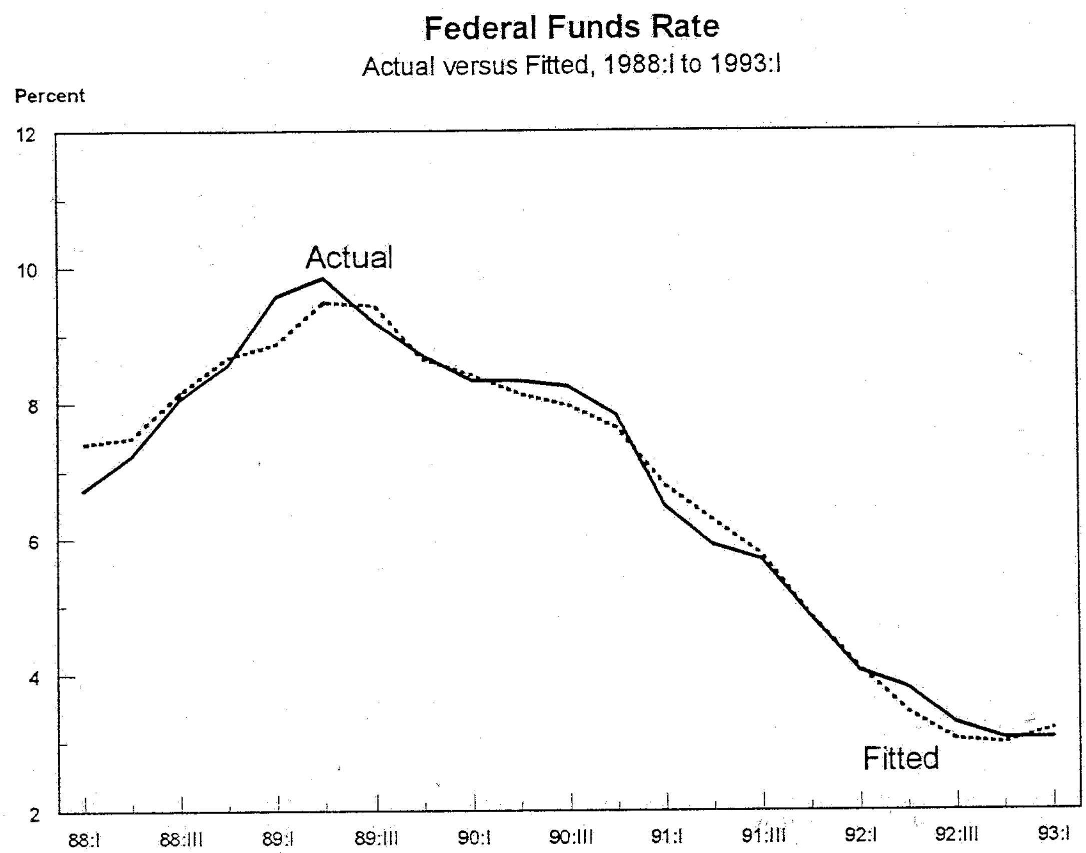
The bottom panel of Figure 10 shows the optimal policy frontier from Figure 3, along with the estimate of the unconditional variance implied by the reaction function for 1988 to 1993. The unconditional variance outcome implied by the model with the late '80s and early '90s reaction function lies a bit further from the frontier than the outcome implied by the model for a fixed reaction function for the post-1982 period. Given the degrees of freedom available to estimate the three parameters of the reaction function, these results should be taken with a grain of salt. Still, they do not suggest that Fed policy may have taken a turn for the worse in the last five years.

