

THE COEXISTENCE OF MONEY AND CREDIT AS MEANS OF PAYMENT*

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Abstract

This paper studies the choice of payment instruments in a simple model where both money and credit can be used as means of payment. We endogenize the acceptability of credit by allowing retailers to invest in a costly record-keeping technology. Our framework captures the two-sided market interaction between consumers and retailers, leading to strategic complementarities that can generate multiple steady-state equilibria. In addition, limited commitment makes debt contracts self-enforcing and yields an endogenous upper bound on credit use. Our model can explain why the demand for credit declines as inflation falls, and how hold-up problems in technological adoption can prevent retailers from accepting credit as consumers continue to coordinate on cash usage. We show that when money and credit coexist, equilibrium is generically inefficient and changes to the debt limit are not neutral. We also discuss the extent to which our model can reconcile some key patterns in the use of cash and credit in retail transactions.

Keywords: coexistence of money and credit, costly record-keeping, endogenous credit

JEL Classification Codes: D82, D83, E40, E50

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1 Introduction

Technological improvements in electronic record-keeping have made credit cards as ubiquitous as cash as means of payment in many OECD countries. Indeed much of the U.S. economy runs on debt, with vast increases in both the usage and availability of unsecured debt ever since the development of credit cards featuring revolving lines of credit.¹ According to the U.S. Survey of Consumer Finances (2007), nearly three-quarters of all U.S. households owned at least one credit card with nearly half of all households carrying outstanding balances on these accounts. Moreover, approximately 27% of U.S. households report simultaneously revolving credit card debt and holding sizeable amounts of low-return liquid assets such as cash (Telyukova (2013)). This suggests that while consumers are increasingly relying on credit cards to facilitate transactions, they are still not completely abandoning cash. Indeed there is a substantial share of transactions such as mortgage and rent payments that cannot be paid by credit card, so the choice of payment instruments by consumers also depends critically on what others accept.

As consumers change the way they pay and businesses change the way they accept payments, it is increasingly important to understand how consumer demand affects merchant behavior and vice versa. In fact, the payment system is a classic example of a two-sided market where both consumers and firms must make choices that affect one other (Rysman (2009), BIS (2012)). This dynamic often generates complementarities and network externalities, which is a key characteristic of the retail payment market.² Moreover, the recent trends in retail payments raise many interesting and challenging questions for central banks and policymakers. In particular, how does the availability of alternative means of payment, such as credit cards, affect the role of money? And if both money and credit can be used, how does policy and inflation affect the money-credit margin?

We investigate the possible substitution away from cash to credit cards using a simple model where money and credit can coexist as means of payment. To capture the two-sided nature of actual payment systems, our model focuses on the market interaction between consumers (buyers, or borrowers) and retailers (sellers, or lenders). A vital distinction between monetary and credit

¹Unsecured credit refers to loans not tied to other assets or secured by the pledge of collateral, such as credit card loans. Consumer debt outstanding, which excludes mortgage debt, totaled over \$2.1 trillion at the end of 2005, which amounts to an average debt of \$9,710 for each U.S. adult. The Federal Reserve (2005) reports that credit card loans account for roughly half of all unsecured debt in the United States. Between 1983 and 1998, outstanding balances on credit cards across U.S. households more than tripled on average. More recently, the number of payments made by general-purpose credit cards rose from 15.2 billion to 19.0 billion between 2003 and 2006 in the U.S. (Gerdes (2008)).

²Network externalities exist when the value of a good or service to a potential user increases with the number of other users using the same product. Credit cards are a classic example of a network good, where its adoption and use can be below the socially optimal level because consumers or firms do not internalize the benefit of their own use on others' use. For evidence and a discussion of the empirical issues, see Gowrisankaran and Stavins (2004) and Chakravorti (2010).

trades is that the former is *quid pro quo* and settled on the spot while the latter involves delayed settlement.³ While many economies now feature the widespread adoption of both money and credit as means of payment, getting money and credit to coexist in theory is a much more delicate issue. Indeed, across a wide class of models, there is a dichotomy between monetary and credit trades, a key insight dating back to at least Kocherlakota (1998): so long as credit is feasible, there is no social role for money, and if money is valued, then credit cannot be sustained.

For credit to have a role, we introduce a costly record-keeping technology that allows transactions to be recorded. A retailer that invests in this technology will thus be able to accept an IOU from a consumer.⁴ Due to limited commitment and enforcement however, lenders cannot force borrowers to repay their debts. In order to motivate voluntary debt repayment, we assume that default by the borrower triggers a punishment that banishes agents from all future credit transactions. In that case, a defaulter can only trade with money. Consequently, debt contracts must be self-enforcing and the possibility of strategic default generates an endogenous upper-bound on credit use.⁵

A key insight of our theory is that both money and credit can be socially useful since some sellers (endogenously) accept both cash and credit while others only take cash.⁶ At the same time, consumers make their portfolio decisions taking into account the rate of return on money and the fraction of the economy with access to credit. While credit allows retailers to sell to illiquid consumers or to those paying with future income, money can still be valued since it allows consumers to self-insure against the possibility of not being able to use credit in some transactions. This need for liquidity therefore explains why individuals would ever use both, an insight of the model that we think is fundamental in capturing the deep reason behind the coexistence of money and credit.⁷

³This distinction separates debit cards, which are “pay now” cards, from credit cards, or “pay later” cards. For debit cards, funds are typically debited from the cardholder’s account within a day or two of purchase, while credit cards allow consumers to access credit lines at their bank which are repaid at a future date.

⁴Garcia-Swartz, Hahn, and Layne-Farrar (2006) find that the merchant’s cost of a typical credit transaction in the U.S. is about seven times higher than their costs for accepting cash. This higher cost of accepting credit is borne by the merchant in the forms of merchant fees that typically are not paid for explicitly by buyers. In practice, the buyer often pays the same purchase price using cash or credit, an outcome supported by no-surcharge regulations that prohibit merchants from passing through merchant fees to customers who prefer to pay by credit card.

⁵This is in the spirit of Kehoe and Levine (1993) and Alvarez and Jermann (2000) where defaulters are banished from future credit transactions, but not from spot trades. Kocherlakota and Wallace (1998) consider an alternative formalization where default triggers permanent autarky. In our framework, permanent autarky as punishment would help relax credit constraints since it increases the penalty for default.

⁶Our analysis differs from the usual cash-good/credit-good models due to the presence of limited commitment and limited enforcement. In e.g. Lacker and Schreft (1996), the absence of these frictions means that endogenous borrowing limits do not arise.

⁷Using data from the Survey of Consumer Finances, Telyukova (2013) finds that the demand for precautionary liquidity can account for nearly 44% to 56% of the “credit card debt puzzle,” the fact that households simultaneously

In addition, inflation has two effects when enforcement is limited: a higher inflation rate both lowers the rate of return on money and makes default more costly. This relaxes the credit constraint and induces agents to shift from money to credit to finance their consumption. Consequently, consumers decrease their borrowing as inflation falls. When the monetary authority implements the Friedman rule, deflation completely crowds out credit and there is a flight to liquidity where all borrowing and lending ceases to exist. In that case, efficient monetary policy drives out credit.⁸ When borrowers are not patient enough, inflation can have a hump-shaped effect on welfare and there is a strictly positive inflation rate that maximizes welfare in a money and credit economy. While equilibrium is inefficient when both money and credit are used, the first-best allocation can still be achieved in a pure credit economy, provided that agents are patient enough. However equilibrium is not socially efficient since sellers must incur the real cost of technological adoption.

The channel through which monetary policy affects macroeconomic outcomes is through buyers' choice of portfolio holdings, sellers' decision to invest in the record-keeping technology, and the endogenously determined credit constraint. If sellers must invest ex-ante in a costly technology to record credit transactions, there are strategic complementarities between the seller's decision to invest and the buyer's ability to repay. When more sellers accept credit, the gain for buyers from using and redeeming credit increases, which relaxes the credit constraint. At the same time, an increase in the buyer's ability to repay raises the incentive to invest in the record-keeping technology and hence the fraction of credit trades. This complementarity leads to feedback effects that can generate multiple equilibria, including outcomes where both money and credit are used.

Moreover, this channel mimics the mechanism behind two-sided markets in actual payment systems as described by McAndrews and Wang (1998): merchants want to accept credit cards that have many cardholders, and cardholders want cards that are accepted at many establishments. Just as in our model, the payment network benefits the merchant and the consumer jointly, leading to similar complementarities and network externalities highlighted in the industrial organization literature. At the same time, consumers may still coordinate on using cash due to a hold-up problem in technological adoption. Since retailers do not receive the full surplus associated with technological adoption, they fail to internalize the total benefit of accepting credit. The choice of payment instruments will therefore depend on fundamentals, as well as history and social conventions.

revolve credit card debt while holding money in the bank. The role of money in providing self-insurance against heterogeneity in monitoring also appears in Sanches and Williamson (2010) and Gomis-Porqueras and Sanches (2013).

⁸In our model, the presence of multiple steady-state equilibria where either money, credit, or both are used makes the choice of optimal policy difficult to analyze in full generality. If the monetary authority must choose an inflation rate before it knows which equilibrium will obtain, policy will affect the equilibrium selection process. In models where the fraction of credit trades is fixed, limited commitment and imperfect enforcement can also lead to a positive optimal inflation rate; see e.g. Berentsen, Camera, and Waller (2007), Antinolfi, Azariadis, and Bullard (2009), and Gomis-Porqueras and Sanches (2013).

This paper proceeds as follows. Section 1.1 reviews the related literature. Section 2 describes the environment with limited enforcement, and Section 3 defines equilibrium where an exogenous fraction of sellers accept credit. Section 4 determines the endogenous debt limit and characterizes equilibrium. Section 5 endogenizes the fraction of credit trades and discusses multiplicity. Section 6 investigates the effect of inflation on social welfare. Finally, Section 7 concludes.

1.1 Related Literature

Within modern monetary theory, there is a strong tradition of studying the coexistence of money and credit, starting with Shi (1996) and Kocherlakota and Wallace (1998). More recently, there are several models featuring divisible money and centralized credit markets using the Lagos and Wright (2005) environment.⁹ Telyukova and Wright (2008) rationalize the credit card debt puzzle in a model where there is perfect enforcement and segmentation of monetary and credit trades, in which case monetary policy has no effect on credit use. Sanches and Williamson (2010) get money and credit to coexist in a model with imperfect memory, limited commitment, and theft, while Bethune, Rocheteau, and Rupert (2013) develop a model with credit and liquid assets to examine the relationship between unsecured debt and unemployment. However in all these approaches, only an exogenous subset of agents can use credit while the choice of using credit is endogenous in this paper.¹⁰

Our model of endogenous record-keeping is based on the model of money and costly credit in Nosal and Rocheteau (2011), though a key novelty is that we derive an endogenous debt limit under limited commitment instead of assuming that loan repayments are perfectly enforced. Dong (2011) also introduces costly record-keeping, but focuses on the buyer's choice of payments used in bilateral meetings.

In contrast with previous studies, we show that money is not crowded out one-for-one when credit is also used. In particular, Gu, Mattesini, and Wright (2013) show that changes to the debt limit are neutral in an environment with complete access to record-keeping. In our model, as credit limits change, the fraction of sellers accepting credit also changes, which affects the value of

⁹In another approach, Berentsen, Camera, and Waller (2007) considers record-keeping of financial transactions and models credit as bank loans or deposits in the form of money. However money is the only means of payment since goods transactions remain private information for banks. See also Chiu, Dong, and Shao (2012), which compares the welfare effects of inflation in a nominal versus real loan economy.

¹⁰In a similar vein, Schreft (1992) and Dotsey and Ireland (1995) introduce costs paid to financial intermediaries to endogenize the composition of trades that use money or credit, and Freeman and Kydland (2000) feature a fixed record-keeping cost for transactions made with demand deposits. Our formalization is also reminiscent of Townsend (1989) and Williamson (1987)'s costly state verification assumption. More recently, a related assumption is also made in Lester, Postlewaite, and Wright (2012) where sellers have to incur a fixed cost in order to authenticate and hence accept an asset in trade.

money and the net benefit of using credit. As a result, money is not crowded out one-for-one when credit is also used. Another important difference is that our model assumes that a defaulter is permanently excluded from credit trades but can still use money. In that case, increases in the cost of holding money, which affects the existence of monetary equilibrium, can help discipline credit market behavior by raising the cost of default and hence relaxing debt limits.

Another related paper is Gomis-Porqueras and Sanches (2013), which discusses the role of money and credit in a model with anonymity, limited commitment, and imperfect record-keeping. A key difference in the set-up is that they adopt a different pricing mechanism by assuming a buyer-take-all bargaining solution. While this difference may seem innocuous, our assumption of proportional bargaining allows us to take the analysis further in two important ways.¹¹ By giving the seller some bargaining power, proportional bargaining allows us to endogenize the fraction of sellers that accept credit by allowing them to invest in costly record keeping. This also allows us to discuss hold-up problems on the seller's side which will lead to complementarities with the buyer's borrowing limit. This generates interesting multiplicities and network effects that the previous study cannot discuss.

This paper also relates with a growing strand in the industrial organization literature that examines the costs and benefits of credit cards to network participants, including recent work by Wright (2003, 2011) and Rochet and Tirole (2002, 2003, 2006). Chakravorti (2003) provides a theoretical survey of the industrial organization approach to credit card networks, and Rysman (2009) gives an overview of the economics of two-sided markets. However, this literature abstracts from a critical distinction between monetary and credit transactions by ignoring the actual borrowing component of credit transactions. An exception however is Chakravorti and To (2007), which develops a theory of credit cards in a two-period model with delayed settlement. However since money is not modeled, the model cannot examine issues of coexistence and substitutability between cash and credit, which is a key contribution of the present paper.

2 Model

Time is discrete and continues forever. The economy consists of a continuum $[0, 2]$ of infinitely lived agents, evenly divided between *buyers* (or consumers) and *sellers* (or retailers). Each period is divided into two sub-periods where economic activity will differ. In the first sub-period, agents

¹¹More generally, proportional bargaining guarantees that trade is pairwise Pareto efficient and has several desirable features that cannot be guaranteed with Nash bargaining, as discussed in Aruoba, Rocheteau, and Waller (2007). First, it guarantees the concavity of agents' value functions. Second, the proportional solution is monotonic and hence does not suffer from a shortcoming of Nash bargaining that an agent can end up with a lower individual surplus even if the size of the total surplus increases.

meet pairwise and at random in a decentralized market, called the *DM*. Sellers can produce output, $q \in \mathbb{R}_+$, but do not want to consume, while buyers want to consume but cannot produce. Agents' identities as buyers or sellers are permanent, exogenous, and determined at the beginning of the DM. In the second sub-period, trade occurs in a frictionless centralized market, called the *CM*, where all agents can consume a numéraire good, $x \in \mathbb{R}_+$, by supplying labor, y , one-for-one using a linear technology.

Instantaneous utility functions for buyers, (U^b), and sellers, (U^s), are assumed to be separable between sub-periods and linear in the CM:

$$\begin{aligned} U^b(q, x, y) &= u(q) + x - y, \\ U^s(q, x, y) &= -c(q) + x - y. \end{aligned}$$

Functional forms for utility and cost functions in the DM, $u(q)$ and $c(q)$ respectively, are assumed to be C^2 with $u' > 0$, $u'' < 0$, $c' > 0$, $c'' > 0$, $u(0) = c(0) = c'(0) = 0$, and $u'(0) = \infty$. Also, let $q^* \equiv \{q : u'(q^*) = c'(q^*)\}$. All goods are perishable and agents discount the future between periods with a discount factor $\beta = \frac{1}{1+r} \in (0, 1)$.

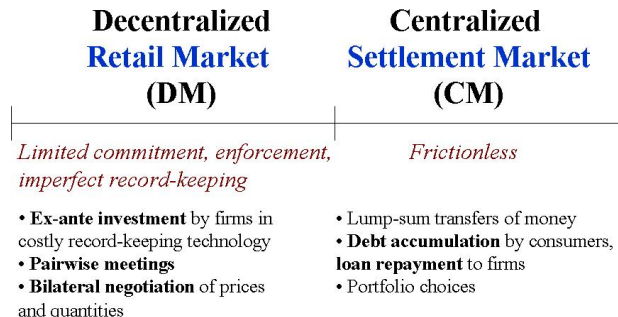
The only asset in this economy is fiat money, which is perfectly divisible, storable, and recognizable. Money $m \in \mathbb{R}_+$ is valued at ϕ , the price of money in terms of numéraire. Its aggregate stock in the economy, M_t , can grow or shrink each period at a constant gross rate $\gamma \equiv \frac{M_{t+1}}{M_t}$. Changes in the money supply are implemented through lump-sum transfers or taxes in the CM to buyers. In the latter case, we assume that the government has enough enforcement in the CM so that agents will repay the lump-sum tax.¹²

To purchase goods in the DM, both monetary and credit transactions are feasible due to the availability of a record-keeping technology that can record agent's transactions and enforce repayment. However this technology is only available to a fraction $\Lambda \in [0, 1]$ of sellers, while the remaining $1 - \Lambda$ sellers can only accept money.¹³ For example, investment in this technology is infinitely costly for a fraction $1 - \Lambda$ of firms while costless for the remaining Λ firms. In Section 5, we

¹²While the government can never observe agents' real balances, it has the authority to impose arbitrarily harsh penalties on agents who do not pay taxes when $\gamma < 1$. Alternatively, Andolfatto (2008, 2013) considers an environment where the government's enforcement power is limited and the payment of lump-sum taxes is voluntary. Penalty for failing to pay taxes in the CM is permanent exclusion from the DM. Along these lines, Appendix B determines the size of the tax obligation that individuals are willing to honor voluntarily, in which case the Friedman rule is infeasible since it requires taxation and individuals may choose to renege on taxes if it is too high.

¹³The Bank of Canada's 2006 national survey of merchants on their preferred means of payment for point-of-sale transactions reports that nearly all merchants accept cash while 92% accept both cash and credit cards (Arango and Taylor (2008)). The study also finds that record-keeping and other technological costs associated with accepting credit are incurred by the seller. For example, the ad valorem fee on credit card transactions is incurred by merchants, which in practice includes processing fees and interchange fees.

Figure 1: **Timing of Representative Period**



endogenize Λ by considering an alternative cost function where individual sellers have heterogenous costs of investing.

We assume that contracts written in the DM can be repaid in the subsequent CM. Buyers can issue $b \in \mathbb{R}_+$ units of one-period IOUs that we normalize to be worth one unit of the numéraire good.¹⁴ Since agents lack commitment, potential borrowers must be punished if they do not deliver on their promise to repay. We assume that any default is publicly recorded by the record-keeping technology and triggers punishment that leads to permanent exclusion from the credit system. In that case, a borrower who defaults can only use money for all future transactions.¹⁵

The timing of events in a typical period is summarized in Figure 1. At the beginning of the DM, a buyer matches with a seller with probability σ , where the buyer has $b \in \mathbb{R}_+$ units of IOUs and $m \in \mathbb{R}_+$ units of money, or equivalently, $z \equiv \phi m \in \mathbb{R}_+$ units of real balances. Terms of trade

¹⁴One interpretation of our model is that sellers with access to the record-keeping technology can make loans directly to the buyer without interacting with an intermediary, such as a bank or credit card issuer. Equivalently, the seller and credit card issuer is modeled as a consolidated entity. It would be equivalent to assume that the buyer draws on a line of credit from a third-party credit card issuer or intermediary in the CM, who then pays the seller for the purchase if the seller has made the ex-ante investment to accept credit. While beyond the scope of the present paper, having a more active role for intermediaries would potentially allow for a more fruitful analysis of bank competition and pricing issues.

¹⁵The record-keeping technology detects default with probability one. Introducing imperfect monitoring where default is only detected probabilistically would all else equal decrease the cost of default and hence tighten credit limits. See e.g. the analysis in Gu, Mattesini, and Wright (2013). In addition, while we assume that a defaulter is excluded from using credit but can still use money, one can also assume that punishment for default is permanent autarky. In Appendix B, we derive the debt limit assuming that punishment for default is permanent autarky and discuss the implications of this assumption.

are determined using a proportional bargaining rule. In the CM, buyers produce the numéraire good, redeem their loan, and acquire money, while sellers can purchase the numéraire with money and can get their loan repaid.

3 Equilibrium

The model can be solved in four steps. First, we characterize properties of agents' value functions in the CM. Using these properties, we then determine the terms of trade in the DM. Third, we determine the buyer's choice of asset holdings, and in Section 4 we characterize equilibrium with the endogenous debt limit. Then in Section 5, we determine Λ endogenously by allowing sellers to invest in the costly record-keeping technology. We focus on stationary equilibria where real balances are constant over time.

3.1 Centralized Market

In the beginning of the CM, agents consume the numéraire good x , supply labor y , and readjust their portfolios. Let $W^b(z, -b)$ denote the value function of a buyer who holds z units of real balances and has issued b units of IOUs in the previous DM. Variables with a prime denote next period's choices. The buyer's maximization problem at the beginning of the CM, $W^b(z, -b)$, is

$$W^b(z, -b) = \max_{x, y, z' \geq 0} \left\{ x - y + \beta V^b(z') \right\} \quad (1)$$

$$\text{s.t. } x + b + \phi m' = y + z + T, \quad (2)$$

$$z' = \phi' m', \quad (3)$$

where V^b is the buyer's continuation value in the next DM and $T \equiv (\gamma - 1)\phi M$ is the lump-sum transfer from the government (in units of numéraire). According to (2), the buyer finances his net consumption of numéraire ($x - y$), the repayment of his IOUs (b), and his following period real balances ($\phi m'$) with his current real balances (z) and the lump-sum transfer (T). Substituting $m' = z'/\phi'$ from (3) into (2), and then substituting $x - y$ from (2) into (1) yields

$$W^b(z, -b) = z - b + T + \max_{z' \geq 0} \left\{ -\gamma z' + \beta V^b(z') \right\}. \quad (4)$$

The buyer's lifetime utility in the CM is the sum of his real balances net of any IOUs to be repaid, the lump-sum transfer from the government, and his continuation value at the beginning of the next DM net of the investment in real balances. The gross rate of return of money is $\frac{\phi_{t+1}}{\phi_t} = \frac{M_t}{M_{t+1}} = \gamma^{-1}$.

Hence in order to hold z' units of real balances in the following period, the buyer must acquire $\gamma z'$ units of real balances in the current period.

Notice that $W^b(z, -b)$ is linear in the buyer's current portfolio: $W^b(z, -b) = z - b + W^b(0, 0)$. In addition, the choice of real balances next period is independent of current real balances. Similarly, the value function $W^s(z, b)$ of a seller who holds z units of real balances and b units of IOUs can be written:

$$W^s(z, b) = z + b + \beta V^s(0),$$

where $V^s(0)$ is the value function of a seller at the beginning of the following DM since they have no incentive to accumulate real balances in the DM.

3.2 Terms of Trade

We now turn to the terms of trade in the DM. Agents meet bilaterally, and bargain over the units of real balances or IOUs to be exchanged for goods. We adopt Kalai (1977)'s proportional bargaining solution where the buyer proposes a contract (q, b, d) , where q is the transfer of output from the seller to buyer, d is the transfer of real balances from the buyer to seller, and b is the amount borrowed by the buyer, such that he receives a constant share $\theta \in (0, 1)$ of the match surplus, while the seller gets the remaining share, $1 - \theta > 0$.¹⁶

We will show that the terms of trade depend only on buyers' portfolios and what sellers accept. First consider a match where the seller accepts credit. To apply the pricing mechanism, notice that the surplus of a buyer who gets q for payment $d + b$ to the seller is $u(q) + W^b(z - d - b) - W^b(z) = u(q) - d - b$, by the linearity of W^b . Similarly, the surplus of a seller is $-c(q) + d + b$. The bargaining problem then becomes

$$(q, d, b) = \arg \max_{q, d, b} \{u(q) - d - b\} \quad (5)$$

$$\text{s.t. } -c(q) + d + b = \frac{1 - \theta}{\theta} [u(q) - d - b] \quad (6)$$

$$d \leq z \quad (7)$$

$$b \leq \bar{b}. \quad (8)$$

According to (5) – (8), the buyer's offer maximizes his trade surplus such that (i) the seller's payoff is a constant share $\frac{1 - \theta}{\theta}$ of the buyer's payoff, (ii) the buyer cannot transfer more money than he has, and (iii) the buyer cannot borrow more than he can repay. Condition (7) is a feasibility constraint on the amount the buyer can transfer to the seller, while condition (8) is the buyer's

¹⁶There are also strategic foundations for the proportional bargaining solution. In Dutta (2012), Kalai (1977)'s solution emerges as a unique equilibrium outcome in a limiting case of a Nash demand game.

incentive constraint that motivates voluntary debt repayment. The threshold \bar{b} is an equilibrium object and represents the endogenous borrowing limit faced by the buyer, which is taken as given in the bargaining problem but is determined endogenously in the next section.

Combining the feasibility constraint (7) and the buyer's incentive constraint (8) results in the payment constraint

$$d + b \leq z + \bar{b}, \quad (9)$$

which says the total payment to the seller, $d + b$, cannot exceed the buyer's payment capacity, which is $z + \bar{b}$ when the seller accepts credit. The solution to the bargaining problem will depend on whether the payment constraint, (9), binds. If (9) does not bind, then the buyer will have sufficient wealth to purchase the first-best level of output, q^* . In that case, payment to the seller will be exactly

$$d + b = (1 - \theta)u(q^*) + \theta c(q^*).$$

If (9) binds, the buyer does not have enough payment capacity and will borrow up to their credit limit and pay the rest with any cash on hand:

$$z + \bar{b} = (1 - \theta)u(q^c) + \theta c(q^c), \quad (10)$$

where $q^c \equiv q(z + \bar{b}) < q^*$.

If the seller does not have access to record-keeping, credit cannot be used. In that case, the bargaining problem can be described by (5) – (7) with $b = \bar{b} = 0$. If $z \geq z^* \equiv (1 - \theta)u(q^*) + \theta c(q^*)$, the buyer is not constrained and borrows enough to obtain q^* . Otherwise, the buyer just hands over his real balances,

$$z = (1 - \theta)u(q) + \theta c(q), \quad (11)$$

where $q \equiv q(z) < q^*$.

3.3 Decentralized Market

We next characterize agents' value functions in the DM. After simplification, the expected discounted utility of a buyer holding z units of real balances at the beginning of the period is:

$$V^b(z) = \sigma(1 - \Lambda)\theta[u(q) - c(q)] + \sigma\Lambda\theta[u(q^c) - c(q^c)] + z + W^b(0, 0), \quad (12)$$

where we have used the bargaining solution and the fact that the buyer will never accumulate more balances than he would spend in the DM. According to (12), a buyer in the DM is randomly matched with a seller who does not have access to record-keeping with probability $\sigma(1 - \Lambda)$, receives

θ of the match surplus, $u(q) - c(q)$, and can only pay with money. With probability $\sigma\Lambda$, a buyer matches with a seller with access to record-keeping, in which case he gets θ of $u(q^c) - c(q^c)$ and can pay with both money and credit. The last two terms result from the linearity of W^b and is the value of proceeding to the CM with one's portfolio intact.

3.4 Optimal Portfolio Choice

Next, we determine the buyer's choice of real balances. Given the linearity of W^b , the buyer's bargaining problem (5) – (7), and substituting $V^b(z)$ from (12) into (4), the buyer's choice of real balances must satisfy:

$$\max_{z \geq 0} \{-iz + \sigma(1 - \Lambda)\theta[u(q) - c(q)] + \sigma\Lambda\theta[u(q^c) - c(q^c)]\}, \quad (13)$$

where $i = \frac{\gamma - \beta}{\beta}$ is the nominal interest rate on an illiquid bond and represents the cost of holding real balances. As a result, the buyer chooses his real balances z in order to maximize his expected surplus in the DM net of the cost of holding money, i .

Since the objective function (13) is continuous and maximizes over a compact set, a solution exists. We further assume $u(q^*) - z(q^*) > 0$ in order to guarantee the existence of a monetary equilibrium. In the Appendix, we show that (13) is concave. The first-order condition for problem (13) when $z \geq 0$ is

$$-i + \sigma(1 - \Lambda)\frac{\theta[u'(q) - c'(q)]}{\theta c'(q) + (1 - \theta)u'(q)} + \sigma\Lambda\frac{\theta[u'(q^c) - c'(q^c)]}{\theta c'(q^c) + (1 - \theta)u'(q^c)} \leq 0, \quad (14)$$

where we have used that under proportional bargaining,

$$\frac{dq}{dz} = \frac{1}{z'(q)} = \frac{1}{\theta c'(q) + (1 - \theta)u'(q)},$$

$$\frac{dq^c}{dz} = \frac{1}{z'(q^c)} = \frac{1}{\theta c'(q^c) + (1 - \theta)u'(q^c)}.$$

Condition (14) is satisfied with equality if $z > 0$. Notice that an increase in the debt limit, \bar{b} , will reduce z , but from (14), is not completely offset by a decline in the price of money, ϕ , due to the $(1 - \Lambda)$ of trades that also take place with money but not credit. To see this, differentiate (14) to obtain

$$\frac{dz}{d\bar{b}} = - \left[1 + \frac{(1 - \Lambda)S''(z)}{\Lambda S''(z + \bar{b})} \right]^{-1} \in (-1, 0).$$

As a result, changes in the debt limit do not imply a one-for-one change in real balances when

$\Lambda \in (0, 1)$. Notice however that when $\Lambda = 1$, $\frac{dz}{db} = -1$, in which case any increase in the debt limit will crowd out real balances one-for-one as in Gu, Mattesini, and Wright (2013). We discuss in detail this special case of perfect record-keeping in Section 4.3.

Finally, notice that under perfect enforcement, buyers are never constrained by $b \leq \bar{b}$ and can borrow as much as they want to finance consumption of the first-best, q^* . When $z > 0$ under perfect enforcement, the third term on the left-hand-side of (14) equals to zero since at q^* , $u'(q^*) = c'(q^*)$. In that case, the right-hand-side is increasing with Λ , meaning that an increase in the fraction of credit trades Λ decreases $q(z)$ and hence real balances z .

4 Limited Enforcement and Credit Limits

When the government's ability to force repayment is limited, borrowers may have an incentive to renege on their debts. In order to support trade in a credit economy, we assume that punishment for default entails permanent exclusion from the credit system. In that case, debt contracts must be self-enforcing, and a borrower who defaults can no longer use credit and can only use money for all future transactions.

The borrowing limit, \bar{b} , is determined in order to satisfy the buyer's incentive constraint to voluntarily repay his debt in the CM:

$$W^b(z, -b) \geq \widetilde{W}^b(z),$$

where $W^b(z, -b)$ is the value function of a buyer who repays his debt at the beginning of the CM, and $\widetilde{W}^b(z)$ is the value function of a buyer who defaults. By the linearity of W^b , the value function of a buyer who repays his debt in the CM is

$$W^b(z, -b) = z - b + W^b(0, 0).$$

On the other hand, the value function of a buyer who defaults, $\widetilde{W}^b(z)$, must satisfy

$$\begin{aligned} \widetilde{W}^b(z) &= z + T + \max_{z' \geq 0} \left\{ -\gamma z' + \beta \widetilde{V}^b(z') \right\} \\ &= z + \widetilde{W}^b(0), \end{aligned}$$

where $\tilde{z} \geq 0$ is the choice of real balances for a buyer without access to credit such that

$$-i + \sigma \frac{\theta[u'(\tilde{q}) - c'(\tilde{q})]}{\theta c'(\tilde{q}) + (1 - \theta)u'(\tilde{q})} \leq 0, \quad (15)$$

and with equality if $\tilde{z} > 0$. In addition, \tilde{q} solves $\tilde{z} = (1 - \theta)u(\tilde{q}) + \theta c(\tilde{q})$ if $\tilde{z} < (1 - \theta)c(q^*) + \theta u(q^*)$ and $\tilde{z} = (1 - \theta)c(q^*) + \theta u(q^*)$ if $\tilde{z} \geq (1 - \theta)c(q^*) + \theta u(q^*)$. By the linearity of $W^b(z, -b)$ and $\widetilde{W}^b(z)$, a buyer will repay his debt if

$$b \leq \bar{b} \equiv W^b(0, 0) - \widetilde{W}^b(0),$$

where \bar{b} is the endogenous debt limit. In other words, the amount borrowed can be no larger than the cost of defaulting, which is the difference between the lifetime utility of a buyer with access to credit and the lifetime utility of a buyer permanently excluded from using credit.