7 Conclusions

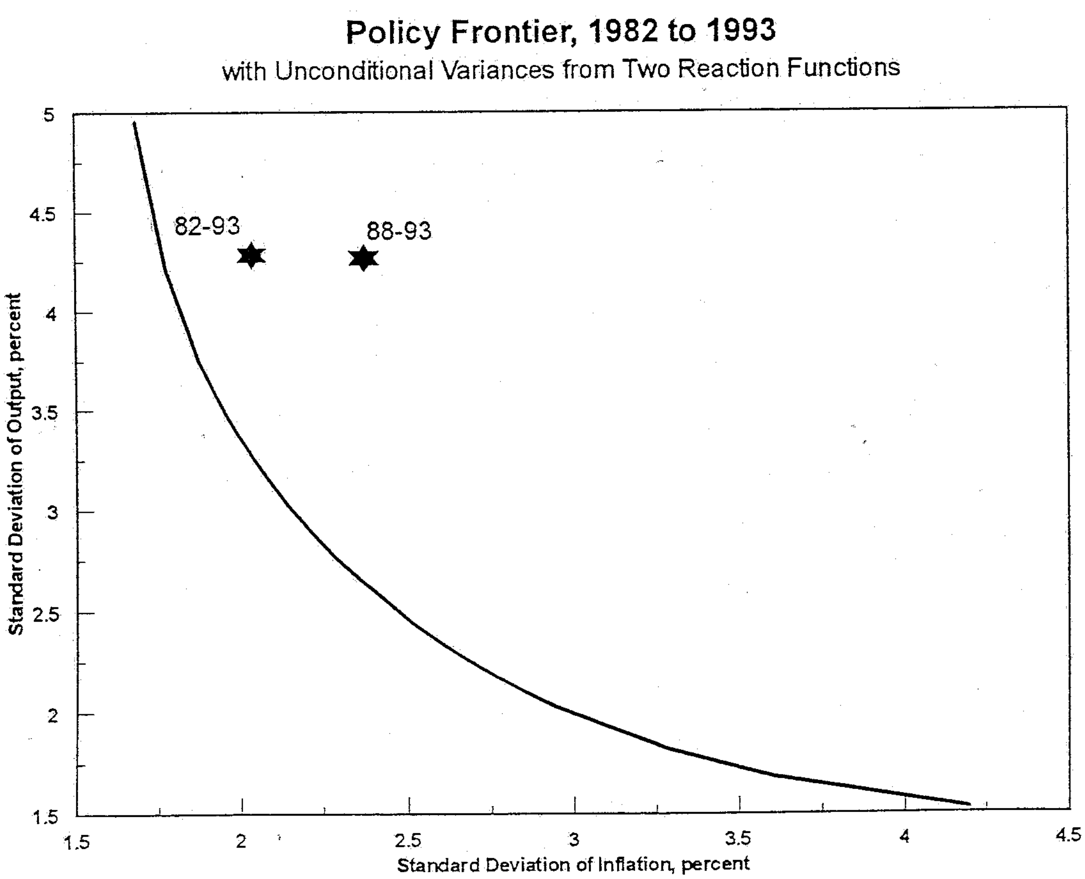
- The contracting specification is quite stable across three distinct monetary policy regimes. The parameters that characterize excess demand sensitivity and the shape of the contract price distribution appear to have changed insignificantly since the mid 1960s. The aggregate demand (I–S) specification appears to have shifted somewhat; in the pared-down specification used here, the shift appears in the coefficients on lagged output, not in the interest elasticity.

- Using the model specification for the post-nonborrowed reserves operating procedure period, an optimal policy frontier is computed. The frontier indicates that the actual performance of the economy lies fairly close to the frontier. In addition, the shape of the frontier implies that a reduction in the standard deviation of inflation below 2 percent entails an enormous increase in output variability. Similarly, reducing the standard deviation of output below 2 percent entails a large increase in inflation variability.

Figure 10



Federal Funds Rate
Actual versus Fitted, 1988:I to 1993:I
Percent
12
10
8
6
4
2
Actual
Fitted
88:I
88:III
89:I
89:III
90:I
90:III
91:I
91:III
92:I
92:III
93:I



Policy Frontier, 1982 to 1993
with Unconditional Variances from Two Reaction Functions
5
4.5
4
3.5
3
2.5
2
1.5
82-93
88-93
Standard Deviation of Output, percent
1.5
2
2.5
3
3.5
4
4.5
Standard Deviation of Inflation, percent

- Moving onto the frontier would entail a substantial change in policy responses. In particular, monetary policy would have to respond more vigorously to deviations of output from potential, regardless of its preferences over inflation and output deviations.

- Policy frontiers for alternative specifications—an MPS Phillips curve and a Taylor nominal contracting model—show that the qualitative feature of sharp trade-offs below a threshold for either inflation or output variability is preserved across models. This consistency was noted in Taylor (1992).

- The frontier for the Taylor (1980) model lies well inside the frontiers for the MPS or the real contracting model. The model implies that significantly smaller standard deviations are attainable with appropriate monetary policy. While it is costly to decrease the standard deviation of inflation below a threshhold, that threshhold is about 0.5 percent. Thus, from the perspective of the Taylor model, the actual outcomes for inflation and output variability must be viewed as much less of a success story.

- I suggest that a macro model must accurately reproduce the dynamics in the data, summarized by the rate of convergence to equilibrium (the size of the dominant root(s) in the system), to produce a plausible estimate of the policy frontier. I argue that the real contracting model more accurately replicates the dynamics in the data, particularly the persistence of inflation, and thus provides a more accurate depiction of the trade-offs facing policymakers. Conversely, the standard nominal contracting model, by understating the persistence of inflation, understates the unconditional variance of inflation, thus shifting the policy frontier in toward the origin.

APPENDIX

Computations

Each of the forward-looking models in this study can be cast in the format

$$\sum_{i=-\tau}^{0} H_i x_{t+i} + \sum_{i=1}^{\theta} H_i E_t(x_{t+i}) = \epsilon_t \tag{17}$$

where τ and θ are positive integers, x_t is a vector of variables, and the H_i are conformable square coefficient matrices. The expectation operator $E_t(\cdot)$ denotes mathematical expectation conditioned on the process history through period t,

$$E_t(x_{t+i}) = E(x_{t+i} \mid x_t, x_{t-1}, \ldots)$$

The random shock ϵ_t is independently and identically distributed $N(0, \Omega)$. The covariance matrix Ω is singular whenever equation 17 includes identities.