Lemma 1. *The equilibrium debt limit, \bar{b} , is a solution to*

$$r\bar{b} = \max_{z \geq 0} \{-iz + \sigma\theta [(1 - \Lambda)S(z) + \Lambda S(z + \bar{b})]\} - \max_{\tilde{z} \geq 0} \{-i\tilde{z} + \sigma\theta S(\tilde{z})\} \equiv \Omega(\bar{b}), \quad (16)$$

where $r = \frac{1-\beta}{\beta}$, and $S(\cdot) \equiv u[q(\cdot)] - c[q(\cdot)]$.

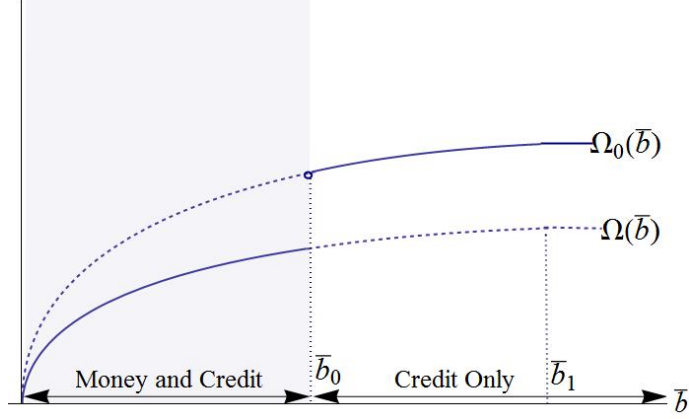
The left-side of (16), $r\bar{b}$, represents the return from borrowing a loan of size \bar{b} . The right-side, $\Omega(\bar{b})$, is the flow cost of defaulting, which equals the surplus from not having access to credit. To characterize equilibrium under limited enforcement, we start by establishing some key properties of the function $\Omega(\bar{b})$.

Lemma 2. *The function $\Omega(\bar{b}) \equiv \max_{z \geq 0} \{-iz + \sigma\theta [(1 - \Lambda)S(z) + \Lambda S(z + \bar{b})]\} - \max_{\tilde{z} \geq 0} \{-i\tilde{z} + \sigma\theta S(\tilde{z})\}$ has the following key properties:*

1. $\Omega(0) = 0$,
2. $\Omega'(0) = i\Lambda \geq 0$,
3. $\Omega'(\bar{b}) \begin{cases} > 0 & \text{when } \bar{b} < (1 - \theta)u(q^*) + \theta c(q^*) \\ = 0 & \text{when } \bar{b} \geq (1 - \theta)u(q^*) + \theta c(q^*), \end{cases}$
4. $\Omega(\bar{b})$ is a concave function if $\bar{b} < (1 - \theta)u(q^*) + \theta c(q^*)$, and is linear if $\bar{b} \geq (1 - \theta)u(q^*) + \theta c(q^*)$,
5. When $\Lambda \in (0, 1)$, $\Omega(\bar{b})$ is continuous for all $z > 0$ and becomes discontinuous at \bar{b}_0 , above which $z = 0$.
6. When $\Lambda = 1$, $\Omega(\bar{b})$ is continuous for all $z \geq 0$.

To describe how the debt limit affects the value of money and DM output, we first define two critical values for the debt limit. For money to be valued, the size of the loan must be no greater than the buyer's payment capacity: $z = (1 - \theta)u(q^c) + \theta c(q^c) - \bar{b} > 0$ if and only if

Figure 2: Flow Cost of Default in Monetary and Credit Trades



$\bar{b} < (1 - \theta)u(q^c) + \theta c(q^c)$. Hence the value $\bar{b}_0 \equiv (1 - \theta)u(q^c) + \theta c(q^c)$ is the threshold for the debt limit, above which money is no longer valued and solves

$$r\bar{b}_0 = \sigma\theta\Lambda S(\bar{b}_0).$$

The value $\bar{b}_1 \equiv (1 - \theta)u(q^*) + \theta c(q^*)$ is the threshold for the debt limit, above which the buyer can borrow enough to finance consumption of the first-best, q^* . For all $\bar{b} \geq \bar{b}_1$, $r\bar{b} = \sigma\theta\Lambda S(q^*)$.

Lemma 3. *Equilibrium with limited enforcement will be such that*

1. If $\bar{b} \in [0, \bar{b}_0)$, then $z > 0$ and $q(z + \bar{b}) < q^*$,
2. If $\bar{b} \in [\bar{b}_0, \bar{b}_1)$, then $z = 0$ and $q(\bar{b}) < q^*$,
3. If $\bar{b} \in [\bar{b}_1, \infty)$, then $z = 0$ and $q(\bar{b}) = q^*$.

At $\bar{b} = 0$, credit is not used and the buyer can only use money. Since $\Omega(0) = 0$, an equilibrium without credit always exists. The function $\Omega(\bar{b})$ represents the flow cost of not having access to credit and is increasing in the size of the loan, \bar{b} .

Figure 2 shows that there are three qualitatively different regions. When $\bar{b} \in [0, \bar{b}_0)$, money is valued and the right side of (16) is continuous, increasing, and strictly concave. In this range, the buyer can use both money and credit, but cannot borrow enough to obtain the first-best, q^* . Above \bar{b}_0 , money is no longer valued, in which case the right side of (16) is given by $\Omega_0(\bar{b}) \equiv \sigma\theta\Lambda S(\bar{b})$ for $\bar{b} \in [\bar{b}_0, \bar{b}_1)$. Here, only credit is used and the buyer still cannot borrow enough to finance q^* . Finally when $[\bar{b}_1, \infty)$, the buyer is no longer constrained by his wealth and can borrow enough

to obtain the first-best, q^* . In that case, the flow cost of default becomes constant and equal to $\Omega_0^* \equiv \sigma\theta\Lambda S^*$, where $S^* \equiv u(q^*) - c(q^*)$. Hence in Figure 2, money is valued only in the shaded region where $\bar{b} < \bar{b}_0$, while equilibrium is non-monetary for all $\bar{b} \geq \bar{b}_0$.

Furthermore since $S(\cdot)$ is a concave function, the slope of $\Omega(\bar{b})$ at $\bar{b} = 0$ which is $\sigma\theta\Lambda S'(z)$, is strictly less than the slope of $\Omega_0(\bar{b})$ at $\bar{b} = 0$ which is $\sigma\theta\Lambda S'(0) = \sigma\Lambda\frac{\theta}{1-\theta}$. Consequently, the cost of default when money is valued, $\Omega(\bar{b})$, is less than the cost of defaulting with no money, $\Omega_0(\bar{b})$. In addition, when $\Lambda \in (0, 1)$, both $\Omega(\bar{b})$ and $\Omega_0(\bar{b})$ are strictly concave.¹⁷ This is shown in Figure 2 where $\Omega(\bar{b})$ lies strictly below $\Omega_0(\bar{b})$. Intuitively, the off-equilibrium-path punishment for default becomes harsher when money is no longer valued since the marginal cost of not having access to credit is larger under permanent autarky.

Definition 1. *Given Λ , a steady-state equilibrium with limited enforcement is a list $(q, q^c, z, \tilde{z}, \bar{b})$ that satisfy (10), (11), (14), (15), and (16).*

We now turn to characterizing three types of steady-state equilibria that can arise in the model: (i) a pure credit equilibrium, (ii) a pure monetary equilibrium, and (iii) a money and credit equilibrium.

4.1 Pure Credit Equilibrium

A non-monetary equilibrium with credit exists when $z = \tilde{z} = 0$ and $\bar{b} \in [\bar{b}_0, \infty)$. When money is not valued, the debt limit \bar{b} must satisfy

$$r\bar{b} = \sigma\theta\Lambda S(\bar{b}) \equiv \Omega_0(\bar{b}). \quad (17)$$

A necessary condition for there to be credit is that the slope of $r\bar{b}$ is less than the slope of $\Omega_0(\bar{b})$ at $\bar{b} = 0$, or

$$r < \sigma\Lambda\frac{\theta}{1-\theta}. \quad (18)$$

When the fraction of sellers accepting credit is exogenous, there exists a threshold for the fraction of credit trades, below which $\bar{b} = 0$. From (18), credit is feasible if

$$\Lambda > \frac{r(1-\theta)}{\sigma\theta} \equiv \bar{\Lambda}.$$

Figure 3 shows the determination of the debt limit. Notice that an equilibrium without credit always exists since $\bar{b} = 0$ is always a solution to (16). This captures the idea that an equilibrium

¹⁷See Section 4.3 for an analysis of the model with $\Lambda = 1$, which will have qualitatively different properties from the model with $\Lambda \in (0, 1)$.

without credit is self-fulfilling and can arise under the expectation that borrowers will not repay their debts in the future.

In addition, there exists a critical value for the rate of time preference, \bar{r} , below which the debt limit stops binding and borrowers can borrow enough to purchase the first-best, q^* . The borrowing constraint will not bind if $\bar{b} \geq (1 - \theta)u(q^*) + \theta c(q^*)$, in which case \bar{r} must satisfy $\bar{r}[(1 - \theta)u(q^*) + \theta c(q^*)] = \sigma\theta\Lambda[u(q^*) - c(q^*)]$. Hence borrowers will be unconstrained if

$$r \leq \frac{\sigma\theta\Lambda[u(q^*) - c(q^*)]}{(1 - \theta)u(q^*) + \theta c(q^*)} \equiv \bar{r}.$$

Accordingly, the first-best is more likely to be attained if agents are more patient, trading frictions are small, buyers have enough market power in the DM, or the fraction of the economy with access to record-keeping is large. Since $\bar{r} < \frac{\sigma\Lambda\theta}{1-\theta}$ is always satisfied whenever a pure credit equilibrium exists, the borrowing constraint binds if $\bar{r} < r < \sigma\Lambda\frac{\theta}{1-\theta}$ and does not bind if $r < \bar{r} < \sigma\Lambda\frac{\theta}{1-\theta}$.

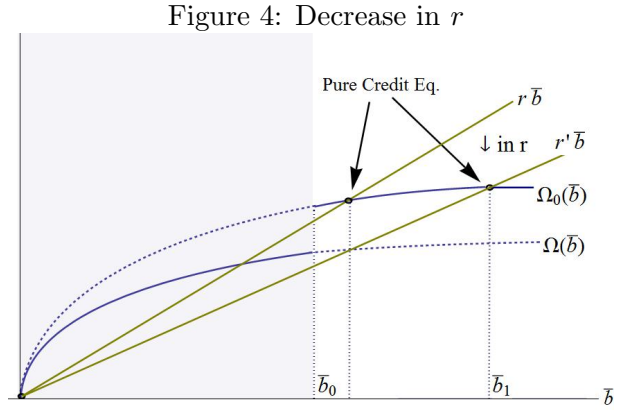
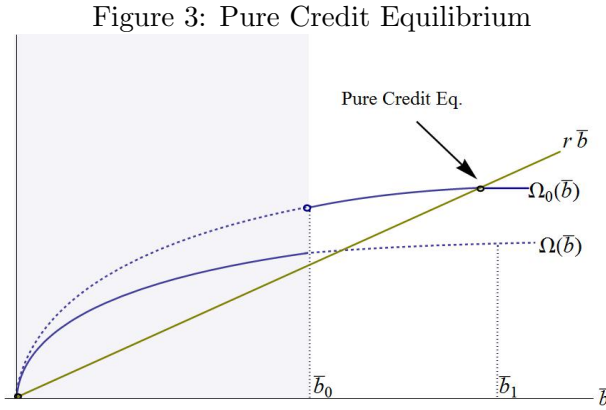
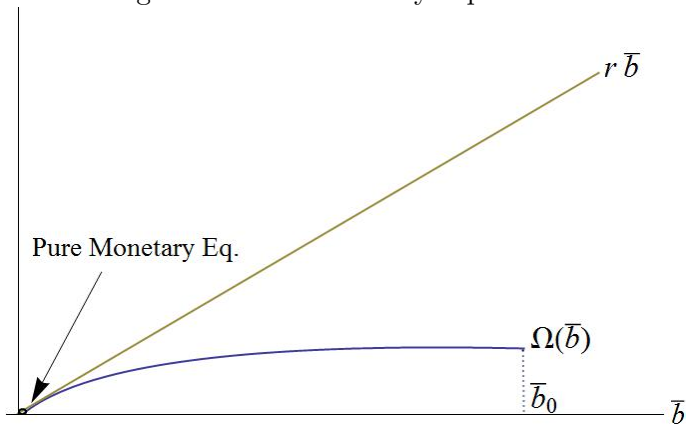


Figure 4 depicts the pure credit equilibrium and shows the effects of a decrease in r , or as agents become more patient. When r decreases to $r' = \bar{r}$, the debt limit increases to \bar{b}_1 and quantity traded increases from $q < q^*$ to q^* . Intuitively, the borrowing limit relaxes as agents become more patient since buyers can credibly promise to repay more. If on the other hand r increases above $\sigma\Lambda\frac{\theta}{1-\theta}$, the borrowing limit is driven to zero as borrowers do not care about the future enough to sustain credit use.

More generally, $\Omega_0(\bar{b})$ shifts up as the measure of sellers with access to record-keeping, Λ , increases, trading frictions, σ^{-1} , decrease, or the buyer's bargaining power, θ , increases, each of which relaxes the debt limit and thereby increasing \bar{b} . Moreover, notice that since money is not valued in a pure credit equilibrium, inflation has no effect on the debt limit or equilibrium allocations.

Figure 5: Pure Monetary Equilibrium



4.2 Pure Monetary Equilibrium

In a pure monetary equilibrium, money is valued ($z > 0$) while credit is not used ($\bar{b} = 0$). At $\bar{b} = 0$, $\Omega(0) = 0$ by Lemma 2. Further, since $S(z)$ is concave and $S'(0) = \frac{1}{(1-\theta)}$, money is valued if and only if

$$\begin{aligned} i &= \sigma\theta S'(z), \\ \Leftrightarrow i &< \sigma\theta S'(0), \\ \Leftrightarrow i &< \frac{\sigma\theta}{1-\theta} \equiv \bar{i}. \end{aligned}$$

The critical value, \bar{i} is the upper-bound for the cost of holding money, above which money is no longer valued.¹⁸

In addition, an equilibrium with money and credit is not feasible if $i\Lambda < r$, so that the slope of $\Omega(\bar{b})$ at $\bar{b} = 0$ is less than the slope of $r\bar{b}$. Consequently, there exists a critical value $\underline{i} \equiv \frac{r}{\Lambda}$, below which credit is not incentive-feasible given money is valued. Figure 5 plots $\Omega(\bar{b})$ as a function of \bar{b} when $i < \underline{i}$ and $i < \bar{i}$. In that case, $\Omega(\bar{b})$ intersects $r\bar{b}$ from below, and the unique monetary equilibrium is one where $\bar{b} = 0$.