The generalized saddlepath procedure of Anderson and Moore (1985) is used to solve equation 17 for expectations of the future in terms of expectations of the present and the past. That procedure computes the vector autoregressive representation of the solution path,

$$E_t(x_{t+k}) = \sum_{i=-\tau}^{-1} B_i E_t(x_{t+k+i}), \qquad k > 0 \tag{18}$$

which may be used to substitute for the expectations in equation 17 to obtain the "observable structure" of the model,[21]

$$\sum_{i=-\tau}^{0} S_i x_{t+i} = \epsilon_t \tag{19}$$

The likelihood function of each subsample model is evaluated using the observable structure and the realization of the data. For the first subsample, the initial values of unobservable variables such as the ex ante real interest rate are set to initial guesses. For subsequent subsamples, the initial values of unobservable variables are set equal to the solution values from the previous subsample.

Several assumptions about the covariance structure of the subsample error terms are entertained. The simplest is that each subsample k has its own covariance matrix Ω_k. However, for the middle sample, this entails estimating 10 covariance matrix elements with 12 observations. To avoid this serious degrees of freedom problem, the assumption employed in the estimation reported in the paper is that the residual *correlations* are fixed across all subsamples, while the *variances* are allowed to vary across subsamples. Thus only the four innovation variances for the middle subsample must be estimated; the correlations (and implicitly, the covariances) are estimated with pooled information from all three samples. For each iteration of the likelihood evaluation, the full-sample average residual covariance matrix is estimated, and the implied residual correlation matrix is computed. Then subsample covariance matrices are computed as

$$\Omega_k = \sigma_k \Lambda \sigma_k \tag{20}$$

where σ_k is the subsample k diagonal matrix of standard deviations. The

[21] See Fuhrer and Moore (1993a) for more details of computation.

likelihood function for each subsample is computed as described in Fuhrer and Moore (1993a); the subsample likelihoods are added, and the total likelihood is maximized with a sequential quadratic programming algorithm using numerical derivatives (see Gill, Murray, and Wright 1981).

Computing the unconditional variances for the observable variables in the models requires a few more steps. Premultiplying the observable structure by $-S_0^{-1}$, yields the *reduced form* of the structural model,

$$x_t = \sum_{i=-\tau}^{-1} B_i x_{k+i} + B_0 \epsilon_t \tag{21}$$

The coefficient matrices $\{B_i : i = -\tau, \ldots, -1\}$ in equation 21 are identical to those in equation 18, while B_0 is simply S_0^{-1}.

The companion system of the reduced form is

$$\begin{bmatrix} x_t \\ x_{t-1} \\ \vdots \\ x_{t-\tau+1} \end{bmatrix} = \begin{bmatrix} B_{-1} & B_{-2} & \cdots & B_{-\tau} \\ I & & & 0 \\ & \ddots & & \vdots \\ & & I & 0 \end{bmatrix} \begin{bmatrix} x_{t-1} \\ x_{t-2} \\ \vdots \\ x_{t-\tau} \end{bmatrix} + \begin{bmatrix} B_0 \epsilon_t \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{22}$$

In a more compact notation, the companion system is

$$y_t = A y_{t-1} + \eta_t \tag{23}$$

where $y_t = [x_t, \ldots, x_{t-\tau+1}]'$, and $\eta_t = [B_0 \epsilon_t, 0, \ldots, 0]'$. Recursively substituting equation 23 into itself,

$$y_{t+k} = A^k y_t + \sum_{i=1}^{k} A^{k-i} \eta_{t+i} \tag{24}$$

Because η_t is uncorrelated over time, the covariance matrix of the

k-period-ahead forecasts of y_t is

$$V_t(y_{t+k}) = \sum_{i=0}^{k-1} A^i \Psi (A^i)' \tag{25}$$

where Ψ is the covariance matrix of η_t. In a stationary model, as k goes to infinity the conditional covariance matrix $V_t(y_{t+k})$ converges to Γ_0, the unconditional covariance matrix of y_t.

Note that the structural model includes the log price level, an I(1) variable. In the structural models successive terms of $V_t(y_{t+k})$ are computed until the conditional variances of the I(0) variables converge to constants. At this point the conditional variances of the I(1) variables are increasing at a linear rate. When the conditional variances of the stationary variables converge, the sum in equation 25 is treated as if it were Γ_0, the unconditional covariance matrix of y_t.

Given the estimate of the unconditional covariance and the state-space transition matrix A, the autocovariance function of y_t may be computed recursively as [22]

$$\Gamma_k = A\Gamma_{k-1}, \qquad k > 0 \tag{26}$$

[22]see the appendix in Fuhrer and Moore (1993a) for details.

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