¹⁸In a monetary model with bargaining where z depends on q , one must also check if there are corner solutions, i.e. $z = 0$, which may not be ruled out even with the Inada condition $u'(0) = \infty$. Why would buyers ever choose $z = 0$? While the buyer's marginal utility is infinite at $z = 0$, marginal cost can be infinite as well. Here, marginal cost is given by the term $\frac{dz}{dq} = z'(q) = \theta c'(q) + (1-\theta)u'(q)$. By the envelope theorem, the term $\frac{u'(q)-c'(q)}{\theta c'(q)+(1-\theta)u'(q)}$ at $z = 0$ becomes $\frac{1}{1-\theta}$. To see this another way, one can substitute $z(q) = (1-\theta)u(q) + \theta c(q)$ into the objective function and rearrange to obtain $\max_{q \in [0, q^*]} \{[\sigma\theta - i(1-\theta)]u(q) - [i + \sigma]\theta c(q)\}$. Using the fact that $u'(0) = \infty$, a necessary and sufficient condition for $z > 0$ under proportional bargaining is that $[\sigma\theta - i(1-\theta)] > 0$, or $i < \sigma \frac{\theta}{1-\theta}$.

The next proposition characterizes how a key policy variable, the money growth rate γ , affects the existence of a monetary equilibrium.

Proposition 1. Define $\underline{\gamma} \equiv \beta(1 + \underline{i})$ and $\bar{\gamma} \equiv \beta(1 + \bar{i})$, where $\underline{i} \equiv \frac{r}{\Lambda}$ and $\bar{i} \equiv \frac{\sigma\theta}{1-\theta}$. If $\underline{\gamma} < \bar{\gamma}$, then $r < \sigma\Lambda\frac{\theta}{1-\theta} \Leftrightarrow \Lambda > \bar{\Lambda}$ and the following steady-state equilibria are possible:

1. If $\gamma = \beta$, a pure monetary equilibrium with $q = q^*$, $z = \tilde{z} = (1 - \theta)u(q^*) + \theta c(q^*)$, and $\bar{b} = 0$ exists uniquely.
2. If $\gamma \in (\beta, \underline{\gamma})$, a pure monetary equilibrium with $q < q^*$, $z = \tilde{z} < (1 - \theta)u(q^*) + \theta c(q^*)$, and $\bar{b} = 0$ exists.
3. If $\gamma \in (\underline{\gamma}, \bar{\gamma})$, a pure monetary equilibrium coexists with a pure credit equilibrium. If in addition, $r \in (0, \bar{r}]$, equilibrium with credit is unconstrained with $q^c = q^*$ and $\bar{b} = (1 - \theta)u(q^*) + \theta c(q^*)$. If $r \in (\bar{r}, \Lambda\bar{i})$, equilibrium with credit is constrained with $q^c < q^*$ and $\bar{b} < (1 - \theta)u(q^*) + \theta c(q^*)$.
4. If $\gamma \geq \bar{\gamma}$, a pure monetary equilibrium ceases to exist, and a pure credit equilibrium will exist. If $r \in (0, \bar{r}]$, then $q^c = q^*$ and $\bar{b} = (1 - \theta)u(q^*) + \theta c(q^*)$. If $r \in (\bar{r}, \Lambda\bar{i})$, then $q^c < q^*$ and $\bar{b} < (1 - \theta)u(q^*) + \theta c(q^*)$.

The first part of Proposition 1 is very intuitive and simply says that when $\gamma = \beta$, the rate of return on money is high enough so that there is no need to use credit. At the Friedman rule, deflation completely crowds out credit since the incentive to renege is too high to support voluntary debt repayment. Efficient monetary policy drives out credit, and money alone is enough to finance the first-best.¹⁹

In addition, the Friedman rule is sufficient but not necessary to permit the uniqueness of a pure monetary equilibrium. Proposition 1 also shows that so long as $\gamma < \underline{\gamma}$, credit can never be sustained in a monetary equilibrium since the incentive to renege is too high. To take the most extreme case, suppose that $\Lambda = 1$ so that record-keeping is perfect and all sellers accept credit. Given money is valued, credit cannot be sustained if $i < r$, or equivalently, $\gamma < \underline{\gamma} = 1$. Even though all sellers accept credit, buyers choose to only hold real balances since the incentive to renege on debt repayment is too high.

It is possible for a pure monetary equilibrium to coexist with a pure credit equilibrium when $\gamma \in (\underline{\gamma}, \bar{\gamma})$. In this region, the cost of holding money is high enough for debt repayment to be

¹⁹As we show in Appendix B however, the Friedman rule may not be feasible since it requires taxation and individuals may choose to renege on their tax obligation if it is too high. In that case, the only way to achieve the first-best will be with credit.

feasible but low enough so that money is still valued. When $\gamma > \bar{\gamma}$, money is too costly to hold and only credit is feasible. The first-best allocation can be achieved provided that agents are patient enough, or if $r \in (0, \bar{r}]$. This can be implemented with any inflation rate such that $\gamma > \bar{\gamma} \equiv \beta(1 + \bar{i})$. In that case, equilibrium is unconstrained and a pure credit economy ensures that agents trade the first-best level of output.

4.3 Money and Credit Equilibrium

In a monetary equilibrium with credit, $\bar{b} \in (0, \bar{b}_0)$, $z > 0$, and $\tilde{z} > 0$. We first derive existence conditions for a money and credit equilibrium in the model with $\Lambda \in (0, 1)$, and then turn to a very special case of the model when $\Lambda = 1$. In either case we must check that two conditions are satisfied for a money and credit equilibrium to exist: (1) credit is incentive-feasible, given that money is valued and (2) money is valued, given a debt limit.

First, given money is valued, there will be a positive debt limit if the slope of $\Omega(\bar{b})$ at $\bar{b} = 0$, $\Omega'(0) = \sigma\theta\Lambda S'(z)$, is greater than the slope of $r\bar{b}$, or

$$i\Lambda > r,$$

where we have used the fact that when $z > 0$, $i = \sigma\theta S'(z)$ at $\bar{b} = 0$. As a result, a necessary condition for $\bar{b} > 0$ given $z > 0$ is

$$i > \underline{i} \equiv \frac{r}{\Lambda}. \quad (19)$$

The critical value \underline{i} is the lower bound on the nominal interest rate, above which credit is incentive feasible. Intuitively, condition (19) says that the cost of holding money cannot be too low so that buyers would prefer to renege on repayment, and buyers have to be patient enough to care about the possibility of future punishment.

Second, given \bar{b} , there will be an interior solution for z if and only if

$$\begin{aligned} i &< \sigma\theta[(1 - \Lambda)S'(0) + \Lambda S'[\bar{b}(i, \Lambda)]], \\ i &< \sigma(1 - \Lambda)\frac{\theta}{1 - \theta} + \sigma\Lambda\theta S'[\bar{b}(i, \Lambda)], \end{aligned}$$

where $\bar{b}(i, \Lambda)$ implicitly defines \bar{b} as a function of exogenous parameters, i and Λ . Consequently, $z > 0$ if and only if

$$i < \tilde{i}, \quad (20)$$

where \tilde{i} solves

$$\tilde{i} = \sigma(1 - \Lambda)\frac{\theta}{1 - \theta} + \sigma\Lambda\theta S'[\bar{b}(\tilde{i}, \Lambda)]. \quad (21)$$

The determination of \tilde{i} is illustrated in Figure 6, which plots the right-hand-side of (21) against i . In the shaded region, $i \in (\underline{i}, \tilde{i})$, in which case $z > 0$ and $\bar{b} > 0$.

Finally, $\tilde{z} > 0$ if and only if $i < \bar{i} \equiv \frac{\sigma\theta}{1-\theta}$. Notice however that $i < \bar{i}$ is implied by the condition $i < \tilde{i}$: when $i > \underline{i}$, $\tilde{i} = (1 - \Lambda)\bar{i} + \sigma\theta\Lambda S'[\bar{b}(\tilde{i}, \Lambda)] < \bar{i}$. This can also be seen in Figure 6, where $\tilde{i} < \bar{i}$. Hence, $i < \tilde{i}$ implies the condition $i < \bar{i}$ is satisfied. Consequently, a necessary condition for a monetary equilibrium with credit to exist is $i \in (\underline{i}, \tilde{i})$.

Figure 6: Determination of \tilde{i} when $\Lambda \in (0, 1)$ and $i \in (\underline{i}, \tilde{i})$

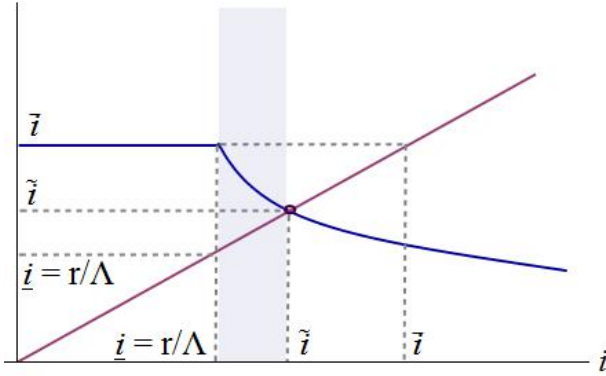
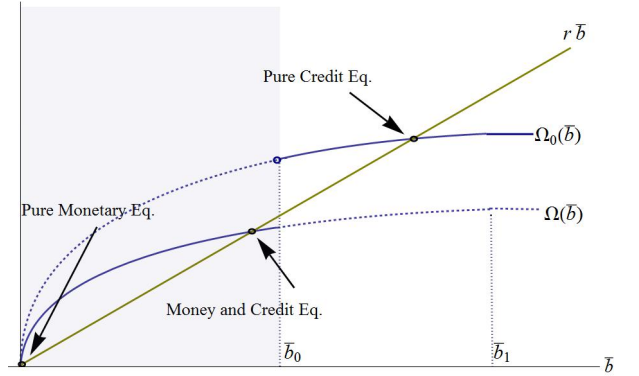


Figure 7: Pure Monetary, Money-Credit, and Pure Credit Eq. when $\Lambda \in (0, 1)$ and $i \in (\underline{i}, \tilde{i})$



A money and credit equilibrium when $\Lambda \in (0, 1)$ is depicted in Figure 7. In order for there to be credit, condition (19) implies that $\Omega(\bar{b})$ must intersect $r\bar{b}$ from above. Notice that a pure monetary equilibrium exists whenever there is an equilibrium with both money and credit since the condition for a monetary equilibrium, $i < \bar{i}$, is always satisfied if $i \in (\underline{i}, \tilde{i})$ and that there is always a solution to (16) with $\bar{b} = 0$. Since only a fraction of sellers accept credit, money maintains a social role since it allows buyers to insure against the possibility of not being able to use credit in some transactions.

We now turn to discussing a special case of the model with perfect record-keeping. The determination of the debt limit with $\Lambda = 1$ is depicted in Figure 9.²⁰ As before, the flow cost of default $\Omega(\bar{b})$ depends on whether or not money is valued. When $i < \bar{i}$, money is valued and $\Omega(\bar{b})$ is linear for all $\bar{b} < \bar{b}_0 = \tilde{z}$. In that case, a buyer chooses his real balances so that his total wealth is the same as a buyer who defaults: since $z + \bar{b} = \tilde{z}$ from (14) and (15), $\Omega(\bar{b}) = i\bar{b}$, which is linear with a slope of i . When $i \geq \bar{i}$, money is no longer valued, in which case $\bar{b} \geq \bar{b}_0$ and the flow cost of default becomes $\Omega(\bar{b}) = \sigma\theta S(\bar{b})$, which is strictly concave if $\bar{b} \in [\bar{b}_0, \bar{b}_1)$ and linear if $\bar{b} \geq \bar{b}_1 = (1 - \theta)u(q^*) + \theta c(q^*)$.

When $\Lambda = 1$, a necessary condition for credit is that the slope of $\Omega(\bar{b}) = \sigma\theta S(\bar{b})$ at $\bar{b} = 0$ is

²⁰For a similar analysis in a model of liquid assets and credit with perfect record-keeping, see Bethune, Rocheteau, and Rupert (2013). See also Gu, Mattesini, and Wright (2013).

Figure 8: $\underline{i} = \tilde{i} < \bar{i}$ at $i = r$ when $\Lambda = 1$

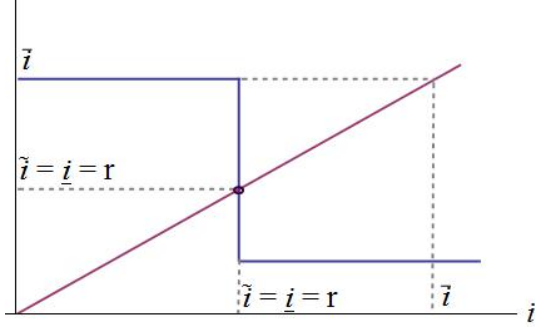
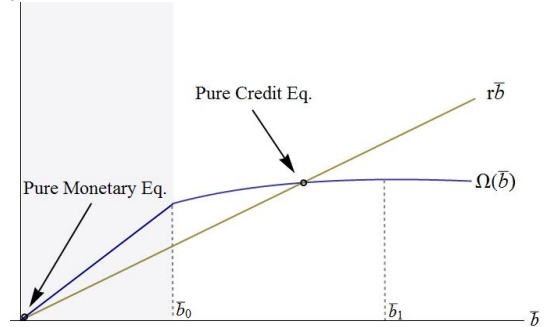


Figure 9: Pure Monetary and Pure Credit Eq. when $\Lambda = 1$



greater than the slope of $r\bar{b}$. If money is not valued, $\bar{b} > 0$ if $r < \frac{\sigma\theta}{1-\theta} = \bar{i}$. In addition, there will be a unique positive debt limit if $i > \underline{i} = r$ so that $\Omega(\bar{b}) = i\bar{b}$ intersects with $r\bar{b}$ from above, as in Figure 9. In that case, there will be a unique $\bar{b} > \bar{b}_0$ that solves (16). However since $\bar{b} > \bar{b}_0$, money is no longer valued. If instead $i < r$, the only solution to (16) is $\bar{b} = 0$, in which case $\Omega(\bar{b}) = i\bar{b}$ would intersect with $r\bar{b}$ from below. Finally if $i = r$, the debt limit is indeterminate and there are a continuum of solutions $\bar{b} \in [0, \bar{b}_0]$ that solves (16).

Can a money and credit equilibrium exist under perfect record-keeping? We prove that in the special case of our model with $\Lambda = 1$, the answer is no. This is illustrated in Figure 8, which depicts the determination of \tilde{i} when $\Lambda = 1$. Notice that there are a continuum of solutions at $\underline{i} = r$ since the debt limit is indeterminate when $i = r$. The 45-degree line intersects with the right-hand-side of (21) at $\tilde{i} = \underline{i} = r$, in which case $\underline{i} = \tilde{i} < \bar{i}$. At $i = r$, we therefore have $\tilde{i} = \underline{i}$, meaning that the existence condition for a money and credit equilibrium can no longer be satisfied.

The following proposition summarizes the existence conditions for a money and credit equilibrium and highlights the special case of the model with $\Lambda = 1$, in which case a money and credit equilibrium ceases to exist.

Proposition 2. *When $i \in (\underline{i}, \tilde{i})$ and $\Lambda \in (0, 1)$, a money and credit equilibrium exists. In addition, a money and credit equilibrium will coexist with a pure monetary equilibrium and a pure credit equilibrium. If $\Lambda = 1$, there can be a pure credit equilibrium where $\bar{b} > 0$ and $z = \tilde{z} = 0$, a pure monetary equilibrium where $\bar{b} = 0$ and $z > 0$, or a non-monetary equilibrium without credit, but there cannot be an equilibrium where both money and credit are used.*

Proposition 2 highlights an important dichotomy between monetary and credit trades when record-keeping is perfect ($\Lambda = 1$): there can be trades with credit only or trades with money only, but never trades with both money and credit. This special case also points to the difficulty of getting money and credit to coexist when all trades are identical and record-keeping is perfect:

either only credit is used as money becomes inessential, or only money is used since the incentive to renege on debt repayment is too high. This also captures the insight by Kocherlakota (1998) that there is no social role for money in an economy with perfect record-keeping.

Having characterized existence properties of a money and credit equilibrium, we now turn to discussing some comparative statics for effects on the debt limit, which the table below summarizes.

$\frac{\partial \bar{b}}{\partial \Lambda}$	$\frac{\partial \bar{b}}{\partial \sigma}$	$\frac{\partial \bar{b}}{\partial i}$	$\frac{\partial \bar{b}}{\partial r}$	$\frac{\partial \bar{b}}{\partial \theta}$
+	+	+	-	+

An increase in the fraction of the economy with access to record-keeping, Λ , increases the right-hand-side of (16), which shifts $\Omega(\bar{b})$ up and induces an increase in \bar{b} . When more sellers accept credit, the gain for buyers from using and redeeming credit increases, which relaxes the debt limit. The increase in Λ can be high enough so that credit starts to drive money out of circulation. This can cause the money and credit equilibrium to disappear, in which case there will be a pure monetary equilibrium and a pure credit equilibrium.

An increase in inflation (analogously, i) generates a similar qualitative effect. In this way, inflation has two effects in this model: first, is the usual effect on reducing the purchasing power of money, which reduces trade and hence welfare; second, is the effect on reducing agents' incentive to default. Intuitively, an increase in the inflation tax relaxes the credit constraint by increasing the cost of default, since defaulters need to bring enough money to finance their consumption.

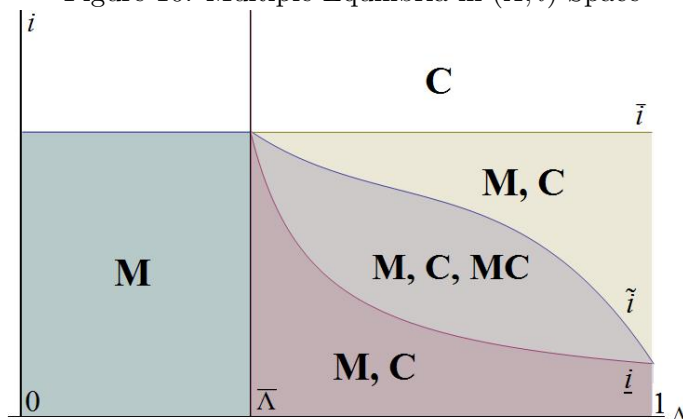
In sum, the debt limit depends on the fraction of credit trades, the extent of trading frictions, the rate of return on money, agents' patience, and the buyer's bargaining power. The larger the fraction of sellers that accept credit, the lower the rate of return on money, or the more patient agents become, the less likely the credit constraint will be binding. In these cases, the buyer can credibly promise to repay more, which induces cooperation in credit arrangements thereby relaxing the debt limit.

4.4 Multiple Equilibria

A particularly striking feature of the model is that there can be a multiplicity of equilibria even without any changes in fundamentals. The next proposition establishes the possible cases for multiple equilibria, which the remainder of this subsection discusses.

Proposition 3. *When $i \geq \bar{i}$, equilibrium will be non-monetary and there will either be (i) autarky where neither money nor credit is used if $\Lambda \in [0, \bar{\Lambda}]$ or (ii) a pure credit equilibrium if $\Lambda \in (\bar{\Lambda}, 1]$. When $i < \bar{i}$, a pure monetary equilibrium either (iii) exists uniquely if $\Lambda \in [0, \bar{\Lambda}]$, (iv) coexists with a pure credit equilibrium if $\Lambda \in (\bar{\Lambda}, 1]$, or (v) coexists with both a pure credit equilibrium and a money and credit equilibrium if and only if $i \in (\underline{i}, \tilde{i})$.*

Figure 10: Multiple Equilibria in (Λ, i) -Space



Proposition 3 is illustrated in Figure 10, which plots existence conditions for different types of equilibria in (Λ, i) -space.²¹ We have shown in the previous sub-sections that a necessary condition for credit is $\Lambda > \bar{\Lambda}$, a pure monetary equilibrium will exist only $i < \bar{i} \equiv \frac{\sigma\theta}{1-\theta}$, and both money and credit to be used if $i \in (\underline{i}, \tilde{i})$ and $\Lambda \in (0, 1)$.

Figure 10 also shows how payment systems depend not just on fundamentals but also on histories and social conventions. Suppose that inflation is initially low and the economy is in an equilibrium where a pure monetary equilibrium coexists with a pure credit equilibrium (region M, C). As inflation increases above \bar{i} , the pure monetary equilibrium disappears and only credit is used (region C). But when inflation goes back down to its initial level, it is possible that agents may still coordinate on the pure credit equilibrium. The economy therefore displays hysteresis and inertia: when there are many possible types of equilibria, social conventions and histories can dictate the equilibrium that prevails.

When agents get less patient (r increases), both the threshold for credit to be used, $\bar{\Lambda}$, and the condition for both money and credit to be used, $\underline{i} = \frac{r}{\Lambda}$, increases. In Figure 10, the vertical line $\bar{\Lambda}$ shifts to the right while the curve \underline{i} shifts up. An increase in r therefore decreases the possibility of any equilibrium with credit. Intuitively, less patient buyers find it more difficult to credibly promise to repay their debts, which decreases their borrowing limit \bar{b} .

²¹The types of equilibria in Figure 10 are a pure credit (C) equilibrium, a pure monetary (M) equilibrium, and a mixed equilibrium where both money and credit are used (MC).

5 Costly Record-Keeping

We now consider the choice of accepting credit by making $\Lambda \in [0, 1]$ endogenous. In order to accept credit, sellers must invest ex-ante in a costly record-keeping technology that records and authenticates an IOU proposed by the buyer.²² The per-period cost of this investment in terms of utility is $\kappa > 0$, which is drawn from a cumulative distribution $F(\kappa) : \mathbb{R}_+ \rightarrow [0, 1]$. Sellers are heterogenous according to their record-keeping cost and are indexed by κ .²³ To ensure an interior equilibrium, assume $\kappa = 0$ for a positive measure of agents and that κ is arbitrarily high for a positive measure of agents. Hence for some sellers this cost will be close to zero, so that they will always accept credit, while for others this cost will be very large and they will never accept credit. The distribution of costs across sellers is known by all agents and is assumed to be continuous.

At the beginning of each period, sellers decide whether or not to invest. When making this decision, sellers take as given buyer's choice of real balances, z , and the debt limit, \bar{b} . The seller's problem is given by

$$\max\{-\kappa + \sigma(1 - \theta)S(z + \bar{b}), \sigma(1 - \theta)S(z)\}. \quad (22)$$

According to (22), if the seller decides to invest, he incurs the disutility cost $\kappa > 0$ that allows him to extend a loan to the buyer. In that case, the seller extracts a constant fraction $(1 - \theta)$ of the total surplus, $S(z + \bar{b}) \equiv u[q(z + \bar{b})] - c[q(z + \bar{b})]$. If the seller does not invest, then he can only accept money, and gets $(1 - \theta)$ of $S(z) \equiv u[q(z)] - c[q(z)]$. Since total surplus is increasing in the buyer's total wealth $z + \bar{b}$, $S(z + \bar{b}) > S(z)$.

There exists a threshold for the record-keeping cost, $\bar{\kappa}$ below which sellers invest in the record-keeping technology and above which they do not invest. From (22), this threshold is given by

$$\bar{\kappa} \equiv \sigma(1 - \theta)[S(z + \bar{b}) - S(z)], \quad (23)$$

and gives the seller's expected benefit of accepting credit. Since $S(z + \bar{b})$ increases with \bar{b} , the seller's expected benefit $\bar{\kappa}$ increases with \bar{b} . Given κ , let $\lambda(\kappa) \in [0, 1]$ denote an individual seller's

²²This cost can also reflect issues of fraud and information problems that permeate the credit industry such as credit card fraud, identity theft, and the need to secure confidential information. Besides being a costly drain on banks and retailers that accept credit, these problems may erode consumer confidence in the credit card industry. See Roberds (1998) for a discussion and Kahn and Roberds (2008) and Roberds and Schreft (2009) for recent formalizations of identify theft and Li, Rocheteau, and Weill (2013) for a model of fraud.

²³Arango and Taylor (2008) find that merchants perceive cash as the least costly form of payment while credit cards stand out as the most costly due to relatively high processing fees.

decision to invest. This decision problem is given by

$$\lambda(\kappa) = \begin{cases} 1 & \left\{ \begin{array}{l} < \\ = \\ > \end{array} \right. \bar{\kappa}. \\ [0, 1] & \\ 0 & \end{cases} \quad (24)$$

Condition (24) simply says that all sellers with $\kappa < \bar{\kappa}$ will invest in the costly record-keeping technology, since the benefit exceeds the cost; sellers with $\kappa > \bar{\kappa}$ do not invest; and any seller with $\kappa = \bar{\kappa}$ will invest with an arbitrary probability since they are indifferent.

Consequently, since $F(\kappa)$ is continuous, the aggregate measure of sellers that invest is

$$\Lambda \equiv \int_0^\infty \lambda(\kappa) dF(\kappa) = F(\bar{\kappa}). \quad (25)$$

That is, the measure of sellers that invest is given by the measure of sellers with $\kappa \leq \bar{\kappa}$.

Definition 2. *A steady-state equilibrium with limited enforcement and endogenous record-keeping is a list $(q, q^c, z, \bar{z}, \bar{b}, \Lambda)$ that satisfy (10), (11), (14), (15), (16), and (25).*

To determine equilibrium when Λ is endogenous, we first characterize the debt limit, given sellers' investment decisions. Next, we determine sellers' investment decisions, given the debt limit. Finally, we jointly determine \bar{b} and Λ in equilibrium.

5.1 Debt Limit, \bar{b}

Given sellers' investment decisions, Λ , credit must be incentive feasible and satisfy (16).

The following lemma summarizes properties of the equilibrium correspondence for the debt limit, which describes how \bar{b} depends on the measure of sellers who invest.

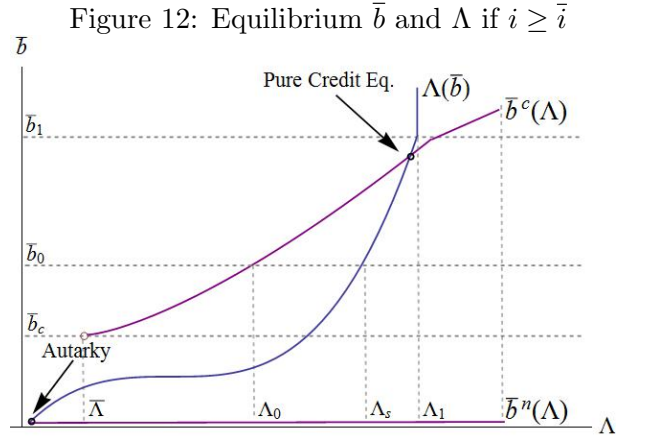
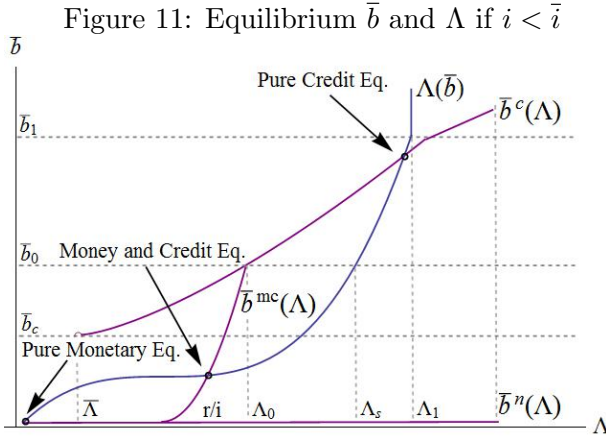
Lemma 4. *When $i < \bar{i}$ ($\Leftrightarrow \frac{r}{i} > \bar{\Lambda}$), the equilibrium correspondence for \bar{b} consists of three curves: $\bar{b}^n = 0$ (no-credit curve), $\bar{b}^{mc} \in (0, \bar{b}_0)$ (money-credit curve), and $\bar{b}^c \in (\bar{b}_c, \infty)$ (pure credit curve). Let the measure of sellers that accept credit at \bar{b}_0 be defined as $\Lambda_0 \equiv \frac{r\bar{b}_0}{\sigma\theta S(\bar{b}_0)}$, and let \bar{b}_c be defined as the threshold for the debt limit, above which $\Lambda > \bar{\Lambda}$.*

1. For all $\Lambda \in [0, 1]$, there exists an equilibrium without credit with $\bar{b}^n = 0$.
2. When $\Lambda \in (\frac{r}{i}, \Lambda_0]$, the debt limit $\bar{b}^{mc} \in (0, \bar{b}_0)$ is strictly increasing and convex in Λ .
3. When $\Lambda \in (\bar{\Lambda}, 1]$, the debt limit $\bar{b}^c \in (\bar{b}_c, \bar{b}_1)$ is strictly increasing and convex in Λ , and linear for $\bar{b}^c \in [\bar{b}_1, \infty)$.

5.2 Measure of Sellers Who Invest, Λ

Given \bar{b} , sellers must decide whether to invest in the costly technology to record credit transactions. The following lemma summarizes how sellers' investment decisions, Λ , depend on the debt limit.

Lemma 5. *The equilibrium condition for Λ is continuous and strictly increasing in \bar{b} when $\bar{b} \in [0, \bar{b}_1)$ and constant at $\Lambda_1 \leq 1$ when $\bar{b} \in [\bar{b}_1, \infty)$.*



Together, Lemma 4 and Lemma 5 allow us to characterize equilibrium as a function of the measure of sellers who invest, Λ , and the debt limit, \bar{b} .

Proposition 4. *When \bar{b} and Λ are endogenous, there are multiple steady-state equilibria characterized by the cases below.*

1. If $i < \bar{i}$ ($\Leftrightarrow \frac{r}{i} > \bar{\Lambda}$), there exists (i) a pure monetary equilibrium where $z > 0$, $\bar{b} = 0$, and $\Lambda = 0$; (ii) a pure credit equilibrium where $z = 0$, $\bar{b} > 0$, and $\Lambda \leq \Lambda_1 \in (0, 1]$; and (iii) a money and credit equilibrium where $z > 0$, $\bar{b} > 0$, and $\Lambda < \Lambda_0 \in (0, 1)$.
2. If $i \geq \bar{i}$ ($\Leftrightarrow \frac{r}{i} \leq \bar{\Lambda}$), there exists (i) a non-monetary equilibrium without credit where $z = 0$, $\bar{b} = 0$, and $\Lambda = 0$; and (ii) a pure credit equilibrium where $z = 0$, $\bar{b} > 0$, and $\Lambda \leq \Lambda_1 \in (0, 1]$.

The first part of Proposition 4 is illustrated in Figure 11. When $i < \bar{i}$, the equilibrium conditions for \bar{b} and Λ intersect three times. Hence there are three different types of equilibria: (i) a pure monetary equilibrium where only money is used ($\Lambda = 0$), (ii) a money and credit equilibrium where a fraction $\Lambda < \Lambda_0 \in (0, 1)$ of sellers accept both money and credit while the remaining $(1 - \Lambda)$ sellers only accept money, and (iii) a pure credit equilibrium where money is not valued and a fraction $\Lambda \leq 1$ of sellers accept credit while the remaining $(1 - \Lambda)$ do not. Similarly, Figure 12 illustrates

the second part of Proposition 4. When $i \geq \bar{i}$, money is not valued and there can only exist (i) a pure credit equilibrium and (ii) a non-monetary equilibrium without credit.

The multiplicity of equilibria arises through the general equilibrium effects in the trading environment that produce strategic complementarities between buyers' and sellers' decisions.²⁴ As is evident from agents' upward-sloping reaction functions in Figure 11, what the seller accepts affects how much debt the buyer can repay and vice versa. When more sellers invest in the costly record-keeping technology, the gain for buyers from using credit also increase. As default becomes more costly, the incentive to renege falls which raises the debt limit. At the same time, when more sellers accept credit, then money is needed in a smaller fraction of matches. So long as it is costly to hold money, buyers will therefore carry fewer real balances. This in turn gives sellers even more incentive to accept credit, which in turn raises the debt limit and hence reduces the buyer's real balances.

6 Welfare

We now turn to examining some of the model's normative implications and begin by comparing the different types of equilibria in terms of social welfare. Society's welfare is measured as the steady-state sum of buyers' and sellers' utilities in the DM: $\mathcal{W} \equiv (1 - \beta)V^b(z) + (1 - \beta)V^s(0)$. This is given by

$$\mathcal{W} \equiv \sigma[\Lambda S(z + \bar{b}) + (1 - \Lambda)S(z)] - k \quad (26)$$

where $k \equiv \int_0^{\bar{\kappa}} \kappa dF(\kappa)$ is defined as the aggregate record-keeping cost averaged across individual sellers. Table 1 summarizes social welfare across these types of equilibria.

EQUILIBRIUM	WELFARE
Pure Monetary	$\mathcal{W}^m = \sigma S(z)$
Pure Credit	$\mathcal{W}^c = \sigma \Lambda S(\bar{b}) - k$
Money and Credit	$\mathcal{W}^{mc} = \sigma[\Lambda S(z + \bar{b}) + (1 - \Lambda)S(z)] - k$

Figure 13 plots social welfare in the three types of equilibria as a function of the money growth rate, γ . So long as $\Lambda > \bar{\Lambda}$, a pure credit equilibrium can exist for all $i > 0$, or equivalently, for all $\gamma > \beta$. Welfare in a pure credit economy is independent of the money growth rate, so that \mathcal{W}^c in Figure 13 is horizontal for all $\gamma > \beta$.

²⁴Strategic complementarities between the seller's decision to invest and the buyer's choice of real balances would still exist even under perfect enforcement where borrowers can always borrow enough to finance purchase of the first-best. See Nosal and Rocheteau (2011) for an analysis assuming loan repayments are always perfectly enforced.

However at $\gamma = \beta$, an efficient economy can run without credit and a pure monetary economy exists uniquely. In that case, welfare is maximized at the Friedman rule, $\gamma = \beta$, with social welfare given by $\mathcal{W}^m = \sigma S^*$, where $S^* \equiv u(q^*) - c(q^*)$. As γ increases however, it may be possible that $\mathcal{W}^c > \mathcal{W}^m$. This will occur if $\gamma > \gamma_c$, where γ_c is the critical value for the money growth rate, above which $\mathcal{W}^c > \mathcal{W}^m$. Moreover when Λ is endogenous, welfare in a pure credit equilibrium under any inflation rate will always be socially inefficient and dominated by a pure monetary equilibrium at the Friedman rule since sellers must incur the real cost of technological adoption.

Consequently, for the pure credit equilibrium to dominate the pure monetary equilibrium in welfare terms, the inflation rate must be high enough, as illustrated in Figure 13, or the aggregate record-keeping cost must be low enough— that is, $\mathcal{W}^c > \mathcal{W}^m$ if $k < \sigma[\Lambda S(\bar{b}) - S(z)]$. However, even with a high enough inflation rate or low enough record keeping cost, it is still possible for the welfare-dominated monetary equilibrium to prevail due to a rent-sharing externality: since sellers must incur the full cost of technological adoption but only obtain a fraction $(1 - \theta)$ of the total surplus, they fail to internalize the full benefit of accepting credit. Consequently, there can be coordination failures and excess inertia in the decision to accept credit, in which case the economy can still end up in the Pareto-inferior monetary equilibrium.

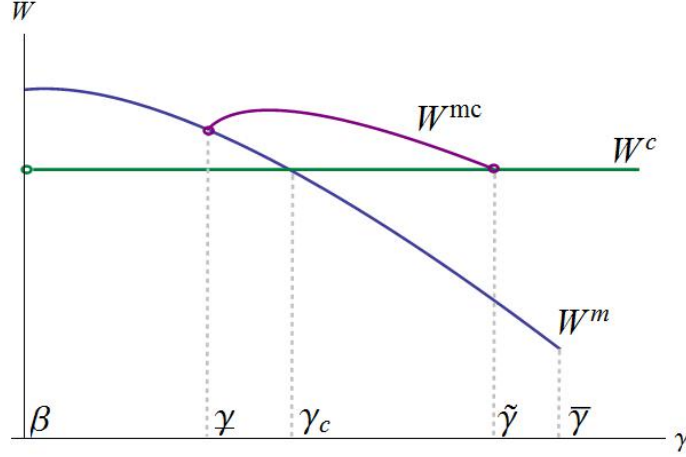
We can also compare welfare in a money and credit economy with welfare in a pure monetary economy and pure credit economy. Recall that a necessary condition for a money and credit equilibrium is $i \in (\underline{i}, \tilde{i})$, or equivalently, $\gamma \in (\underline{\gamma}, \tilde{\gamma})$. When $i = \underline{i}$ or $\gamma = \underline{\gamma}$, credit is no longer feasible given money is valued. In that case, $\bar{b} = 0$ and $z > 0$ and welfare becomes $\mathcal{W}^{mc} = \sigma S(z) = \mathcal{W}^m$. Alternatively when $i = \tilde{i}$ or $\gamma = \tilde{\gamma}$, money is no longer valued given a positive debt limit, in which case $z = 0$, $\bar{b} > 0$, and welfare becomes $\mathcal{W}^{mc} = \sigma \Lambda S(\bar{b}) - k = \mathcal{W}^c$. In the example in Figure 13, we can therefore have $\mathcal{W}^{mc} > \mathcal{W}^m$ and $\mathcal{W}^{mc} > \mathcal{W}^c$ for $\gamma \in (\underline{\gamma}, \tilde{\gamma})$. More generally, we also show in Appendix A that at γ_c , welfare in a money and credit equilibrium dominates welfare in a pure credit equilibrium so long as Λ is not too large.

Effect of Inflation on Welfare

We now consider the effect of inflation on social welfare for the three types of equilibria examined above. To fix ideas, we start by assuming Λ is exogenous. The presence of multiple equilibria for the same fundamentals makes the choice of optimal policy difficult to analyze in full generality since we must deal with the issue of equilibrium selection. In regions with multiplicity, we will assume agents coordinate on a particular equilibrium and then analyze the optimal policy of that equilibrium.

When $i < \tilde{i}$ and $\Lambda < \bar{\Lambda}$, there exists an equilibrium where agents only use money. In that case, social welfare is decreasing in the inflation rate: $\frac{d\mathcal{W}_m}{di} = \sigma S'(z) \frac{dz}{di} < 0$ since from (14), $\frac{dz}{di} < 0$.

Figure 13: Welfare in a Pure Monetary vs. Money and Credit vs. Pure Credit Economy



Since inflation is a tax on money holdings, an increase in inflation will reduce the purchasing power of money, and hence output and welfare. If lump-sum taxes can be enforced, the optimal policy in a pure monetary equilibrium corresponds to the Friedman rule. Appendix B analyzes the case assuming tax liabilities are not perfectly enforced, in which case there is a lower bound on the deflation rate, above which tax repayment is incentive-feasible.

Now suppose that $\Lambda > \bar{\Lambda}$ and $i \geq \bar{i}$ so that the economy is in a pure credit equilibrium. Since money is not valued in a pure credit equilibrium, inflation has no effect on welfare as shown in Figure 13.

Finally, suppose that $i \in (\underline{i}, \tilde{i})$, and agents coordinate on the money and credit equilibrium. In that case, the overall effect of inflation on welfare is ambiguous and depend on two counteracting effects: a real balance effect and the debt limit effect. In a money and credit equilibrium, we verify in Appendix A that the effect of inflation on welfare is given by

$$\frac{dW_{mc}}{di} = \sigma \Lambda S'(z + \bar{b}) \left[\frac{dz}{di} + \left(\frac{dz}{d\bar{b}} + 1 \right) \frac{d\bar{b}}{di} \right] + \sigma(1 - \Lambda) S'(z) \frac{dz}{di}, \quad (27)$$

where $\frac{dz}{di} = [\sigma\theta((1 - \Lambda)S''(z) + \Lambda S''(z + \bar{b}))]^{-1} < 0$, $\frac{dz}{d\bar{b}} = - \left[1 + \frac{(1 - \Lambda)S''(z)}{\Lambda S''(z + \bar{b})} \right]^{-1} \in (-1, 0)$, and $\frac{d\bar{b}}{di} = \frac{\tilde{z} - z}{r - \sigma\theta\Lambda S'(z + \bar{b})} > 0$. Generally, the term

$$\left[\frac{dz}{di} + \left(\frac{dz}{d\bar{b}} + 1 \right) \frac{d\bar{b}}{di} \right]$$

can be positive or negative, which determines whether $\frac{dW_{mc}}{di}$ is positive or negative. Indeed the sign of (27) depends on the value of Λ , the relative change in the marginal surpluses of using both money and credit versus using money only, the magnitude of the effective of inflation on real balances ($\frac{dz}{di} < 0$), and the magnitude of the effect of inflation on the debt limit ($\frac{db}{di} > 0$). In the $(1 - \Lambda)$ of transactions involving money only, inflating is simply a tax on buyers' real balances, which decreases welfare. However in the Λ of transactions with both money and credit, an increase in inflation can be welfare improving by relaxing agents' borrowing constraints. Intuitively, higher inflation makes default more costly which reduces the incentive to default, raises the debt limit, and increases welfare.

Figure 13 also shows that in a money and credit economy, there may be an interior money growth rate strictly above the Friedman rule that maximizes \mathcal{W}^{mc} . However notice that maximum welfare in a money and credit economy is still strictly dominated by welfare in a pure monetary economy at the Friedman rule. In that case, money works too well and there can be no socially useful role for credit. The Friedman rule is still the globally optimal monetary policy, which achieves both the first-best and saves society on record-keeping costs.

When Λ is endogenous, the positive effect of inflation on welfare is amplified and generates additional feedback effects. Since an increase in inflation raises the debt limit, sellers now have an even greater incentive to accept credit, which further relaxes borrowing constraints. When the debt limit relaxes to the point where it no longer binds, all sellers accept credit and money is no longer valued. In that case, agents can still trade the first-best level of output even when monetary authorities do not implement the Friedman rule. Notice however this equilibrium is not socially efficient since sellers must still incur the real cost of technological adoption.

7 Conclusion

As many economies now increasingly rely on credit cards as both a payment instrument and a means to borrow against future income, it is increasingly important to understand how individuals substitute between cash and credit. Despite the increasing availability of unsecured lending such as credit cards loans, consumers still demand paper currency and liquid assets for certain transactions that simply cannot be paid for with credit. That money and credit coexist appears to be the norm rather than the exception in many economies, and one goal of our paper is to try and delve deeper into understanding why.

To that end, we build a simple search model where money can have a socially useful role and credit is feasible. In order to capture the two-sided nature of actual payment systems, we jointly model the acceptability of credit by retailers and the portfolio allocation and debt repayment

decisions by consumers. We show that inflation induces individuals to substitute from money to credit for two reasons: a higher inflation rate both lowers the rate of return on money and makes default more costly, which relaxes agents' borrowing limits. When inflation is in an intermediate range and record-keeping is imperfect, both money and credit can coexist since a fraction of the economy only takes cash while the other fraction can accept both.

While credit allows retailers to sell to illiquid consumers or to those paying with future income, money can still be valued since it allows consumers to self-insure against the risk of not being able to use credit in some transactions. So long as inflation is not too high, there is a precautionary demand for liquidity that explains why individuals would still hold money even in an economy with credit. Therefore, money is not crowded out one-for-one with credit due to the demand for cash in the instances where credit cannot be used. This insight captures much of what is observed across many economies, yet is a result that is difficult to obtain in many previous models. We show however that in special case of our model with perfect record-keeping, the usual Kocherakota (1998) wisdom appears: when credit is feasible, there is no social role for money, and when money is valued, credit cannot be sustained.

Our theory also highlights a strategic complementarity between consumers' credit limit and retailers' decision to invest. Multiple equilibria and coordination failures can therefore arise due to the two-sided market nature of payment systems. This potential for coordination failures also raises new concerns for policymakers. In contrast with conventional wisdom, our theory suggests that economies with similar technologies, institutions, and policies can still end up with very different payment systems, some being better in terms of social welfare than others.

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Data Appendix

Here we describe some recent patterns regarding the usage and adoption of cash and credit cards summarized in the Introduction. To motivate the theory, we establish that (1) the salient differences between cash and credit cards for consumers include set-up costs, usage costs, merchant acceptance, and record-keeping; (2) households simultaneously revolve credit card debt while holding liquid assets such as cash; and (3) credit cards are more costly to accept than cash for merchants.

For the United States, consumer-level data on adoption and usage of cash versus credit cards is publicly available through the Federal Reserve Bank of Boston’s Survey of Consumer Payment Choice (SCPC) for 2008 and 2009. For a summary of the survey methodology and results, see Foster, Meijer, Schuh, and Zabek (2011). The SCPC is administered by the RAND Corporation to a subject pool drawn from the RAND American Life Panel. Respondents answer questions focusing on their personal adoption and use of eight different payment instruments (cash, checks, debit cards, credit cards, prepaid cards, online banking bill payment, bank account deduction, and income deduction) in retail and billing settings.²⁵ In 2009, adoption rates were 100% for cash and 78% for credit cards (check adoption was 100% and debit card adoption was 80%).

The SCPC also asks respondents to evaluate payment instruments along several dimensions, such as set-up cost (cost of setting up the payment instrument), acceptance (the level of merchant acceptance), usage cost (cost of use), and record-keeping (the ease of tracking use). Average ratings in 2009 are summarized in Table 2, with higher numbers corresponding to a more favorable view. For instance, cash is viewed relatively unfavorably in terms of record-keeping (cash transactions are anonymous) but more favorably than credit in “set-up cost”, “usage cost”, and “acceptance.” While we do not want to overemphasize these ratings, the qualitative findings help discipline some of our modeling assumptions for the theory that follows.

Table 2: Average Ratings of Payment Instruments by U.S. Consumers (SCPC)

	Set-Up Cost	Usage Cost	Acceptance	Record-Keeping
Cash	4.3	4.3	4.6	2.5
Credit	3.7	2.7	4.5	4.2

The Survey of Consumer Finances (SCF) provides information on financial behavior of U.S. households, including data on consumer adoption of credit cards. General trends in credit card use and ownership is summarized in Table 3. The clear pattern from this table is the increase in the adoption and use of credit cards by U.S. households over the last few decades.

²⁵The SCPC uses sampling weights to match the March Current Population (CPS) so that the weighted SCPC

Table 3: Credit Card Use and Credit Limits for U.S. Households (SCF)

	1989	2004	2007
Population with Credit Card	56%	71%	75%
Population with Positive (Revolving) Balance on Card	29%	39%	50%
Average Credit Limit to Income	0.20	0.46	—

Moreover, Telyukova (2013) uses the 2001 SCF to document that 27% of U.S. households report revolving an average of \$5,766 in credit card debt with an APR of 14% while holding an average of \$7,338 in low-return liquid assets with a 1% average return rate. In addition, 84% of households who revolved credit card debt held liquid assets that could be, but were not, used for credit card debt repayment, a puzzle due to an apparent violation of the no-arbitrage principle. Telyukova (2013) then finds that the need for liquidity can account for this puzzle, a conclusion our theory supports.

Finally, there is also evidence from Garcia-Swartz, Hahn, and Layne-Farrar (2006) that credit cards are more costly to accept than cash for merchants, and that most of the costs associated with credit card adoption are incurred by merchants. In particular, a merchant's cost of a typical cash transaction in the U.S. is about \$0.10 while a typical credit card transaction costs the merchant over \$0.70. In addition, this cost is not carried over to consumers as most of the cost is borne by merchants through the so-called merchant discount, which the merchant pays to the credit card company. Common estimates for this fee in the U.S. is around 2% of the amount purchased with credit cards (Nilson Report (2012)). This fee is not incurred by consumers due to regulations that the merchant cannot charge different prices for the same good or service based on cash versus credit payment. In practice, this no-surcharge rule prohibits sellers from passing through the fees associated with credit card adoption onto consumers that prefer to pay with a debit or credit card in place of cash.

Appendix A

Concavity of Buyer's Objective Function

The buyer's objective function is

$$\Psi(z) = -iz + \sigma\theta\Lambda[u(q^c) - c(q^c)] + \sigma\theta(1 - \Lambda)[u(q) - c(q)],$$

data to be representative of the U.S. population.

where q^c and q are given by $z + \bar{b} = (1 - \theta)u(q^c) + \theta c(q^c)$ and $z = (1 - \theta)u(q) + \theta c(q)$, respectively, and \bar{b} is given by (16). The partial derivative of the buyer's objective function with respect to the choice of real balances is

$$\Psi'(z) = -i + \sigma\theta\Lambda \left[\frac{u'(q^c) - c'(q^c)}{\theta c'(q^c) + (1 - \theta)u'(q^c)} \right] + \sigma\theta(1 - \Lambda) \left[\frac{u'(q) - c'(q)}{\theta c'(q) + (1 - \theta)u'(q)} \right],$$

if $z + \bar{b} < (1 - \theta)u(q^*) + \theta c(q^*)$ and $\Psi'(z) = 0$ if $z + \bar{b} \geq (1 - \theta)u(q^*) + \theta c(q^*)$. Consider the case where $z + \bar{b} < (1 - \theta)u(q^*) + \theta c(q^*)$ and hence $q^c < q^*$ and $q < q^*$. The second partial derivative is

$$\Psi''(z) = \sigma\theta[\Lambda\Delta^c + (1 - \Lambda)\Delta] < 0,$$

where $\Delta^c = \frac{u''(q^c)c'(q^c) - u'(q^c)c''(q^c)}{[\theta c'(q^c) + (1 - \theta)u'(q^c)]^3} < 0$ and $\Delta = \frac{u''(q)c'(q) - u'(q)c''(q)}{[\theta c'(q) + (1 - \theta)u'(q)]^3} < 0$. Hence for all (z, \bar{b}) such that \bar{b} solves (16) and $z + \bar{b} < (1 - \theta)u(q^*) + \theta c(q^*)$, the objective function $\Psi(z)$ is strictly concave.

Proof of Lemma 1

The borrowing limit, \bar{b} , is determined in order to satisfy the buyer's incentive constraint to voluntarily repay his debt in the CM. As a result, a buyer will repay his debt if

$$\begin{aligned} W^b(z, -b) - \widetilde{W}^b(z) &\geq 0, \\ W^b(0, 0) - \widetilde{W}^b(0) &\geq b, \\ \bar{b} &\geq b, \end{aligned}$$

where $\bar{b} \equiv W^b(0, 0) - \widetilde{W}^b(0)$ is the endogenous debt limit determined such that buyers will always repay their debt. Therefore, the repayment constraint takes the form of an upper bound on credit use. To obtain the expression for \bar{b} , we now determine $W^b(0, 0) - \widetilde{W}^b(0)$. The value functions of a buyer who does not default in the CM can be rewritten as

$$\begin{aligned} W^b(0, 0) &= T + \max_{z \geq 0} \left\{ -\gamma z + \beta V^b(z) \right\} \\ &= T + \max_{z \geq 0} \left\{ -\gamma z + \beta \left[\sigma(1 - \Lambda)\theta S(z) + \sigma\Lambda\theta S(z + \bar{b}) + z + W^b(0, 0) \right] \right\}, \end{aligned}$$

where $S(z) \equiv u[q(z)] - c[q(z)]$ is the total trade surplus when sellers only accept money, and $S(z + \bar{b}) \equiv u[q(z + \bar{b})] - c[q(z + \bar{b})]$ is the total trade surplus when both money and credit are

accepted. Rearranging and dividing both sides of the equality by β results in

$$\frac{(1-\beta)}{\beta}W^b(0,0) = \frac{T}{\beta} + \max_{z \geq 0} \left\{ -\frac{(\gamma-\beta)}{\beta}z + \sigma\theta [(1-\Lambda)S(z) + \Lambda S(z+\bar{b})] \right\}.$$

Similarly, the value functions of a buyer who defaults in the CM can be rewritten as

$$\begin{aligned} \widetilde{W}^b(0) &= T + \max_{\tilde{z} \geq 0} \left\{ -\gamma\tilde{z} + \beta\widetilde{V}^b(\tilde{z}) \right\} \\ &= T + \max_{\tilde{z} \geq 0} \left\{ -\gamma\tilde{z} + \beta \left[\sigma\theta S(\tilde{z}) + \tilde{z} + \widetilde{W}^b(0) \right] \right\}, \end{aligned}$$

where $\tilde{z} > 0$ solves

$$i = \sigma \frac{\theta[u'(q(\tilde{z})) - c'(q(\tilde{z}))]}{\theta c'(q(\tilde{z})) + (1-\theta)u'(q(\tilde{z}))}, \quad (28)$$

since a buyer who defaults can only use money irrespective of what the seller accepts. Rearranging and dividing both sides of the equality by β results in

$$\frac{(1-\beta)}{\beta}\widetilde{W}^b(0) = \frac{T}{\beta} + \max_{\tilde{z} \geq 0} \left\{ -\frac{(\gamma-\beta)}{\beta}\tilde{z} + \sigma\theta S(\tilde{z}) \right\}.$$

Therefore, the debt limit, \bar{b} , must satisfy

$$\frac{(1-\beta)}{\beta}\bar{b} = \frac{(1-\beta)}{\beta} \left[W^b(0,0) - \widetilde{W}^b(0) \right].$$

Substituting in the expressions for $W^b(0,0)$ and $\widetilde{W}^b(0)$ then leads to the following expression for \bar{b} :

$$r\bar{b} = \max_{z \geq 0} \left\{ -iz + \sigma\theta [(1-\Lambda)S(z) + \Lambda S(z+\bar{b})] \right\} - \max_{\tilde{z} \geq 0} \left\{ -i\tilde{z} + \sigma\theta S(\tilde{z}) \right\} \equiv \Omega(\bar{b}) \quad (29)$$

where $r = \frac{1-\beta}{\beta}$.

Proof of Lemma 2

First, we show that $\Omega(0) = 0$. If $\bar{b} = 0$, it must be that $z = \tilde{z}$ since

$$-iz + \sigma\theta S(z) = -i\tilde{z} + \sigma\theta S(\tilde{z}).$$

As a result, $\Omega(0) = 0$. Next, to verify that $\Omega(\bar{b})$ is increasing in \bar{b} , differentiate $\Omega(\bar{b})$ with respect to \bar{b} to obtain

$$\begin{aligned}\frac{\partial \Omega(\bar{b})}{\partial \bar{b}} &= \sigma \theta \Lambda S'(z + \bar{b}) \\ &= \sigma \theta \Lambda \frac{u'(q^c) - c'(q^c)}{(1 - \theta)u'(q^c) + \theta c'(q^c)} > 0,\end{aligned}$$

where $q^c \equiv q(z + \bar{b})$ is output traded when the seller accepts both money and credit and $S'(z + \bar{b}) = S'(q^c) \frac{dq^c}{d\bar{b}} = \frac{u'(q^c) - c'(q^c)}{(1 - \theta)u'(q^c) + \theta c'(q^c)}$, where $z > 0$ satisfies

$$i = \sigma \theta \left[(1 - \Lambda) \frac{u(q) - c(q)}{(1 - \theta)u'(q) + \theta c'(q)} + \Lambda \frac{u(q^c) - c(q^c)}{(1 - \theta)u'(q^c) + \theta c'(q^c)} \right].$$

The slope of $\Omega(\bar{b})$ with respect to \bar{b} is strictly positive for all $\bar{b} < (1 - \theta)u(q^*) + \theta c(q^*)$ and becomes zero when $\bar{b} \geq (1 - \theta)u(q^*) + \theta c(q^*)$. Further, the slope of $\Omega(\bar{b})$ at $\bar{b} = 0$ is

$$\begin{aligned}\left. \frac{\partial \Omega(\bar{b})}{\partial \bar{b}} \right|_{\bar{b}=0} &= \sigma \theta \Lambda S'(z) \\ &= i \Lambda \geq 0.\end{aligned}$$

where we have used the fact that when $\bar{b} = 0$, $i = \sigma \theta S'(z) = \sigma \theta S'(q) \frac{dq}{dz} = \sigma \theta \frac{u'(q) - c'(q)}{(1 - \theta)u'(q) + \theta c'(q)}$. A necessary condition for a monetary equilibrium with credit is that the slope of $r\bar{b}$ from (16) is less than the slope of $\Omega(\bar{b})$ at $\bar{b} = 0$. Consequently a monetary equilibrium with credit will exist so long as $r < \Omega'(\bar{b}) = i\Lambda$.

Next, $\Omega(\bar{b})$ is concave since total surplus $S(z + \bar{b})$ as a function of the buyer's liquid wealth, $z + \bar{b}$, is concave and strictly concave if $z + \bar{b} < \theta c(q^*) + (1 - \theta)u(q^*)$. Consequently,

$$\begin{aligned}\frac{\partial^2 \Omega(\bar{b})}{\partial \bar{b}^2} &= \frac{\partial [\sigma \theta \Lambda S'(z + \bar{b})]}{\partial \bar{b}} \\ &= \sigma \theta \Lambda S''(z + \bar{b}) \leq 0,\end{aligned}$$

since $S''(z + \bar{b}) \leq 0$. Finally, $\Omega(\bar{b})$ is continuous for all $z > 0$, and the solution to (14) and (15) must lie in the interval $[0, z^*]$ for $i > 0$. Then from the Theorem of the Maximum, there exists a solution to (14) and (15), and consequently (16). \square

Proof of Proposition 2

The derivation for the necessary condition for a money and credit equilibrium, $i \in (\underline{i}, \tilde{i})$ is in the text. Here we prove the remainder of the statements in Proposition 2.

Given $\Lambda \in (0, 1)$, that a pure monetary equilibrium exists whenever there is an equilibrium with both money and credit can be verified by noting the condition for a monetary equilibrium, $i < \bar{i}$, is always satisfied if $i \in (\underline{i}, \tilde{i})$ and that there is always a solution to (16) with $\bar{b} = 0$. See also Figure 6.

To show that a pure credit equilibrium will also exist when $i \in (\underline{i}, \tilde{i})$ and $\Lambda \in (0, 1)$, recall that the necessary condition for credit given $z = \tilde{z} = 0$ is that the slope of $\Omega_0(\bar{b}) \equiv \sigma\theta\Lambda S(\bar{b})$ is greater than the slope of $r\bar{b}$, or $r < \sigma\Lambda \frac{\theta}{1-\theta}$, which is equivalent to the condition $\underline{i} < \bar{i}$ or $\Lambda > \bar{\Lambda}$. Since $\tilde{i} < \bar{i}$, if $i > \underline{i}$ we have in a money and credit equilibrium

$$\underline{i} < i < \tilde{i} < \bar{i},$$

which implies that the condition for a pure credit equilibrium, $\underline{i} < \bar{i}$, is satisfied.

We now consider the case of $\Lambda = 0$ and $\Lambda = 1$. When $\Lambda = 0$, there is no record-keeping, in which case $\bar{b} = 0$. There will be a pure monetary equilibrium if $i < \bar{i}$ or a non-monetary equilibrium without credit if $i \geq \bar{i}$. When $\Lambda = 1$, record-keeping is perfect and there can be trades with credit only or trades with money only, but never trades with both money and credit.

First, if $\Lambda = 1$ and money is valued, (14) and (15) imply that $i = \sigma\theta S'[q(z + \bar{b})] = \sigma\theta S'[q(\tilde{z})]$, or $q(z + \bar{b}) = q(\tilde{z})$. Then since $z + \bar{b} = \tilde{z} = (1 - \theta)u(q) + \theta c(q)$ from the bargaining solution, the right side of the debt limit (16) becomes $-i[z - \tilde{z}] = -i[\tilde{z} - \bar{b} - \tilde{z}] = i\bar{b}$. Consequently, (16) implies that $r\bar{b} = i\bar{b}$, or $\bar{b} = 0$ if $i \neq r$. Since buyers obtain the same surplus whether or not they default, there cannot exist a positive debt limit that supports voluntary debt repayment. If $i = r$ then any $\bar{b} \in [0, \bar{b}_0]$ is a solution.

Next, if $\Lambda = 1$ and there is a positive debt limit, money cannot be valued. We prove this by contradiction. Suppose that when $\Lambda = 1$ and $\bar{b} > 0$, money is valued. Then, $i = \sigma\theta S'[q(z + \bar{b})] = \sigma\theta S'[q(\tilde{z})]$ and $z + \bar{b} = \tilde{z}$. The right side of (16) becomes $-i[z - \tilde{z}]$, which implies $\bar{b} = 0$, a contradiction. Therefore when $\Lambda = 1$ and $\bar{b} > 0$, money cannot be valued. \square

Proof of Proposition 3

To prove Proposition 3 and Figure 10, we verify some properties of the curves $\underline{i} = \frac{r}{\Lambda}$, $\bar{i} = \frac{\sigma\theta}{1-\theta}$, and $\tilde{i} = (1 - \Lambda)\bar{i} + \sigma\theta\Lambda S[\bar{b}(\tilde{i}, \Lambda)]$.

- When $\Lambda \in [0, \bar{\Lambda})$, $\tilde{i} = \bar{i}$.

- When $\Lambda = \bar{\Lambda}$, $\underline{i} = \tilde{i} = \bar{i}$.
- When $\Lambda \in (\bar{\Lambda}, 1)$, $\underline{i} < \tilde{i} < \bar{i}$.
- When $\Lambda = 1$, $\underline{i} = \tilde{i}$ at $i = r$.
- \tilde{i} is a decreasing function of Λ :

$$\frac{d\tilde{i}}{d\Lambda} = \frac{-\bar{i} + \sigma\theta\Lambda S''[\bar{b}(\tilde{i}, \Lambda)]\frac{d\bar{b}}{d\Lambda} + \sigma\theta S'[\bar{b}(\tilde{i}, \Lambda)]}{1 - \sigma\theta\Lambda S''[\bar{b}(\tilde{i}, \Lambda)]\frac{d\bar{b}}{d\tilde{i}}} < 0,$$

since $S'(\cdot) > 0$, $S''(\cdot) < 0$, $\frac{d\bar{b}}{d\Lambda} > 0$, and $\frac{d\bar{b}}{d\tilde{i}} > 0$.

When $\Lambda < \bar{\Lambda}$ (or, equivalently $\underline{i} > \bar{i}$), we have $\tilde{i} = (1 - \Lambda)\bar{i} + \sigma\theta\Lambda S'(0) = \bar{i}$. Next, we can verify that given $\bar{b} > 0$, $\underline{i} \leq \tilde{i}$ if and only if $\Lambda > \bar{\Lambda}$, or $\underline{i} < \bar{i}$:

$$\begin{aligned}\underline{i} &\leq (1 - \Lambda)\bar{i} + \sigma\theta\Lambda S'[\bar{b}(\tilde{i}, \Lambda)] \\ \underline{i} &< (1 - \Lambda)\bar{i} + \sigma\theta\Lambda S'(0), \\ \underline{i} &< \bar{i}.\end{aligned}$$

At $\Lambda = 1$ however, we know that a money and credit equilibrium will cease to exist. Given \bar{b} , $z = 0$ if and only if $\underline{i} \geq \tilde{i}$ and $z > 0$ if and only if $\underline{i} < \tilde{i}$, where \tilde{i} solves

$$\tilde{i} = \sigma\theta S'[\bar{b}(\tilde{i}, 1)]. \quad (30)$$

Figure 8 shows that $\tilde{i} = \underline{i} = r$, in which case $\underline{i} = \tilde{i} < \bar{i}$. At $i = r$, we therefore have $\tilde{i} = \underline{i}$, meaning that the condition for a money and credit equilibrium can no longer exist. \square

Proof of Lemma 4

To show that the equilibrium debt limit is increasing in Λ when $z > 0$ and $\bar{b} \in (0, \bar{b}_0)$, we differentiate $r\bar{b} = \Omega(\bar{b})$ to obtain

$$\begin{aligned}r d\bar{b} &= \sigma\theta [-S(z)d\Lambda + S(z + \bar{b})d\Lambda + \Lambda S'(z + \bar{b})d\bar{b}], \\ \frac{d\bar{b}}{d\Lambda} \Big|_{\bar{b} < \bar{b}_0} &= \frac{\sigma\theta [S(z + \bar{b}) - S(z)]}{r - \sigma\theta\Lambda S'(z + \bar{b})} > 0,\end{aligned}$$

since $S(z + \bar{b}) > S(z)$ and $r > \sigma\theta\Lambda S'(\bar{b}_0)$ when $\bar{b} > 0$. Next, we verify that \bar{b} is a strictly convex function of Λ :

$$\left. \frac{d^2\bar{b}}{d\Lambda^2} \right|_{\bar{b} < \bar{b}_0} = \frac{(\sigma\theta)^2 [S(z + \bar{b}) - S(z)] S'(z + \bar{b})}{[r - \sigma\theta\Lambda S'(z + \bar{b})]^2} > 0$$

since $S'(z + \bar{b}) > 0$ for $\bar{b} < \bar{b}_1$.

Similarly, the equilibrium debt limit when $z = 0$ and $\bar{b} \in [\bar{b}_0, \bar{b}_1)$ is also an increasing and strictly convex function of Λ :

$$\left. \frac{d\bar{b}}{d\Lambda} \right|_{\bar{b} \in [\bar{b}_0, \bar{b}_1)} = \frac{\sigma\theta S(\bar{b})}{r - \sigma\theta\Lambda S'(\bar{b})} > 0,$$

$$\left. \frac{d^2\bar{b}}{d\Lambda^2} \right|_{\bar{b} \in [\bar{b}_0, \bar{b}_1)} = \frac{(\sigma\theta)^2 S(\bar{b}) S'(\bar{b})}{[r - \sigma\theta\Lambda S'(\bar{b})]^2} > 0,$$

since $S'(\bar{b}) > 0$.

Finally, when $\bar{b} \geq \bar{b}_1$, \bar{b} is linear in Λ : $\left. \frac{d\bar{b}}{d\Lambda} \right|_{\bar{b} \geq \bar{b}_1} > 0$ and $\left. \frac{d^2\bar{b}}{d\Lambda^2} \right|_{\bar{b} \geq \bar{b}_1} = 0$ since $S'(\bar{b}_1) = 0$. \square

Proof of Lemma 5

When $\bar{b} = 0$, the seller's expected benefit of accepting credit is zero: $\bar{\kappa} = 0$. Consequently, no sellers will invest: $\Lambda = 0$.

When $\bar{b} \in (0, \bar{b}_0)$, the expected benefit of investing is

$$\bar{\kappa} = \sigma(1 - \theta)[S(z + \bar{b}) - S(z)].$$

In this region, the expected benefit of investing is an increasing function of the debt limit, \bar{b} . To verify, differentiate to obtain

$$\left. \frac{d\bar{\kappa}}{d\bar{b}} \right|_{\bar{b} < \bar{b}_0} = \sigma(1 - \theta) \left\{ S'(z + \bar{b}) + [S'(z + \bar{b}) - S'(z)] \frac{dz}{d\bar{b}} \right\} > 0,$$

where $S'(z + \bar{b}) < S'(z)$ and

$$\frac{dz}{d\bar{b}} = - \left[1 + \frac{(1 - \Lambda)S''(z)}{\Lambda S''(z + \bar{b})} \right]^{-1} < 0$$

since $S''(z) < 0$ and $S''(z + \bar{b}) < 0$. Consequently, $\left. \frac{d\bar{\kappa}}{d\bar{b}} \right|_{\bar{b} \leq \bar{b}_0} > 0$. Let Λ_s denote the aggregate measure of sellers who invest at \bar{b}_0 .

When $\bar{b} \in [\bar{b}_0, \bar{b}_1)$, $z = 0$ and the expected benefit of investing is

$$\bar{\kappa} = \sigma(1 - \theta)S(\bar{b}),$$

which is a strictly increasing and concave function of the debt limit:

$$\left. \frac{d\bar{\kappa}}{d\bar{b}} \right|_{\bar{b} \in [\bar{b}_0, \bar{b}_1)} = \sigma(1 - \theta)S'(\bar{b}) > 0,$$

$$\left. \frac{d^2\bar{\kappa}}{d\bar{b}^2} \right|_{\bar{b} \in [\bar{b}_0, \bar{b}_1)} = \sigma(1 - \theta)S''(\bar{b}) < 0,$$

since $S''(\bar{b}) < 0$ for $\bar{b} < \bar{b}_1$.

When $\bar{b} = \bar{b}_1$, money is not valued and buyers can borrow enough to purchase the first-best. In that case, the seller's expected benefit of accepting credit is at its maximum:

$$\bar{\kappa}_{\max} \equiv \sigma(1 - \theta)S^*,$$

where $S^* \equiv S(\bar{b}_1)$. Sellers with a record-keeping cost below $\bar{\kappa}_{\max}$ will invest, and those with a record-keeping cost above $\bar{\kappa}_{\max}$ will not invest. Let $\Lambda_1 \leq 1$ denote the aggregate measure of sellers who invest when $\bar{b} = \bar{b}_1$.

Since $\frac{d\bar{\kappa}_{\max}}{d\bar{b}} = 0$, the aggregate measure of sellers who invest when $\bar{b} > \bar{b}_1$ is $\Lambda_1 \leq 1$. \square

Welfare in a Money and Credit Equilibrium

The welfare function \mathcal{W}^{mc} can be written as:

$$\mathcal{W}^{mc} = \sigma[\Lambda S(z + \bar{b}) + (1 - \Lambda)S(z)] - k.$$

Totally differentiating \mathcal{W}^{mc} with respect to i , we obtain:

$$\frac{d\mathcal{W}^{mc}}{di} = \sigma\Lambda S'(z + \bar{b}) \left[\underbrace{\frac{dz}{di}}_{(-)} + \left(\underbrace{\frac{dz}{d\bar{b}}}_{(-)} + 1 \right) \underbrace{\frac{d\bar{b}}{di}}_{(+)} \right] + \sigma(1 - \Lambda) S'(z) \underbrace{\frac{dz}{di}}_{(-)}.$$

A necessary condition for $\frac{d\mathcal{W}^{mc}}{di} > 0$ is

$$\left[\underbrace{\frac{dz}{di}}_{(-)} + \left(\underbrace{\frac{dz}{d\bar{b}}}_{(+)} + 1 \right) \underbrace{\frac{d\bar{b}}{di}}_{(+)} \right] > 0. \quad (31)$$

First, we verify that $\frac{dz}{d\bar{b}} \in (-1, 0)$. Consider the first-order condition where $z > 0$, (14). Differentiating (14), we obtain

$$\frac{dz}{d\bar{b}} = - \left[1 + \frac{(1 - \Lambda)S''(z)}{\Lambda S''(z + \bar{b})} \right]^{-1} \in (-1, 0),$$

since $S''(z) < 0$ and $S''(z + \bar{b}) < 0$ if $q \equiv q(z) < q^*$ and $q \equiv q(z + \bar{b}) < q^*$. Hence $\frac{dz}{d\bar{b}} + 1 > 0$.

Next, differentiate (14) and the analogous first-order condition for $\tilde{z} > 0$ to get

$$\frac{dz}{di} = [\sigma\theta[(1 - \Lambda)S''(z) + \Lambda S''(z + \bar{b})]]^{-1} < 0$$

and

$$\frac{d\tilde{z}}{di} = [\sigma\theta S''(\tilde{z})]^{-1} < 0.$$

Finally, we obtain $\frac{d\bar{b}}{di}$ by differentiating (16) to get

$$\frac{d\bar{b}}{di} = \frac{\tilde{z} - z}{r - \sigma\theta\Lambda S'(z + \bar{b})} > 0$$

since $\tilde{z} > z$ and $S'(z + \bar{b}) > 0$.

Consequently, $\left[\frac{dz}{di} + \left(\frac{dz}{d\bar{b}} + 1 \right) \frac{d\bar{b}}{di} \right] > 0$ or $\left(\frac{dz}{d\bar{b}} + 1 \right) \frac{d\bar{b}}{di} > -\frac{dz}{di}$ if

$$\left\{ 1 - \left[1 + \frac{(1 - \Lambda)S''(z)}{\Lambda S''(z + \bar{b})} \right]^{-1} \right\} \left[\frac{\tilde{z} - z}{r - \sigma\theta\Lambda S'(z + \bar{b})} \right] > -[\sigma\theta[(1 - \Lambda)S''(z) + \Lambda S''(z + \bar{b})]]^{-1}.$$

Welfare Comparison in a Money and Credit Equilibrium vs. Pure Credit Equilibrium

Here we show that $\mathcal{W}^{mc} > \mathcal{W}^c$ when Λ is exogenous and $k = 0$. First recall that we define γ_c as the critical value for the money growth rate, above which $\mathcal{W}^c > \mathcal{W}^m$. That is, γ_c is implicitly defined by $S[z(\gamma_c)] = \Lambda S(\bar{b})$. At γ_c , welfare in a pure credit equilibrium is given by

$$\mathcal{W}_{\gamma_c}^c = \sigma\Lambda S(\bar{b}) = \sigma S(z)$$

where the second equality follows from the fact that at γ_c , $\mathcal{W}^c = \mathcal{W}^m$. If a money and credit equilibrium exists at γ_c , then welfare is given by

$$\mathcal{W}_{\gamma_c}^{mc} = \sigma [\Lambda S(z + \bar{b}) + (1 - \Lambda)S(z)].$$

When $\frac{dz}{db} = - \left[1 + \frac{(1-\Lambda)S''(z)}{\Lambda S''(z+b)} \right]^{-1} \rightarrow 0$ or $\frac{dz}{db}$ small enough, then at γ_c , $\mathcal{W}_{\gamma_c}^{mc} > \mathcal{W}_{\gamma_c}^c$ since

$$[\Lambda S(z + \bar{b}) + (1 - \Lambda)S(z)] > S(z).$$

Appendix B

Endogenous Limit on Tax Liabilities

In this section, we determine the size of the tax obligation that individuals are willing to honor voluntarily. The idea is that the Friedman rule may be infeasible since it requires taxation and individuals may choose to renege on tax payment if it is too high.

The timing of events in the CM is as in Andolfatto (2008, 2013). First, an agent decides whether to repay his debt, b . We assume that defaulters get caught with probability one, though one can relax this assumption. If the agent repays his debt, he then decides to pay his taxes. If taxes are repaid, agents trade x, y and z in the CM as before. Defaulters in the first or second stages are punished by being excluded from future trades, and the size of the loan, b , and lump-sum transfer, T , are determined endogenously.

We solve by backwards induction and begin with the decision to honor tax obligations. In order for agents to repay the tax, their payoff from doing so must exceed the payoff from not trading in the DM:

$$W^b(z) \geq z + \beta V^b(0).$$

In that case, a buyer who reneges on his tax obligations simply consumes his real balances, z , and exits without accumulating money. This can be reduced to

$$T + \max_{z \geq 0} \{-\gamma z + \beta V^b(z)\} \geq \beta W^b(0).$$

After simplification, the tax payment constraint becomes

$$T + \max_{z \geq 0} \{-\gamma z + \beta[\sigma(1 - \Lambda)\theta[u(q) - c(q)] + \sigma\Lambda\theta[u(q^c) - c(q^c)] + z]\} \geq 0,$$

where $T = (\gamma - 1)\phi M < 0$ when the money supply is contracting. Therefore, the repayment

constraint takes the form of an upper bound on the level of taxes $-T$:

$$-T \leq \beta \max_{z \geq 0} \{-iz + \sigma(1 - \Lambda)\theta[u(q) - c(q)] + \sigma\Lambda\theta[u(q^c) - c(q^c)]\}.$$

Since the lump-sum transfer in the CM is $T = (\gamma - 1)\phi M$, the repayment constraint can equivalently be formulated as a lower bound on the deflation rate:

$$\gamma \geq -\beta(\phi M)^{-1} \max_{z \geq 0} \{-iz + \sigma(1 - \Lambda)\theta[u(q) - c(q)] + \sigma\Lambda\theta[u(q^c) - c(q^c)]\} + 1 \equiv \underline{\gamma}_d.$$

In order for tax repayment to be incentive compatible, the rate of money growth must be $\gamma \in (\underline{\gamma}_d, 1)$.

Permanent Autarky as Punishment for Default

Here we depart from our assumption that a buyer who defaults on debt repayment can still use money, and assume instead that a defaulter is punished with permanent autarky.

As before, the borrowing limit, \bar{b} , is determined in order to satisfy the buyer's incentive constraint to voluntarily repay his debt in the CM, or

$$W^b(z, -b) \geq \widehat{W}^b(z),$$

where $W^b(z, -b)$ is the value function of a buyer who chooses to repay his debt at the beginning of the CM, and $\widehat{W}^b(z)$ is the value function of a buyer who chooses to default. By the linearity of W^b , the value function of a buyer who repays his debt in the CM is

$$W^b(z, -b) = z - b + W^b(0, 0).$$

On the other hand, the value function of a buyer who defaults, $\widehat{W}^b(z)$ must satisfy

$$\widehat{W}^b(z) = z + \widehat{W}^b(0),$$

since by defaulting, he is excluded to permanent autarky and can no longer trade in the DM. As a result, a buyer will repay his debt if

$$\begin{aligned} W^b(z, -b) - \widehat{W}^b(z) &\geq 0, \\ W^b(0, 0) - \widehat{W}^b(0) &\geq b, \\ \bar{b} &\geq b, \end{aligned}$$

where $\bar{b} \equiv W^b(0,0) - \widehat{W}^b(0)$ is the endogenous debt limit determined such that buyers will always repay their debt. To solve for the equilibrium value of \bar{b} , we determine $W^b(0,0) - \widehat{W}^b(0)$. The value functions of a buyer who does not default in the CM can be rewritten as

$$\begin{aligned} W^b(0,0) &= T + \max_{z \geq 0} \left\{ -\gamma z + \beta V^b(z) \right\} \\ &= T + \max_{z \geq 0} \left\{ -\gamma z + \beta \left[\sigma(1-\Lambda)\theta S(z) + \sigma\Lambda\theta S(z+\bar{b}) + z + W^b(0,0) \right] \right\}, \end{aligned}$$

where $S(z) \equiv u[q(z)] - c[q(z)]$ is the total trade surplus when sellers only accept money, and $S(z+\bar{b}) \equiv u[q(z+\bar{b})] - c[q(z+\bar{b})]$ is the total trade surplus when both money and credit are accepted. Rearranging and dividing both sides of the equality by β results in

$$\frac{(1-\beta)}{\beta} W^b(0,0) = \frac{T}{\beta} + \max_{z \geq 0} \left\{ -\frac{(\gamma-\beta)}{\beta} z + \sigma\theta [(1-\Lambda)S(z) + \Lambda S(z+\bar{b})] \right\}.$$

However, the value function of a buyer who defaults in the CM is

$$\begin{aligned} \widehat{W}^b(0) &= T + \beta \widehat{V}^b(0) \\ &= T + \beta \widehat{W}^b(0), \end{aligned}$$

since by defaulting, a buyer can no longer use money nor credit. Rearranging and dividing both sides of the equality by β results in

$$\frac{(1-\beta)}{\beta} \widehat{W}^b(0) = \frac{T}{\beta}.$$

Therefore, the debt limit, \bar{b} , must satisfy

$$\frac{(1-\beta)}{\beta} \bar{b} = \frac{(1-\beta)}{\beta} \left[W^b(0,0) - \widehat{W}^b(0) \right].$$

Substituting in the expressions for $W^b(0,0)$ and $\widehat{W}^b(0)$ then leads to the following expression for \bar{b} :

$$r\bar{b} = \max_{z \geq 0} \left\{ -iz + \sigma\theta [(1-\Lambda)S(z) + \Lambda S(z+\bar{b})] \right\} \equiv \widehat{\Omega}(\bar{b}), \quad (32)$$

where $r = \frac{1-\beta}{\beta}$. In contrast with our baseline assumption that a defaulter can still use money, the cost of default under permanent autarky, $\widehat{\Omega}(\bar{b})$ is larger than the cost of default assuming monetary trades are still allowed, $\Omega(\bar{b})$.